

Graphing and Properties of Hyperbolas

Identify the vertices, foci, and direction of opening of each.

1) $\frac{x^2}{81} - \frac{y^2}{4} = 1$

Vertices: $(9, 0), (-9, 0)$
 Foci: $(\sqrt{85}, 0), (-\sqrt{85}, 0)$
 Opens left/right

2) $\frac{x^2}{121} - \frac{y^2}{81} = 1$

Vertices: $(11, 0), (-11, 0)$
 Foci: $(\sqrt{202}, 0), (-\sqrt{202}, 0)$
 Opens left/right

3) $\frac{y^2}{25} - \frac{x^2}{16} = 1$

Vertices: $(0, 5), (0, -5)$
 Foci: $(0, \sqrt{41}), (0, -\sqrt{41})$
 Opens up/down

4) $\frac{x^2}{121} - \frac{y^2}{36} = 1$

Vertices: $(11, 0), (-11, 0)$
 Foci: $(\sqrt{157}, 0), (-\sqrt{157}, 0)$
 Opens left/right

5) $\frac{(x+2)^2}{169} - \frac{(y+8)^2}{4} = 1$

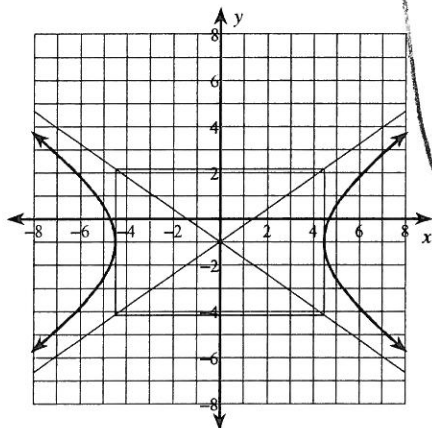
Vertices: $(11, -8), (-15, -8)$
 Foci: $(-2 + \sqrt{173}, -8), (-2 - \sqrt{173}, -8)$
 Opens left/right

6) $\frac{(y+8)^2}{36} - \frac{(x+2)^2}{25} = 1$

Vertices: $(-2, -2), (-2, -14)$
 Foci: $(-2, -8 + \sqrt{61}), (-2, -8 - \sqrt{61})$
 Opens up/down

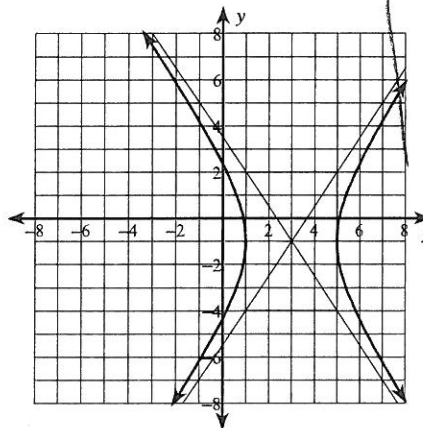
Identify the vertices and foci of each. Then sketch the graph.

$$7) \frac{x^2}{20} - \frac{(y+1)^2}{10} = 1$$



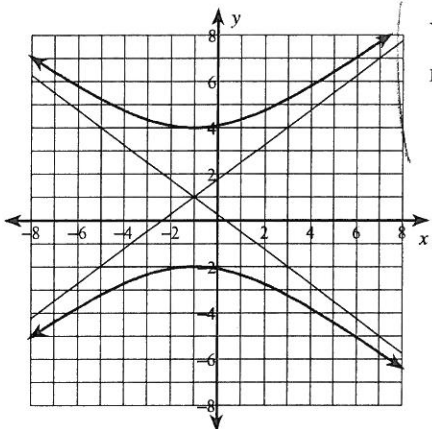
Vertices: $(2\sqrt{5}, -1)$
 $(-2\sqrt{5}, -1)$
 Foci: $(\sqrt{30}, -1)$
 $(-\sqrt{30}, -1)$

