

# 10 B 2 : Normal Distributions (Bell curves)

Name Answer key

Algebra 2E

~~10/10~~ Normal Distributions



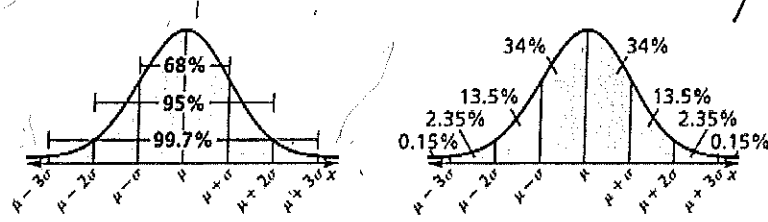
**Normal Distribution:** a bell-shaped curve called a normal curve that is symmetric about the mean.

Areas under a Normal Curve:

"mu"  $\mu$       "sigma"  $\sigma$

A normal distribution with mean  $\mu$  and standard deviation  $\sigma$  has these properties.

1. The total area under the related normal curve is 1.
2. About 68% of the area lies within 1 standard deviation of the mean.  $\mu \pm \sigma$
3. About 95% of the area lies within 2 standard deviations of the mean.  $\mu \pm 2\sigma$
4. About 99.7% of the area lies within 3 standard deviations of the mean.  $\mu \pm 3\sigma$



The second graph shows that 34% of the area lies within 1 standard deviation to the left of the mean, and 34% of the area lies within 1 standard deviation to the right of the mean. It also shows the other partial areas based on the properties of a normal curve.

The areas under a normal curve can be interpreted as probabilities in a normal distribution. So, in a normal distribution, the probability that a randomly chosen  $x$ -value is between  $a$  and  $b$  is given by the area under the normal curve between  $a$  and  $b$ .

Example 1: A normal distribution has a mean  $\mu$  and a standard deviation  $\sigma$ . An  $x$ -value is randomly selected from the distribution.

a. Find  $P(\mu - 2\sigma \leq x \leq \mu)$

47.5%

b. Find  $P(x \geq \mu + \sigma)$

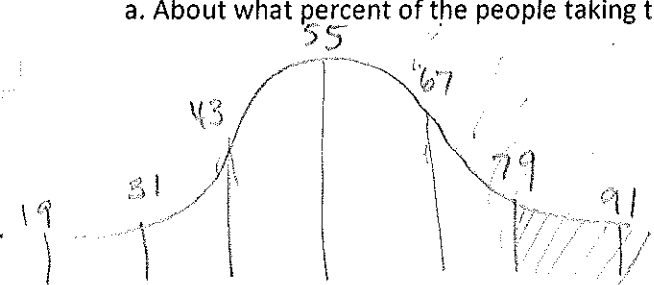
16%

Example 2: The scores for a state's peace officer standards and training test are normally distributed with a mean of 55 and a standard deviation of 12. The test scores range from 0 to 100.

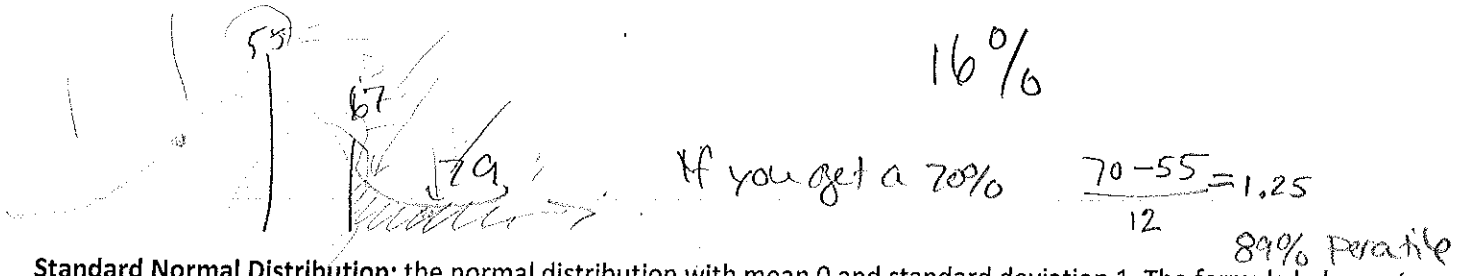
a. About what percent of the people taking the test have scores between 43 and 67?

