CHAPTER 1 The Six Trigonometric Functions

Problem Set 1.1

1. 10° is an acute angle.
The complement of 10° is 80° because $10^\circ + 80^\circ = 90^\circ$.
The supplement of 10° is 170° because $10^\circ + 170^\circ = 180^\circ$.

3. 45° is an acute angle.
The complement of 45° is 45° because $45^\circ + 45^\circ = 90^\circ$.
The supplement of 45° is 135° because $45^\circ + 135^\circ = 180^\circ$.

5. 120° is an obtuse angle.
The complement of 120° is -30° because $120^\circ + (-30)^\circ = 90^\circ$.
The supplement of 120° is 60° because $120^\circ + 60^\circ = 180^\circ$.

7. We can't tell if x° is acute or obtuse (or neither).
The complement of x° is $90^\circ - x$ because $x^\circ + (90^\circ - x^\circ) = 90^\circ$.
The supplement of x° is $180^\circ - x^\circ$ because $x^\circ + (180^\circ - x^\circ) = 180^\circ$.

9.

$\alpha = 180^\circ - (\angle A + \angle D)$	The sum of the angles of a triangle is 180°
$= 180^\circ - (30^\circ + 90^\circ)$	Substitute given values
$= 180^\circ - 120^\circ$	Simplify
$= 60^\circ$	

11.

$\alpha = 180^\circ - (\angle A + \angle D)$	The sum of the angles of a triangle is 180°
$= 180^\circ - (\alpha + 90^\circ)$	Substitute the given values
$= 90^\circ - \alpha$	Simplify right side
$2\alpha = 90^\circ$	Add α to both sides
$\alpha = 45^\circ$	Divide both sides by 2

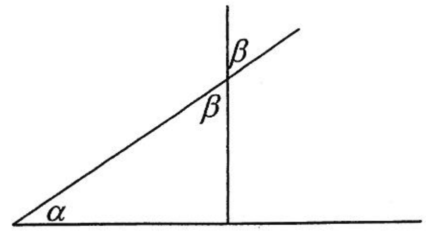
13.

$\angle A = 180^\circ - (\alpha + \beta + \angle B)$	The sum of the angles of a triangle is 180°
$= 180^\circ - (100^\circ + 30^\circ)$	Substitute given values
$= 180^\circ - 130^\circ$	Simplify
$= 50^\circ$	

15. Angles α and β are complementary because

$\alpha+\beta+90^\circ=180^\circ$

$\alpha+\beta=90^\circ$



17.

$\alpha+\beta=90^\circ$	α and β are complementary
$\beta=90^\circ-\alpha$	Subtract α from both sides
$=90^\circ-25^\circ$	Substitute given value
$=65^\circ$	Simplify

19. One complete revolution equals 360°.
Therefore, it rotates 360° in 2 seconds and 180° in 1 second.

21. One complete revolution equals 360°.
In 4 hours, the hour hand revolves $\frac{4}{12}$ or $\frac{1}{3}$ of a revolution.

$\frac{1}{3}$ of $360^\circ=120^\circ$.

23. Let α = the degree measure of each angle

Then $\alpha+\alpha+\alpha=180^\circ$

$3\alpha=180^\circ$

$\alpha=60^\circ$

Therefore, each angle of an equilateral triangle is 60°

25.

$c^2=a^2+b^2$	Pythagorean Theorem
$=4^2+3^2$	Substitute given values
$=16+9$	Simplify
$=25$	

Therefore, $c=\pm5$. Our only solution is $c=5$, because we cannot use $c=-5$.

27.

$a^2+b^2=c^2$	Pythagorean Theorem
$b^2=c^2-a^2$	Subtract a^2 from both sides
$=17^2-8^2$	Substitute given values
$=289-64$	Simplify
$=225$	

Therefore, $b=\pm15$. Our only solution is b=15, because we cannot use $b=-15$.