$$8) \frac{(x-3)^2}{4} - \frac{(y+1)^2}{9} = 1$$



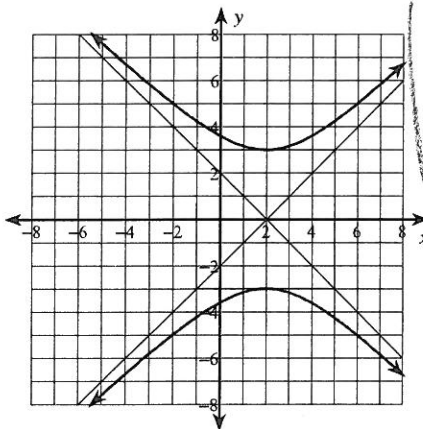
Vertices: $(5, -1)$
 $(1, -1)$
 Foci: $(3 + \sqrt{13}, -1)$
 $(3 - \sqrt{13}, -1)$

$$9) \frac{(y-1)^2}{9} - \frac{(x+1)^2}{16} = 1$$



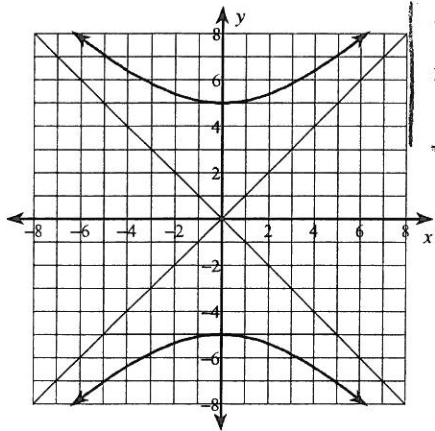
Vertices: $(-1, 4)$
 $(-1, -2)$
 Foci: $(-1, 6)$
 $(-1, -4)$

$$10) \frac{y^2}{9} - \frac{(x-2)^2}{9} = 1$$



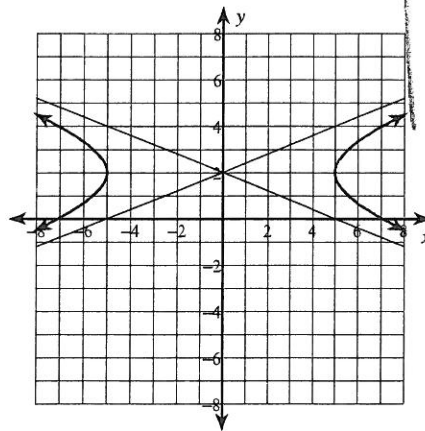
Vertices: $(2, 3)$
 $(2, -3)$
 Foci: $(2, 3\sqrt{2})$
 $(2, -3\sqrt{2})$

$$11) \frac{y^2}{25} - \frac{x^2}{25} = 1$$



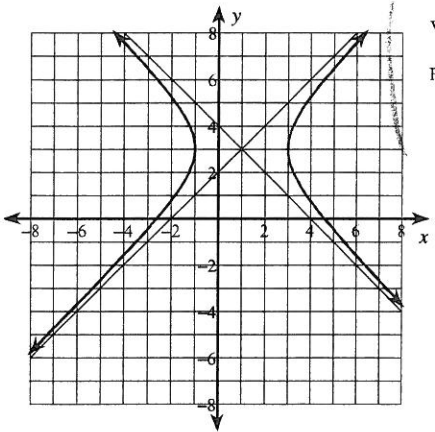
Vertices: (0, 5)
(0, -5)
Foci: (0, $5\sqrt{2}$)
(0, $-5\sqrt{2}$)

$$12) \frac{x^2}{25} - \frac{(y-2)^2}{4} = 1$$



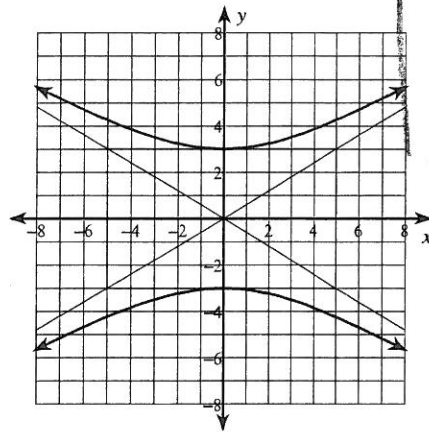
Vertices: (5, 2)
(-5, 2)
Foci: ($\sqrt{29}$, 2)
($-\sqrt{29}$, 2)