68%

What % of testers scored a 79 or higher? 2.5%



b. An agency in the state will only hire applicants with test scores of 67% or greater. About what percent of the people have test scores that make them eligible to be hired by the agency?



**Standard Normal Distribution:** the normal distribution with mean 0 and standard deviation 1. The formula below can be used to transform x-values from a normal distribution with mean  $\mu$  and standard deviation  $\sigma$  into z-values having standard normal distribution.

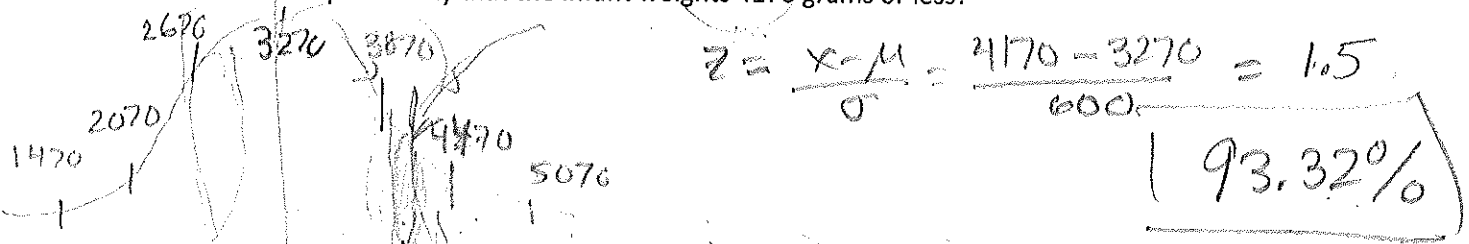
**Formula:**  $z = \frac{x - \mu}{\sigma}$  Subtract the mean from the given x-value, and then divide by the standard deviation.

The z-value for a particular x-value is called the **z-score** for the x-value and is the number of standard deviations the x-value lies above or below the mean  $\mu$ .

Standard Normal Table										
z	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
-3	.0013	.0010	.0007	.0005	.0003	.0002	.0002	.0001	.0001	.0000
-2	.0228	.0179	.0139	.0107	.0082	.0062	.0047	.0035	.0026	.0019
-1	.1587	.1357	.1151	.0968	.0808	.0668	.0548	.0446	.0359	.0287
-0	.5000	.4602	.4207	.3821	.3446	.3085	.2743	.2420	.2119	.1841
0	.5000	.5398	.5793	.6179	.6554	.6915	.7257	.7580	.7881	.8159
1	.8413	.8643	.8849	.9032	.9192	.9332	.9452	.9554	.9641	.9713
2	.9772	.9821	.9861	.9893	.9918	.9938	.9953	.9965	.9974	.9981
3	.9987	.9990	.9993	.9995	.9997	.9998	.9998	.9999	.9999	1.0000

