

Name: \_\_\_\_\_ Per: \_\_\_\_\_ Date: \_\_\_\_\_  
 Serafino • Precalculus S2

## 11B Parametric Equations of Conic Sections

### Notes / Homework

What have we done in this course so far? Trig? Conic sections? Parametric equations?  
 So, turns out we have all the info necessary to write of conic sections IN parametric equations.

Check.  
 Cool.

#### PARABOLAS



$$\begin{aligned} x &= \cos t + h \\ y &= \cos^2 t + k \end{aligned}$$

$$\begin{aligned} x &= \sin t + h \\ y &= \sin^2 t + k \end{aligned}$$

$$\begin{aligned} x &= \cos^2 t + h \\ y &= \cos t + k \end{aligned}$$

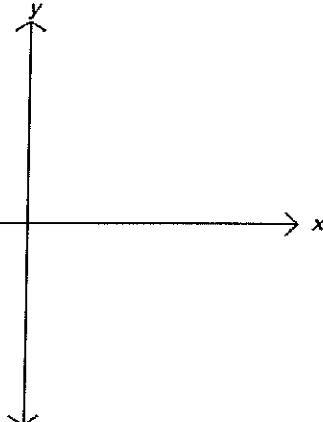
$$\begin{aligned} x &= \sin^2 t + h \\ y &= \sin t + k \end{aligned}$$



What to use: use the same trig function for x and y, but only square one of them.

1.  $x = \sin t + 4$   
 $y = \sin^2 t + 1$

Rectangular Equation:



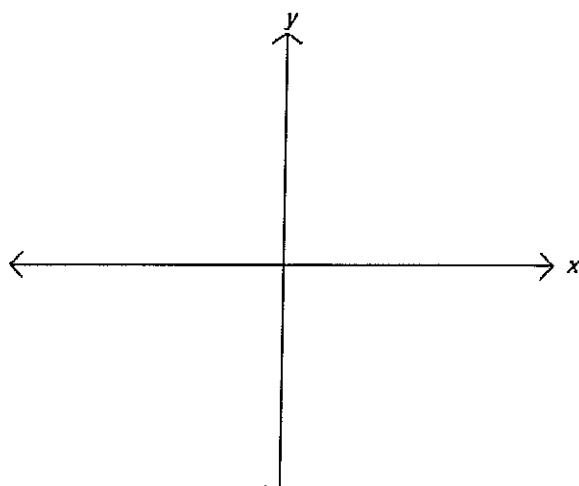
t	x	y

Domain:

Range:

2.  $x = \sin^2 t + 3$   
 $y = \sin t$

Rectangular Equation:



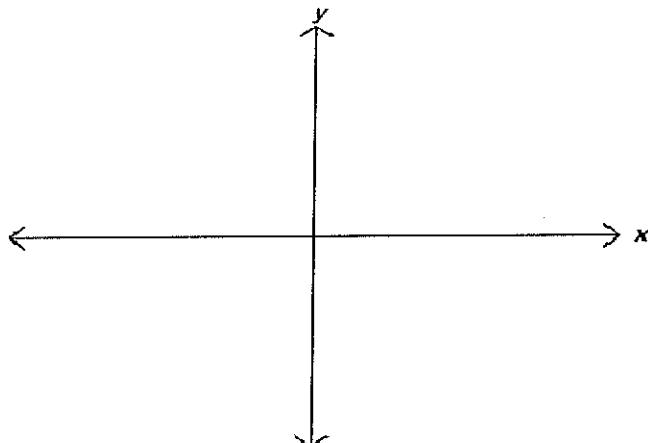
t	x	y

Domain:

Range:

3.  $x = \cos t - 3$   
 $y = 5 \cos^2 t$

Rectangular Equation:



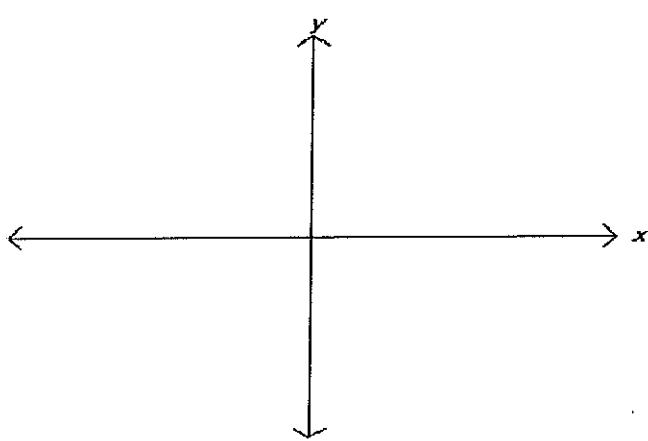
t	x	y

Domain:

Range:

4.  $x = \cos^2 t - 1$   
 $y = \cos t - 4$

Rectangular Equation:



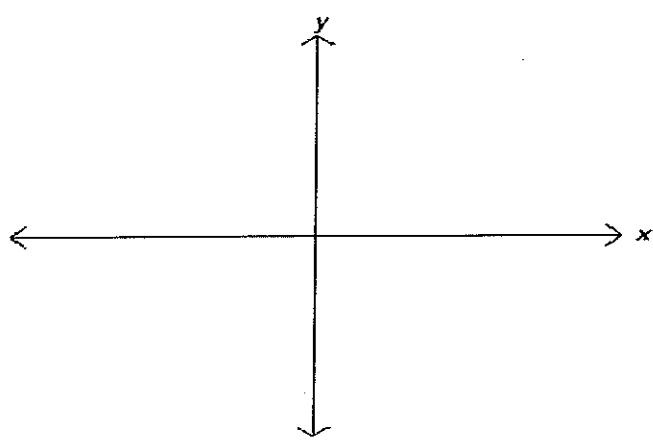
t	x	y

Domain:

Range:

5.  $x = \sin 2t$   
 $y = \sin^2 2t + 4$

Rectangular Equation:



t	x	y

Domain:

Range:

## ELLIPSES & CIRCLES

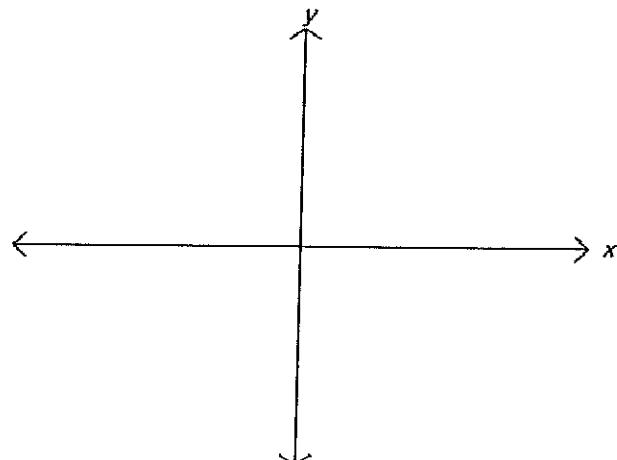
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$$\begin{array}{ll} x = r_1 \cos(t) + h & x = r_1 \sin(t) + h \\ y = r_2 \sin(t) + k & y = r_2 \cos(t) + k \end{array}$$

$$\text{Identity used: } (\sin\theta)^2 + (\cos\theta)^2 = 1$$

6.  $x = 5 \cos t + 3$   
 $y = 3 \sin t - 2$

Rectangular Equation:

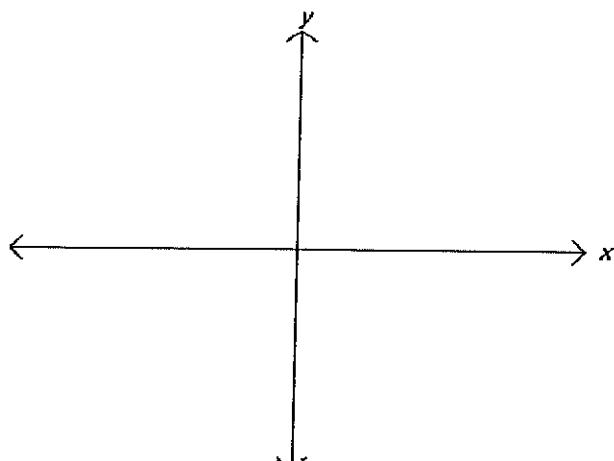


Domain:

Range:

7.  $x = 2 \sin t$   
 $y = \sqrt{10} \cos t$

Rectangular Equation:

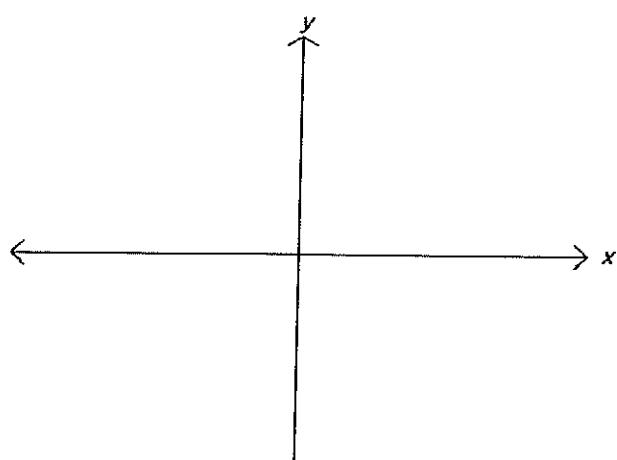


Domain:

Range:

8.  $x = 4 \cos t - 2$   
 $y = 4 \sin t$

Rectangular Equation:

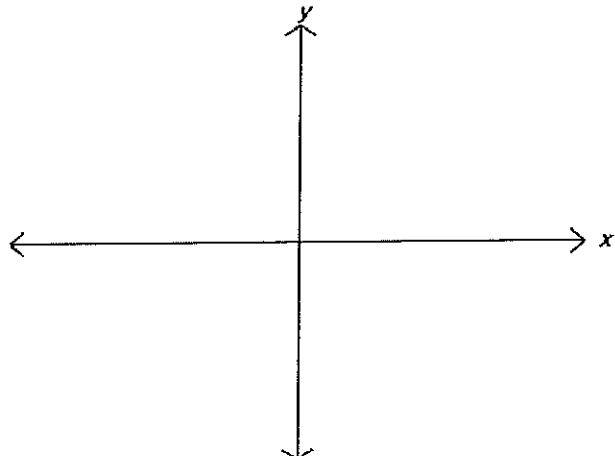


Domain:

Range:

9.  $x = \sqrt{3} \sin t + 4$   
 $y = \sqrt{3} \cos t + 3$   
 $t \in [0, 3\pi/2)$

Rectangular Equation:

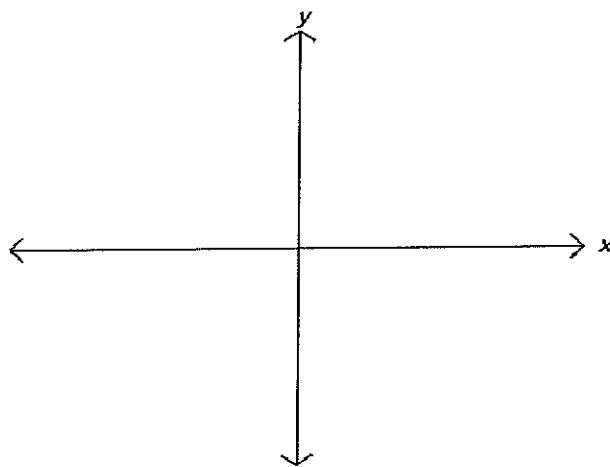


Domain:

Range:

10.  $x = 6 \cos t$   
 $y = 3 \sin t + 1$   
 $t \in [0, 3\pi/2)$

Rectangular Equation:

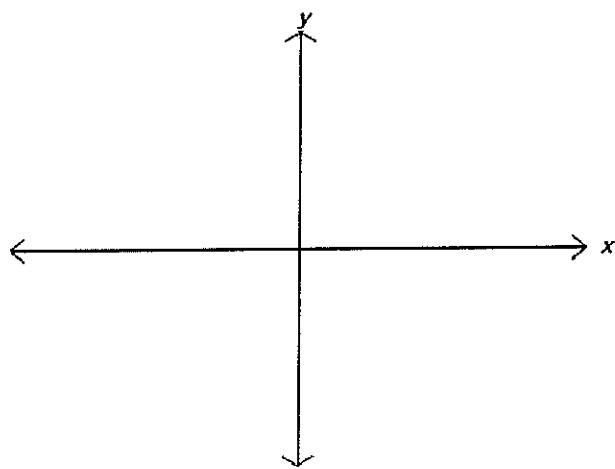


Domain:

Range:

11.  $x = \cos t + 3$   
 $y = 2 \sin t$   
 $t \in (0, \pi/2]$

Rectangular Equation:

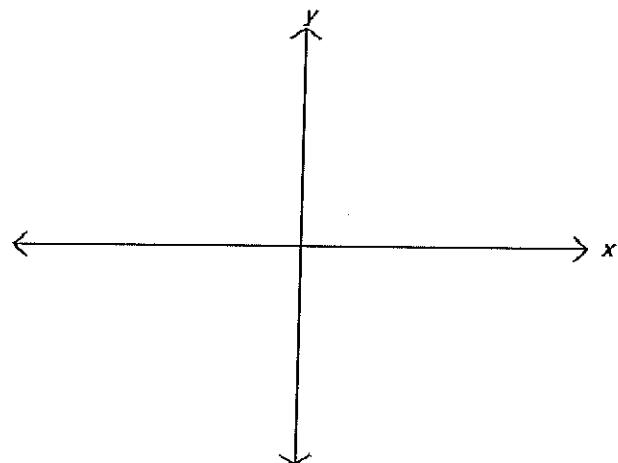


Domain:

Range:

12.  $x = 5 \sin t - 2$   
 $y = 5 \cos t + 3$

Rectangular Equation:

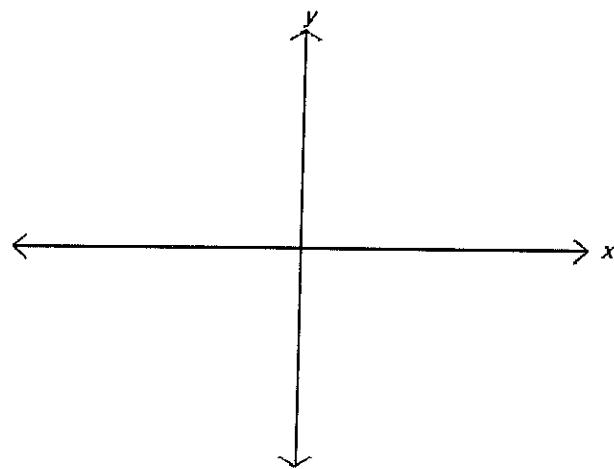


Domain:

Range:

13.  $x = 2\sqrt{3} \sin t - 2$   
 $y = 3\sqrt{2} \cos t + 3$

Rectangular Equation:



Domain:

Range:

14. Write two different sets of parametric equations for a circle with equation  $(x - 2)^2 + (y + 4)^2 = 9$ . Determine which one will move clockwise, and which will move counterclockwise.