$$13) \frac{(x-1)^2}{4} - \frac{(y-3)^2}{4} = 1$$



Vertices: (3, 3)
(-1, 3)
Foci: ($1 + 2\sqrt{2}$, 3)
($1 - 2\sqrt{2}$, 3)

$$14) \frac{y^2}{9} - \frac{x^2}{25} = 1$$



Vertices: (0, 3)
(0, -3)
Foci: (0, $\sqrt{34}$)
(0, $-\sqrt{34}$)

Identify the asymptotes, length of the transverse axis, length of the conjugate axis, length of the latus rectum, and eccentricity of each.

$$15) -10y - y^2 = -4x^2 - 72x - 199$$

$$\text{Asym.: } y = 2x + 13$$

$$y = -2x - 23$$

Transverse Axis: 10 units

Conjugate Axis: 20 units

Latus Rectum: 40 units

Eccentricity: $\sqrt{5} \approx 2.236$

$$16) -y^2 + 12y - 19 = 18x - x^2$$

$$\text{Asym.: } y = x - 3$$

$$y = -x + 15$$

Transverse Axis: 16 units

Conjugate Axis: 16 units

Latus Rectum: 16 units

Eccentricity: $\sqrt{2} \approx 1.414$

Writing Equations of Hyperbolas

Use the information provided to write the standard form equation of each hyperbola.

1) $-x^2 + y^2 - 18x - 14y - 132 = 0$

$$\frac{(y-7)^2}{100} - \frac{(x+9)^2}{100} = 1$$

2) $9x^2 - 4y^2 - 90x + 32y - 163 = 0$

$$\frac{(x-5)^2}{36} - \frac{(y-4)^2}{81} = 1$$

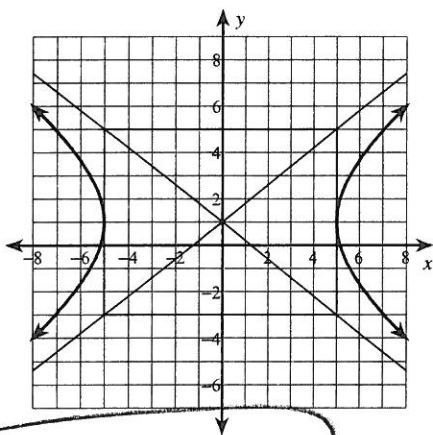
3) $-16x^2 + 9y^2 + 32x + 144y - 16 = 0$

$$\frac{(y+8)^2}{64} - \frac{(x-1)^2}{36} = 1$$

4) $-2x^2 + 3y^2 + 4x - 60y + 268 = 0$

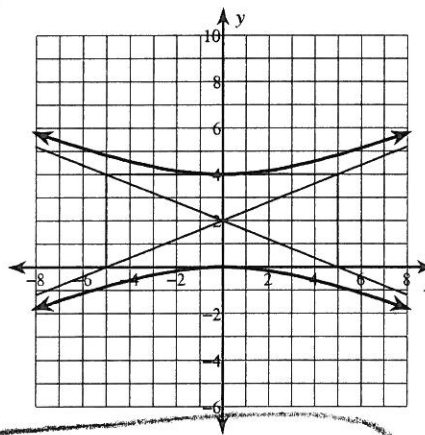
$$\frac{(y-10)^2}{10} - \frac{(x-1)^2}{15} = 1$$

5)



$$\frac{x^2}{25} - \frac{(y-1)^2}{16} = 1$$

6)



$$\frac{(y-2)^2}{4} - \frac{x^2}{25} = 1$$

7) Vertices: (8, 14), (8, -10)