29. $a^2 + b^2 = c^2$ Pythagorean Theorem

$a^2 = c^2 - b^2$ Subtract b^2 from both sides

$= 13^2 - 12^2$ Substitute given values

$= 169 - 144$ Simplify

$= 25$

Therefore, $a = \pm 5$. Our only solution is $a = 5$, because we cannot use $a = -5$.

31. $x^2 = 3^2 + 3^2$ Pythagorean Theorem

$= 9 + 9$ Simplify

$= 18$

Therefore, $x = \pm\sqrt{18} = \pm 3\sqrt{2}$. Our only solution is $x = 3\sqrt{2}$ because we cannot use $x = -3\sqrt{2}$.

Note: This must be a $45^\circ - 45^\circ - 90^\circ$ *triangle.*

33. $x^2 = (2)^2 + \left(2\sqrt{3}\right)^2$ Pythagorean Theorem

$= 4 + 12$ Simplify

$= 16$

Therefore $x = \pm 4$. Our only solution is $x = 4$, because we cannot use $x = -4$.

35. $\left(\sqrt{10}\right)^2 = x^2 + (x+2)^2$ Pythagorean Theorem

$10 = x^2 + x^2 + 4x + 4$ Simplify

$10 = 2x^2 + 4x + 4$ Combine like terms

$0 = 2x^2 + 4x - 6$ Subtract 10 from both sides

$0 = x^2 + 2x - 3$ Divide both sides by 2

$0 = (x+3)(x-1)$ Factor

$x + 3 = 0$ or $x - 1 = 0$ Set each factor equal to zero

$x = -3 \qquad x = 1$

Therefore, $x = 1$ because $x = -3$ is not possible.

37. $(BD)^2 = (CD)^2 + (BC)^2$ Pythagorean Theorem

$5^2 = (CD)^2 + (4)^2$ Substitute given values

$25 = (CD)^2 + 16$ Simplify

$9 = (CD)^2$ Subtract 16 from both sides

$CD = 3$ or $CD = -3$ Take square root of both sides

$CD = 3$ Eliminate negative solution

Therefore, $AC = 2 + 3 = 5$ $AC = AD + DC$

This problem is continued on the next page.

$(AB)^2 = (AC)^2 + (BC)^2$	Pythagorean Theorem
$= 5^2 + 4^2$	Substitute given values
$= 25 + 16$	Simplify
$= \sqrt{41}$	
$AB = \sqrt{41}$ or $AB = -\sqrt{41}$	Take square root of both sides
$AB = \sqrt{41}$	Eliminate negative solution

39.

$(AB + BC)^2 = (CD)^2 + (AD)^2$	Pythagorean Theorem
$(4 + r)^2 = r^2 + 8^2$	Substitute given values
$16 + 8r + r^2 = r^2 + 64$	Simplify
$16 + 8r = 64$	Subtract r^2 from both sides
$8r = 48$	Subtract 16 from both sides
$r = 6$	Divide both sides by 8

41.

This is an isosceles triangle.	Therefore, the altitude must bisect the base
$x^2 = (18)^2 + (13.5)^2$	Pythagorean Theorem
$= 324 + 182.5$	Simplify
$= 506.25$	
$x = 22.5$ or $x = -22.5$	Take square root of both sides
$x = 22.5$ ft	Eliminate negative solution

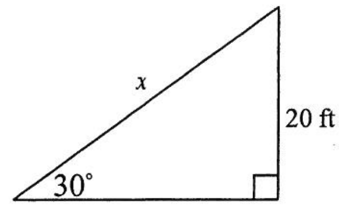
43. The shortest side is 1.
The longest side is twice the shortest side. Therefore, it is 2.
The side opposite the 60° angle is $1\sqrt{3}$ or $\sqrt{3}$.

45. The longest side is 8 which is twice the shortest side.
Therefore, the shortest side is 4.
The side opposite the 60° angle is $4\sqrt{3}$.