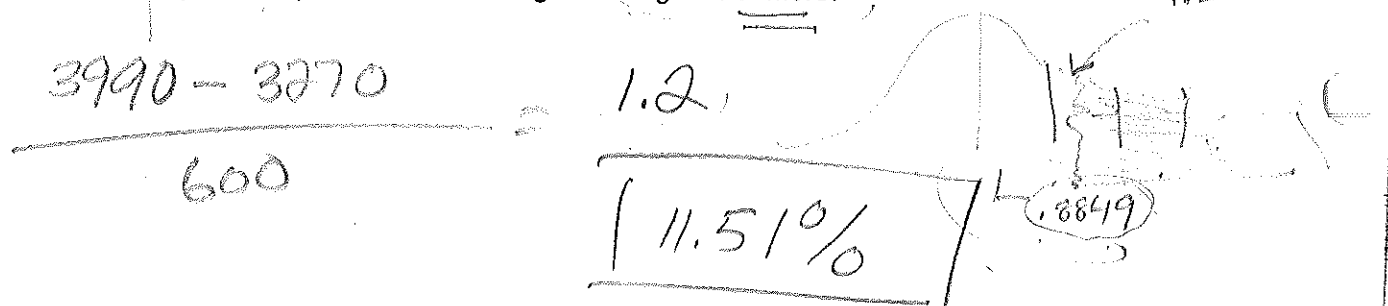
**Example 3:** A study finds that the weights of infants at birth are normally distributed with a mean of 3270 grams and a standard deviation of 600 grams. An infant is randomly chosen.

a. What is the probability that the infant weights 4170 grams or less?



$$z = \frac{x - \mu}{\sigma} = \frac{4170 - 3270}{600} = 1.5$$

b. What is the probability that the infant weights 3990 grams or more?



$$z = \frac{3990 - 3270}{600} = 1.2$$

Not all distributions are normal. In fact, they can be skewed left or skewed right.

Symmetric:

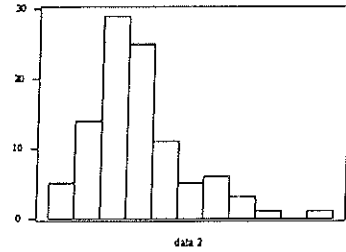
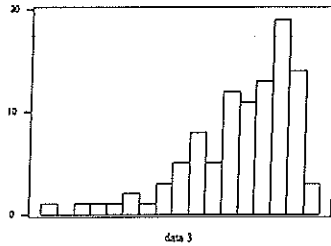
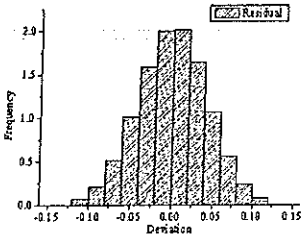
- Histogram has normal distribution
- Mean = median

Skewed Left:

- Not normally distributed
- mean < median

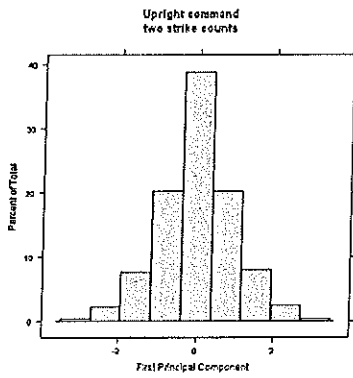
Skewed Right:

- Not normally distributed
- mean > median



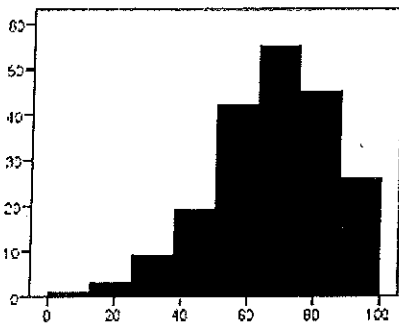
Example 4: Determine whether each histogram has normal distribution.

a.