Conjugate Axis is 6 units long

$$\frac{(y-2)^2}{144} - \frac{(x-8)^2}{9} = 1$$

8) Vertices: $(4, 9 + \sqrt{30})$, $(4, 9 - \sqrt{30})$

Conjugate Axis is $2\sqrt{195}$ units long

$$\frac{(y-9)^2}{30} - \frac{(x-4)^2}{195} = 1$$

9) Vertices: (15, 1), (-1, 1)

Endpoints of Conjugate Axis: (7, 7)
(7, -5)

$$\frac{(x-7)^2}{64} - \frac{(y-1)^2}{36} = 1$$

10) Vertices: (0, -7), (-12, -7)

Endpoints of Conjugate Axis: (-6, 0)
(-6, -14)

$$\frac{(x+6)^2}{36} - \frac{(y+7)^2}{49} = 1$$

- 11) Vertices: $(3, 0), (3, -18)$
 Foci: $(3, -9 + \sqrt{145}), (3, -9 - \sqrt{145})$

$$\frac{(y+9)^2}{81} - \frac{(x-3)^2}{64} = 1$$

- 13) Vertices: $(7, 4), (7, -24)$
 Distance from Center to Focus = $7\sqrt{5}$

$$\frac{(y+10)^2}{196} - \frac{(x-7)^2}{49} = 1$$

- 15) Foci: $(2, -6 + \sqrt{157}), (2, -6 - \sqrt{157})$

$$\text{Asymptotes: } y = \frac{6}{11}x - \frac{78}{11}$$

$$y = -\frac{6}{11}x - \frac{54}{11}$$

$$\frac{(y+6)^2}{36} - \frac{(x-2)^2}{121} = 1$$

- 17) Vertices: $(0, -1), (-20, -1)$

$$\text{Asymptotes: } y = x + 9$$

$$y = -x - 11$$

$$\frac{(x+10)^2}{100} - \frac{(y+1)^2}{100} = 1$$

- 19) Foci: $(-9, -5 + 9\sqrt{2}), (-9, -5 - 9\sqrt{2})$
 Conjugate Axis is 18 units long

$$\frac{(y+5)^2}{81} - \frac{(x+9)^2}{81} = 1$$

- 21) Foci: $(-10, 10 + 4\sqrt{10}), (-10, 10 - 4\sqrt{10})$
 Points on the hyperbola are 24 units closer to one focus than the other

$$\frac{(y-10)^2}{144} - \frac{(x+10)^2}{16} = 1$$

- 12) Vertices: $(-5, 1), (-5, -7)$
 Foci: $(-5, -3 + \sqrt{97}), (-5, -3 - \sqrt{97})$

$$\frac{(y+3)^2}{16} - \frac{(x+5)^2}{81} = 1$$

- 14) Vertices: $(-5, 22), (-5, -4)$
 Distance from Center to Focus = $\sqrt{218}$

$$\frac{(y-9)^2}{169} - \frac{(x+5)^2}{49} = 1$$

- 16) Foci: $(6\sqrt{5}, 10), (-6\sqrt{5}, 10)$

$$\text{Asymptotes: } y = \frac{1}{2}x + 10$$

$$y = -\frac{1}{2}x + 10$$

$$\frac{x^2}{144} - \frac{(y-10)^2}{36} = 1$$

- 18) Vertices: $(13, 0), (-1, 0)$

$$\text{Asymptotes: } y = x - 6$$

$$y = -x + 6$$

$$\frac{(x-6)^2}{49} - \frac{y^2}{49} = 1$$

- 20) Foci: $(8, -5 + \sqrt{53}), (8, -5 - \sqrt{53})$
 Endpoints of Conjugate Axis: $(15, -5)$
 $(1, -5)$

$$\frac{(y+5)^2}{4} - \frac{(x-8)^2}{49} = 1$$

- 22) Center at $(-1, -1)$
 Transverse axis is vertical and 24 units long
 Conjugate axis is 8 units long

$$\frac{(y+1)^2}{144} - \frac{(x+1)^2}{16} = 1$$