Name: ______________________________ Per: ________ Date: ______________
Serafino ▪ Precalculus S2

11A Parametric Equations (V2: New & Improved)

Notes & HW Packet (but correct 4, 5, 6 & 10 on your copy)

We've done a lot of graphing. Linear, quadratic, trig functions, conic sections, etc. The whole time, you've been graphing in two variables (x, y). The two–variable system is called the Cartesian or Rectangular Coordinate System. The coordinate plane is sometimes called the Cartesian Plane or the Rectangular Coordinate Plane.

$y = 3x + 5$ $\qquad$ $y = 5 \sin(2x) - 1$ $\qquad$ $(x + 3)^2 + (y - 4)^2 = 20$


Haha. I have a plane and you don't.

All these equations show the relationship between x and y, and in functions, one variable (y) depends on the other (x). But that's not always how simply the world works. Very often, x and y are related variables that actually depend on some *other* thing. For example, height and weight are related (height effects weight)…but height and weight are both by age or diet. Another example: an airplane's altitude and distance are affected by time or speed. When two variables depend on a third, independent variable, we call the other variable *t, the parameter.*

Writing & Evaluating Parametric Equations: Parametric Equation are written so each variable, x and y, are functions of the parameter. The x–coordinate is a function of t, and the y–coordinate is a different function of t.

Ex 1: The top of slide is 12 feet off the ground and 3 feet in front of the base of the ladder. When sliding, a rider travels 1 ft/sec forward (horizontally) and 2 ft/sec downward (vertically) Let x = horizontal distance from the base of the ladder, let y = altitude, let t = time in seconds

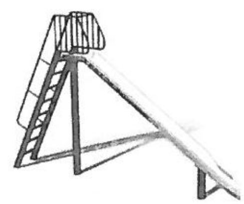
Parametric Equations:

x =

y =

Location of the slider in the first 5 seconds

t	0	1	2	3	4	5
x						
y						



The parameter (t) was given with a restricted domain:

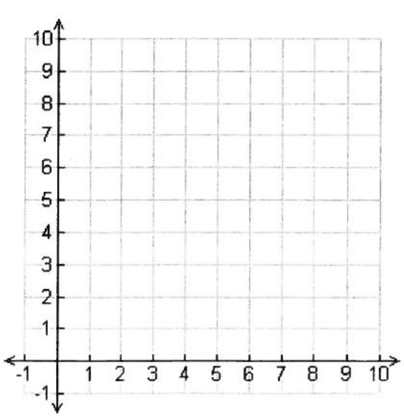
D: $t \in$

What are the range of values for x and y in that interval?

R: $x \in$ $\qquad$ $y \in$

Graphing Parametric Equations: We still graph parametrics in the Cartesian plane. We still want to see the relationship between x and y, but every point will come from evaluating each variable at values for t. $x = f(t)$, $y = g(t)$. Plot the points & connect them in the order in which you plotted them.

Important: When you connect the points, it very important that you show orientation: label critical values of t and show the direction the graph "flows" (with arrows) as t increases. This way, you can actually SEE the influence the parameter has on each variable and on the graph.

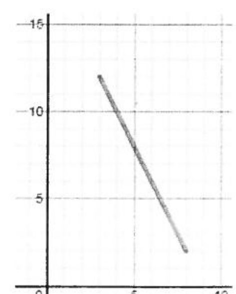


How to graph parametrics in Desmos.com:

- Enter an ordered pair of functions of t.
- Set the domain of the parameter.

$(t+3, \ -2t+12)$

domain: $0 \le t \le 5$



Eliminating the Parameter: It's cool that we can now examine horizontal or vertical distance separately, but we don't always want to work with three variables… sometimes it's simpler to just explore the relationship between x and y, since those are your axes anyway. So, we eliminate t.

Parametric: $x = t + 3$; $y = -2t + 12$; $t \in [0, 5]$

Graph a few the Rectangular equation

Cartesian / Rectangular equation (the usual, y = something)

Step 1) Isolate "t" in the x = equation

Step 2) substitute it into the equation for y =

Rectangular Equation:

It's CRITICAL to state domain (& range) for the rectangular equation
We're still in the "slide" scenario – it's not just an infinite line.

Domain: $x \in$

Range: $y \in$

<u>The graphs are the same:</u> We are still in the rectangular coordinate system, so the two graphs should look IDENTITICAL and have the exact same (x, y) points as the original parametric graph. The only difference is now, we don't need to enter the parameter of time. We can predict the slider's height simply from his/her horizontal distance.

We still use algebra here! Everything you've ever done with any function before can still be done. We can analyze situations and answer questions from parametric equations.