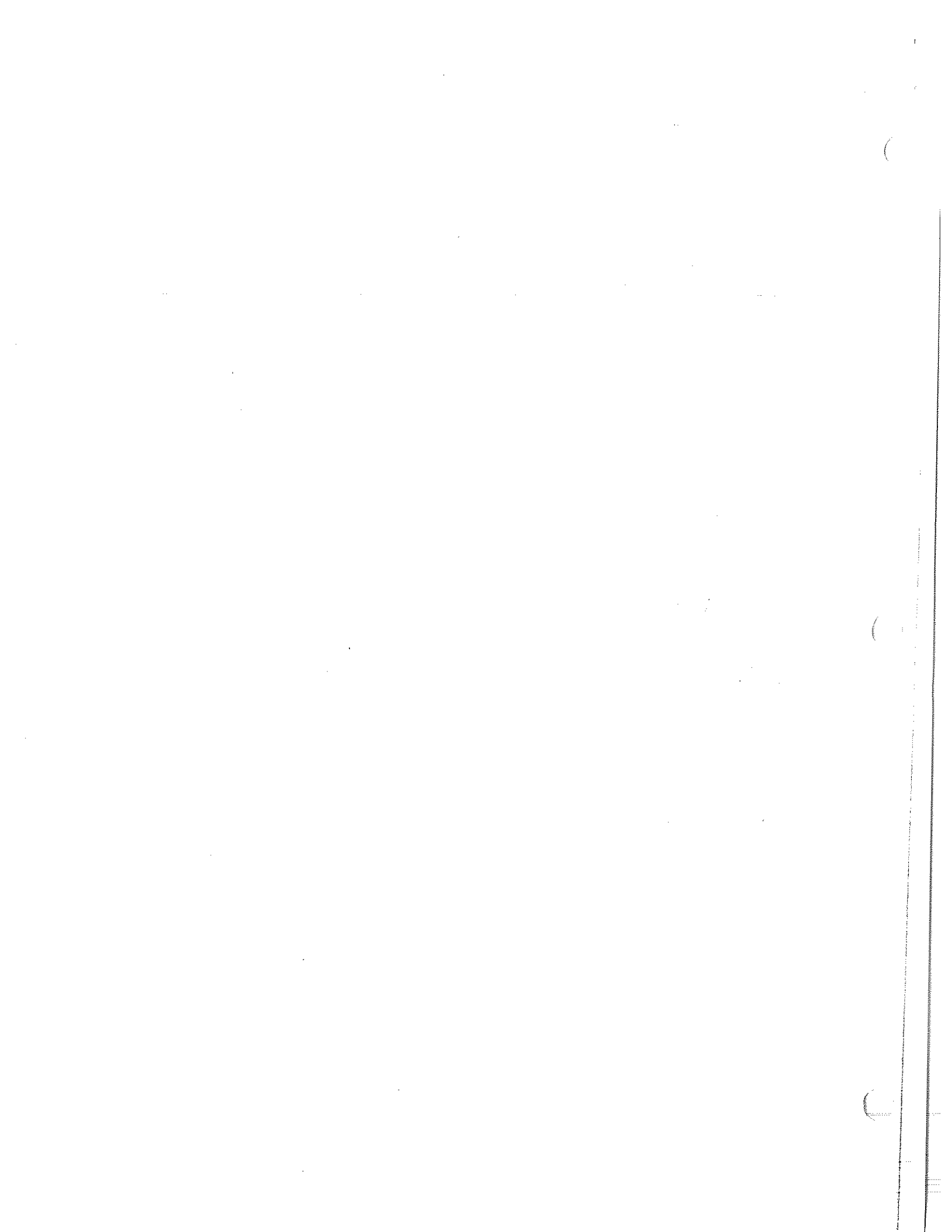


normal, symmetric

b.



no, skewed left



Chapter 7.5

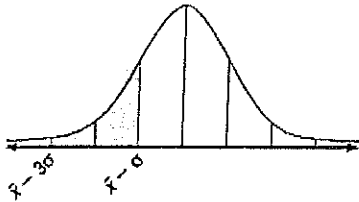
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Algebra 2E

### 10-3 Normal Distribution Homework

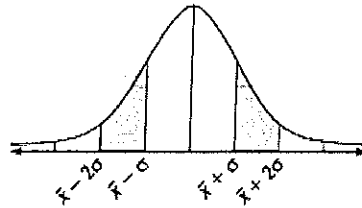
1. For each of the following, give the percent of the area under the normal curve represented by the shaded region(s).

a.