15. Write a parametric equation for  $\frac{(x+5)^2}{8} + \frac{y^2}{12} = 1$  for which t will rotate clockwise

16. Write a parametric equation for  $\frac{(x-5)^2}{16} + \frac{(y+2)^2}{25} = 1$  for which t will rotate counterclockwise.

**HYPERBOLAS**Horizontal

$$x = r_1 \sec(t) + h$$

$$y = r_2 \tan(t) + k$$

$$x = r_1 \csc(t) + h$$

$$y = r_2 \cot(t) + k$$

Vertical

$$x = r_1 \tan(t) + h$$

$$y = r_2 \sec(t) + k$$

$$x = r_1 \cot(t) + h$$

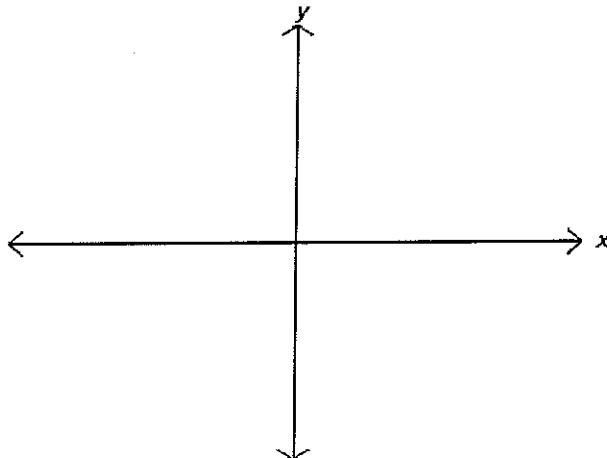
$$y = r_2 \csc(t) + k$$

Identities:  $\sec^2 - \tan^2 = 1$  or  $\csc^2 - \cot^2 = 1$

*Note: Orientation/direction as t increases in hyperbolas is VERY funky. I will show you in class, but you will not be accountable for it on an assessment. It is worth seeing at least once, though, because it's kind of cool.*

17.  $x = 6 \sec t + 3$   
 $y = 3 \tan t - 2$

Rectangular Equation:

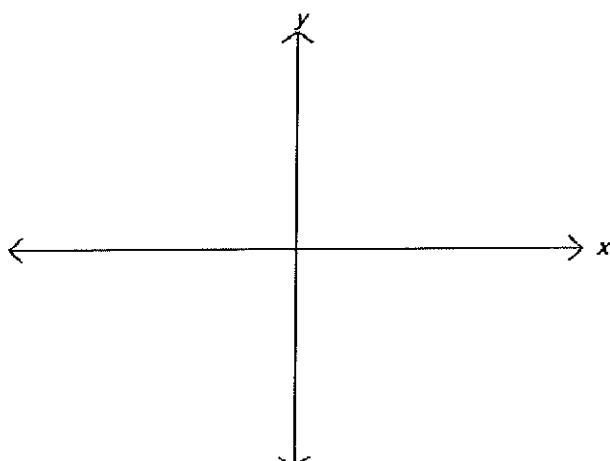


Domain:

Range:

18.  $x = \sqrt{3} \tan t$   
 $y = 5 \sec t - 4$

Rectangular Equation:

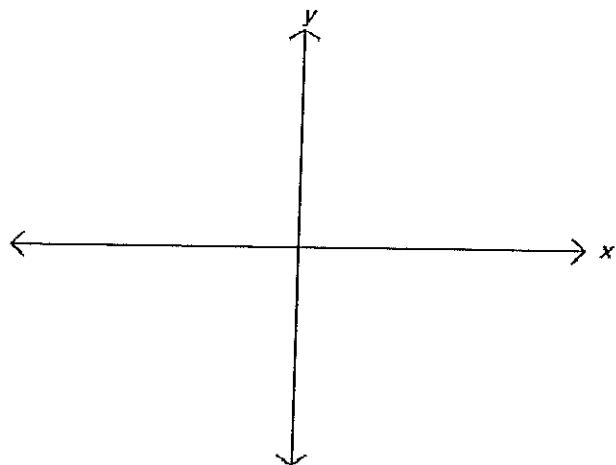


Domain:

Range:

19.  $x = 3 \csc t - 2$   
 $y = 5 \cot t - 4$

Rectangular Equation:

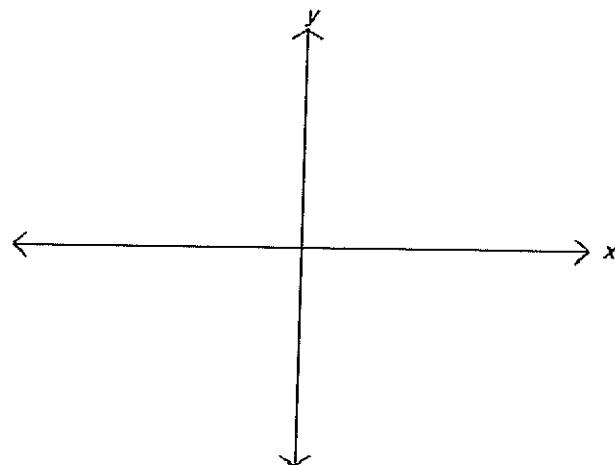


Domain:

Range:

20.  $x = 4 \cot t + 4$   
 $y = 2 \csc t + 3$

Rectangular Equation:



Domain:

Range:

21. Write a parametric equation for  $\frac{(y-8)^2}{36} - \frac{(x+1)^2}{49} = 1$

22. Write a parametric equation for  $\frac{x^2}{20} - \frac{(y+2)^2}{12} = 1$

**Put it all together:**

1. Classify the conic with direction of opening, and write a set of parametric equations for each.

a.  $(x+3)^2 + (y-1)^2 = 16$

e.  $\frac{(x-5)^2}{16} + \frac{(y+2)^2}{25} = 1$

b.  $\frac{(x+5)^2}{8} + \frac{y^2}{12} = 1$

f.  $\frac{(y-8)^2}{36} - \frac{(x+1)^2}{49} = 1$

c.  $\frac{x^2}{20} - \frac{(y+2)^2}{12} = 1$

g.  $x^2 + y^2 - 6x - 2y - 10 = 0$

d.  $(x-1)^2 = (y+5)$

h.  $y^2 = (x+2)$

2. Classify the conic with direction of opening, and write a rectangular equation for each.

a. 
$$\begin{cases} x = \cos(t) + 2 \\ y = \cos^2(t) - 3 \end{cases}$$

f. 
$$\begin{cases} x = \sin^2(t) - 2 \\ y = \sin(t) \end{cases}$$

b. 
$$\begin{cases} x = 3\cos(t) + 2 \\ y = 5\sin(t) - 3 \end{cases}$$

g. 
$$\begin{cases} x = 12\cos(5t) + 4 \\ y = 12\sin(5t) - 1 \end{cases}$$

c. 
$$\begin{cases} x = \sqrt{5}\sin(t) - 4 \\ y = 3\cos(t) + 8 \end{cases}$$

h. 
$$\begin{cases} x = 3\csc(t) + 2 \\ y = 8\cot(t) - 3 \end{cases}$$

d. 
$$\begin{cases} x = 3\tan(2t) \\ y = 5\sec(2t) \end{cases}$$

i. 
$$\begin{cases} x = 4 + 3\cot(t) \\ y = 4 + 5\csc(t) \end{cases}$$

e. 
$$\begin{cases} x = 3\sec(t) + 9 \\ y = 5\tan(t) - 8 \end{cases}$$

j. 
$$\begin{cases} x = 5\cos(t) + 4 \\ y = 5\sin(t) + 4 \end{cases}$$