Use the parametric equations: $x = t + 3$; $y = -2t + 12$

a) When is the rider exactly 5.3 feet above ground?

b) How far in front from the base is the rider after 2.7 seconds?

c) When is the rider the same horizontal and vertical distance?

d) How long would it take the rider to hit the ground?

e) How far from the ladder is the end of the slide?

f) What is the angle of elevation of the slide?

f) How LONG is the slide itself?

g) What is the actual speed of the slider?

The beauty of a parameter is that you can always change the parameter so your INPUT is in the units you want (no annoying conversations).

Ex 2: A helicopter is takes off from a launch pad, moving forward 5 ft/sec and rising 12 ft/sec.

a. Write Parametric equations where t = sec.

b. Write Parametric equations where t = minutes.

c. After 3 minutes, what is its altitude?

d. How long will it take the helicopter to fly over the pilot's house, which is half a mile away?

e. What is the angle of elevation of the helicopter?

f. After 5 minutes, what is the direct distance between the chopper and launch pad?

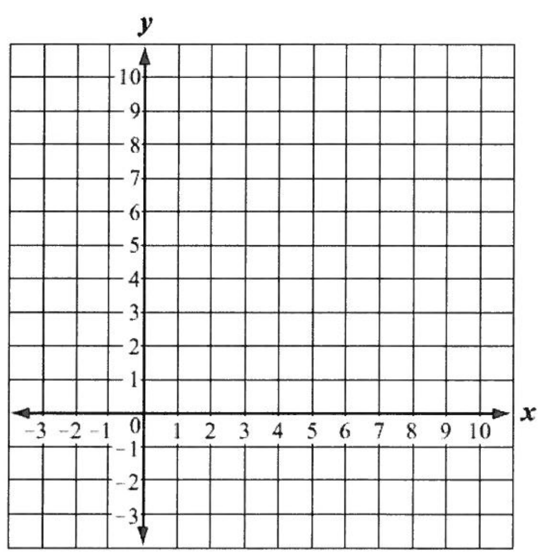
g. Eliminate the parameter to get a rectangular equation & state domain & range.

Ex 3: Diving Darren is standing on a diving board and jumps off into a pool. His path is modeled by the equation:

$x = t$
$y = -t^2 + 2t + 8$

t = seconds of the dive, x = time in seconds passing, y = altitude above the pool,

a. Graph the path of his dive, from when he's on the board until he hits the water. Label t!



b. How high is the diving board?

c. What height is the peak of the dive?

d. How long is he in the air?

e. How many seconds is Darren falling?

Ex 4: A golfer hits a ball, which sails through the air. Let t = time in seconds, let x = horizontal distance from the golfer and let y = vertical height from the ground.

$x = 92t$
$y = -16t^2 + 92t$

$t \in [0, 5]$

t	0	1	2	3	4	5
x						
y						

* *Note: we are not getting a complete picture from these points!*

a) After how many seconds is the ball's maximum height? (Use your knowledge of quadratics).

b) What is the ball's maximum height?

How far from the golfer is the ball at its max height?

c) Eliminate the parameter to write a single, rectangular equation. Then state domain & range.

d) Check your work: evaluate at a previously known value for x to ensure you get the same y.

Ex 5: A research submarine is 200 m below the surface of the water and will to descend with a vertical speed of 1 m/s and horizontal speed of 2 m/s.

a. Write Parametric equations for the submarine's depth and horizontal distance from its current location.

b. Eliminate the Parameter to get a Rectangular Equation for the submarine's path in the first 4 minutes.

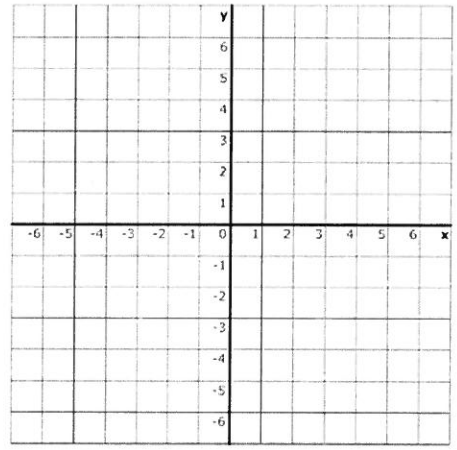
State domain & range.

c. Graph the first 4 minutes of the descent.

Example 5: Equations & Graphs without Application:

For each equation, a) make a table of values b) graph showing orientation c) convert to a Cartesian equation and state the domain and range.