15.85%

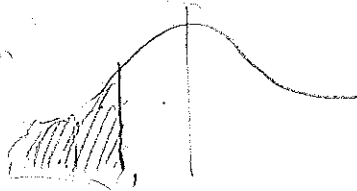
b.



27%

2. A normal distribution has mean  $\mu$  and standard deviation  $\sigma$ . Find the indicated probability for a randomly selected x-value from the distribution.

a.  $P(x \leq \mu - \sigma)$



16%

b.  $P(x \geq \mu - \sigma)$



84%

c.  $P(x \geq \mu + 2\sigma)$

2.5%

d.  $P(\mu - 3\sigma \leq x \leq \mu)$

49.85%

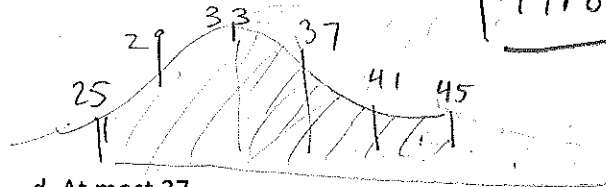
3. A normal distribution has a mean of 33 and a standard deviation of 4. Find the probability that a randomly selected x-value from the distribution is in the given interval.

a. Between 29 and 37

68%

b. Between 33 and 45

49.85%



c. At least 25

97.5%

d. At most 37

84%

4. The wing lengths of houseflies are normally distributed with a mean of 4.6 millimeters and a standard deviation of 0.4 millimeters.

a. About what percent of houseflies have wing lengths between 3.8 millimeters and 5.0 millimeters?

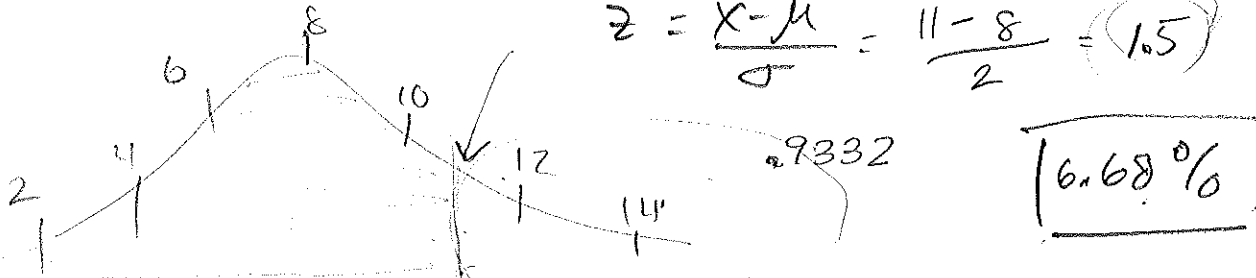
81.5%

b. About what percent of houseflies have wing lengths longer than 5.8 millimeters?

0.15

5. A busy time to visit a bank is during its Friday evening rush hours. For these hours, the waiting times at the drive-through window are normally distributed with a mean of 8 minutes and a standard deviation of 2 minutes. You have no more than 11 minutes to do your banking and still make it to your meeting on time. What is the probability you will be late for the meeting?

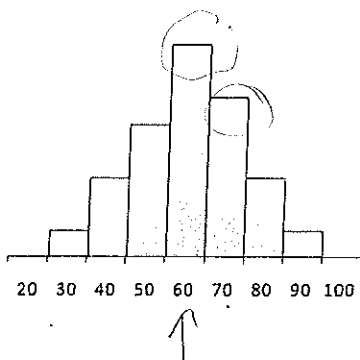
$$z = \frac{x - \mu}{\sigma} = \frac{11 - 8}{2} = 1.5$$



6. A data set has a median of 80 and a mean of 90. Your friend claims that the distribution of the data is skewed left. Is your friend correct? Explain your reasoning.

NO! It's skewed right

7. Determine whether the histogram has a normal distribution. Explain.



yes.