47. Let t = the shortest side, $2t$ = the longest side, and $t\sqrt{3}$ = the side opposite 60°

Therefore, $t\sqrt{3} = 6$	Side opposite 60° is 6
$t = \frac{6}{\sqrt{3}}$	Divide both sides by $\sqrt{3}$
$= \frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$	Rationalize the denominator
$= \frac{6\sqrt{3}}{3} = 2\sqrt{3}$	
Since $t = 2\sqrt{3}$, $2t = 2(2\sqrt{3}) = 4\sqrt{3}$.	The shortest side is $2\sqrt{3}$ and the longest side is $4\sqrt{3}$.

49. The shortest side is 20 feet.
The longest side is twice the shortest side
Therefore, $x = 2(20)$
$x = 40$ feet



51. The tent is made up of 3 congruent rectangles and 2 congruent triangles.
First we'll find the sides of the $30^\circ - 60^\circ - 90^\circ$ triangle.
The side opposite 60° is 4 ft. Let t = the shortest side.

$$t\sqrt{3} = 4$$
$$t = \frac{4}{\sqrt{3}}$$
$$t = \frac{4}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$$

The shortest side is $\frac{4\sqrt{3}}{3}$. The hypotenuse is $2\left(\frac{4\sqrt{3}}{3}\right) = \frac{8\sqrt{3}}{3}$.

Also the base of the triangular sides are $2\left(\frac{4\sqrt{3}}{3}\right) = \frac{8\sqrt{3}}{3}$.

Note: This is an equilateral triangle.

Area of rectangles = length · width

$$\text{Area of rectangles } = 6\left(\frac{8\sqrt{3}}{3}\right)$$
$$= 16\sqrt{3} \text{ ft}^2$$

$$\text{Area of triangles } = \frac{1}{2} \cdot \text{base} \cdot \text{height}$$
$$= \frac{1}{2}\left(\frac{8\sqrt{3}}{3}\right)(4)$$
$$= \frac{16\sqrt{3}}{3} \text{ ft}^2$$

Area of tent = 3 rectangles + 2 triangles

$$= 3\left(16\sqrt{3}\right) + 2\left(\frac{16\sqrt{3}}{3}\right)$$
$$= 48\sqrt{3} + \frac{32\sqrt{3}}{3}$$
$$\approx 101.6 \text{ ft}^2$$

53. $\text{hypotenuse} = \frac{4}{5} \bullet \sqrt{2}$ — Hypotenuse is $t \bullet \sqrt{2}$

$= \frac{4\sqrt{2}}{5}$ — Simplify

55. $\text{hypotenuse} = t\sqrt{2}$ — t is the shorter side

$8\sqrt{2} = t\sqrt{2}$ — Substitute given value

$8 = t$ — Divide both sides by $\sqrt{2}$

57. $\text{hypotenuse} = t\sqrt{2}$ — t is the shorter side

$4 = t\sqrt{2}$ — Substitute given value

$\frac{4}{\sqrt{2}} = t$ — Divide both sides by $\sqrt{2}$

$t = 2\sqrt{2}$ — Rationalize denominator by multiplying numerator and denominator by $\sqrt{2}$

59. We are looking for the hypotenuse of a $45^\circ - 45^\circ - 90^\circ$ triangle where the shorter sides are 1000 feet.

$\text{hypotenuse} = 1000\sqrt{2}$ — Hypotenuse is $t\sqrt{2}$

$\approx 1414 \text{ ft}$ — Rounded to the nearest foot

The bullet travels 1414 feet.

61. (a) $\text{hypotenuse} = t\sqrt{2}$ — t is the edge of the cube

$= 1\sqrt{2}$ — Substitute given value

$= \sqrt{2}$ — Simplify

Therefore, $CH = \sqrt{2}$ inches

(b) $(CF)^2 = (CH)^2 + (FH)^2$ — Pythagorean Theorem

$= \left(\sqrt{2}\right)^2 + (1)^2$ — Substitute given values

$= 2 + 1$ — Simplify

$= 3$

$CF = \pm\sqrt{3}$ — Take easy square root of both sides

$CF = \sqrt{3}$ inches — Eliminate the negative solution