$x = t^2 - 4$
$y = t/2$
$t = [-2, 3)$



Make a table:

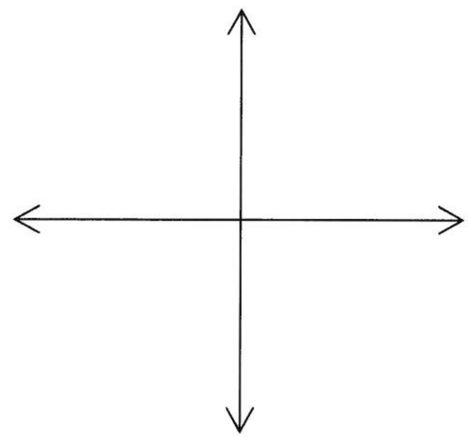
Rectangular Equation:

Domain: Range:

PRACTICE GRAPHING & CONVERTING:

1. $\begin{cases} x = 1 - 2t \\ y = 2 + 2t \end{cases}$

$t \in (0, 3] \cup [4, 5)$

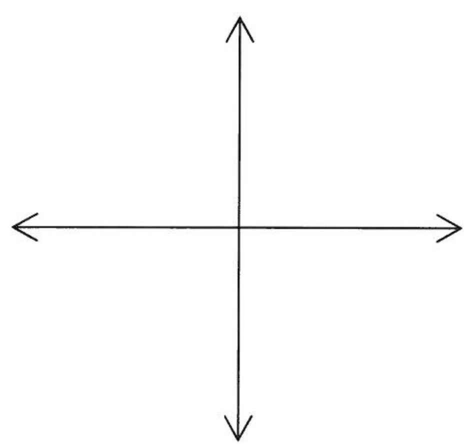


Rectangular Equation:

Domain: Range:

2. $\begin{cases} x = t - 2 \\ y = t^2 \end{cases}$

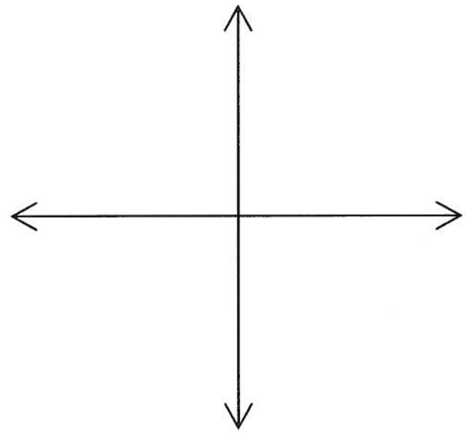
$t \in [-1, \infty)$



Rectangular Equation:

Domain: Range:

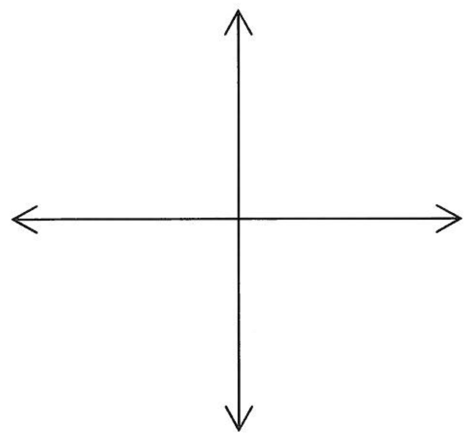
3. $x = \sqrt{t} + 4$
$y = \sqrt{t} - 4$
$t \in (0, \infty)$



Rectangular Equation:

Domain: Range:

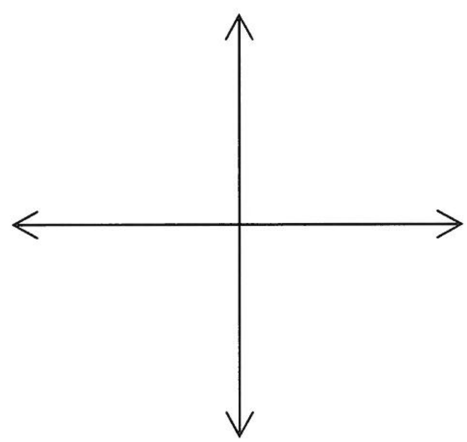
4. $\begin{cases} x = \dfrac{t}{4} \\ y = -3t \end{cases}$
$t \in [-2, 2)$



Rectangular Equation:

Domain: Range:

5. $\begin{cases} x = 20t \\ y = 10t + 10 \end{cases}$
$t \in [-2, 2)$

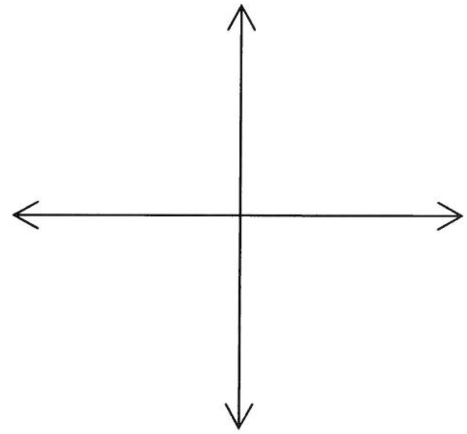


Rectangular Equation:

Domain: Range:

6. $\begin{cases} x = t^2 - 1 \\ y = 2t + 5 \end{cases}$

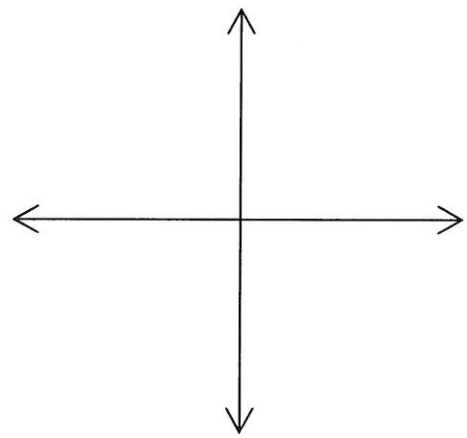
$t \in [-2, 2)$



Rectangular Equation:

Domain: Range:

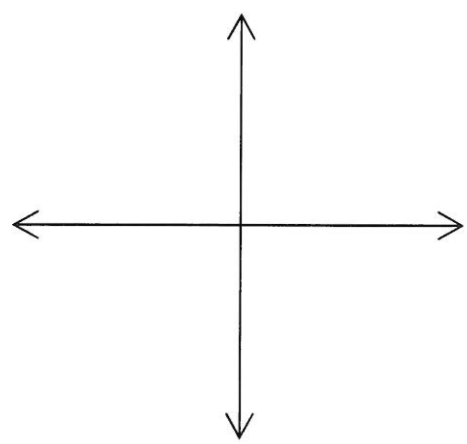
7. $x = -2t + 1$
$y = t - 5$
$t \in [-3, 2]$



Rectangular Equation:

Domain: Range:

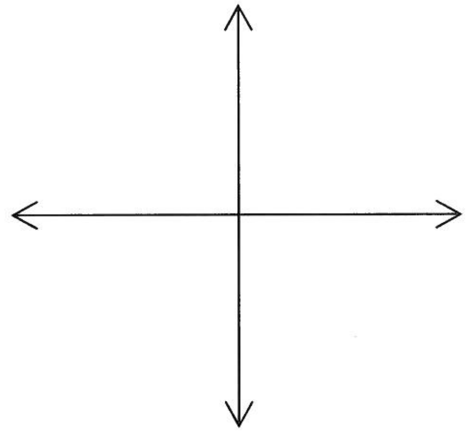
8. $x = -t$
$y = |t - 2|$
$t \in (-4, 4)$



Rectangular Equation:

Domain: Range:

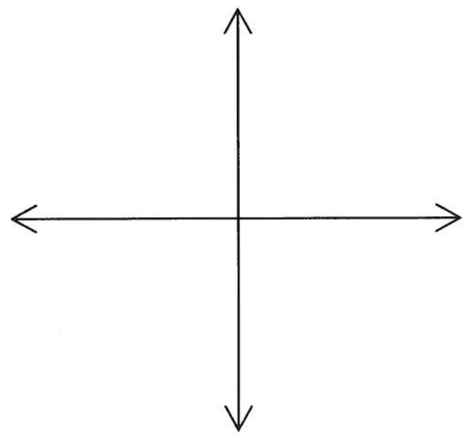
8. $x = 5 - t^2$
$y = t - 3$
$t \in [-2, 3]$



Rectangular Equation:

Domain: Range:

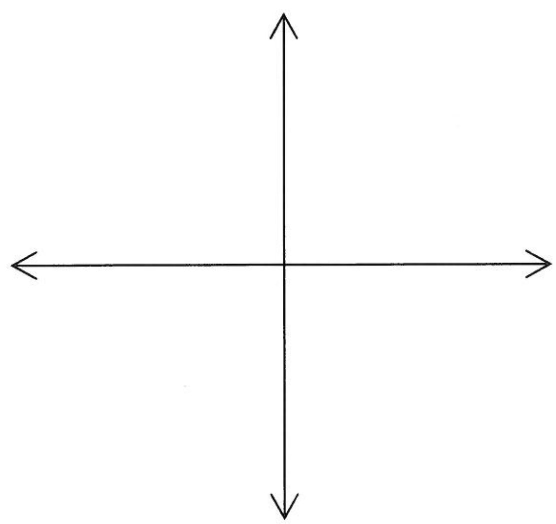
9. $x = \sqrt{t}$
$y = 5 - t$
$t \in [0, \infty)$



Rectangular Equation:

Domain: Range:

☆10. $x = t^2 - 4$
$y = 2t^2$
$t \in$ ~~$[0, \infty)$~~
$(-\infty, \infty)$



Rectangular Equation:

Domain: Range: