Name: Answer key Per: _____ Date: _____
Serafino • Precalculus S2

11B Parametric Equations of Conic Sections

Notes / Homework

What have we done in this course so far? Trig? Conic sections? Parametric equations? Check.
So, turns out we have all the info necessary to write of conic sections IN parametric equations. Cool.

PARABOLAS

∪					∩
$x = \cos t + h$ $y = \cos^2 t + k$	$x = \sin t + h$ $y = \sin^2 t + k$	$x = \cos^2 t + h$ $y = \cos t + k$	$x = \sin^2 t + h$ $y = \sin t + k$	

*What to use: use the same trig function for x and y, but only square one of them.

Vertex: (4, 1)

1. $x = \sin t + 4$
   $y = \sin^2 t + 1$

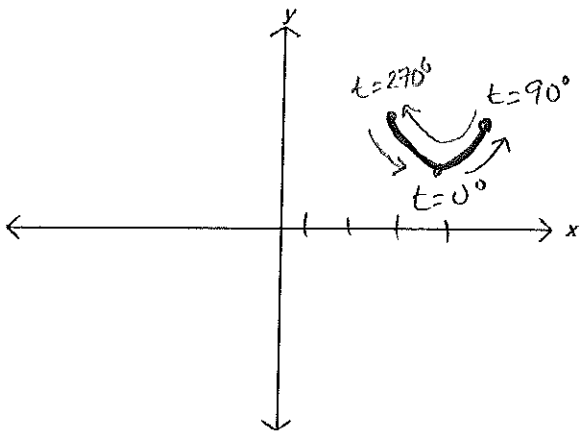
$\sin t = x - 4$

Rectangular Equation:

$y = (x-4)^2 + 1$

Domain: $x \in [3, 5]$

Range: $y \in [1, 2]$



t	x	y
0	4	1
90	5	2

* You only need to plug in 2 t's to get direction, then use symmetry to get the rest.

2. $x = \sin^2 t + 3$
   $y = \sin t$

$(\sin t)^2 = x - 3$

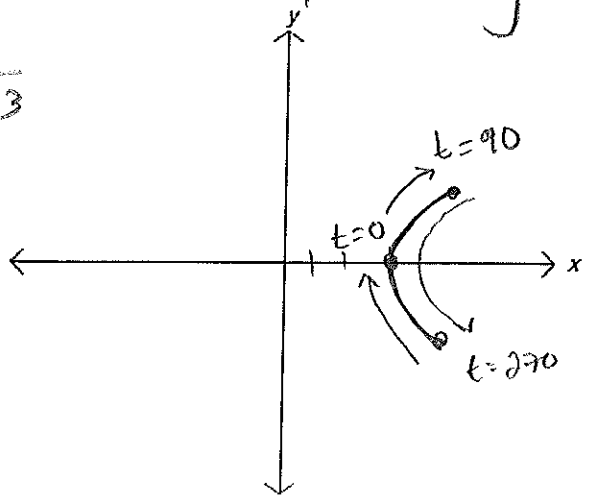
$\sin t = \pm\sqrt{x-3}$

Rectangular Equation:

$y = \pm\sqrt{x-3}$

Domain: $x \in [3, 4]$

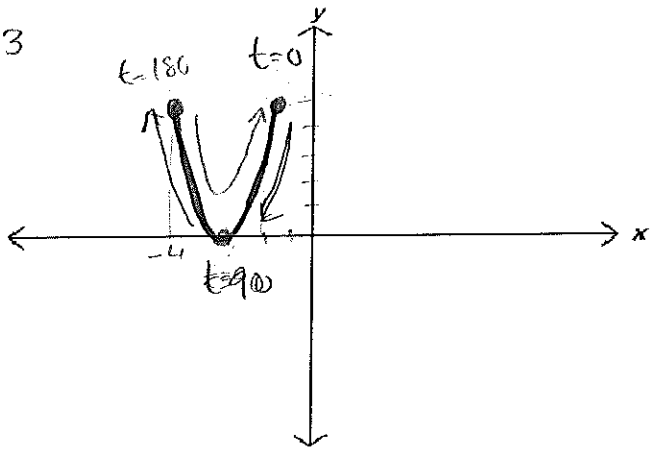
Range: $y \in [-1, 1]$



t	x	y
0	3	0
90	4	1

V(-3,0)

3. $x = \cos t - 3$ $\quad \cos t = x+3$
$y = 5\cos^2 t$



t	x	y
0	-2	5
90	-3	0

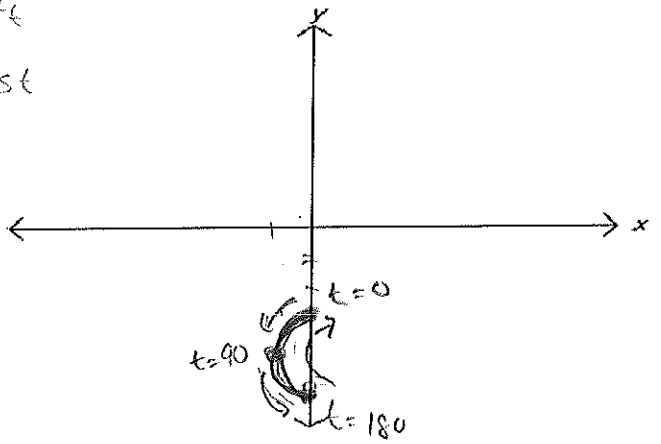
Rectangular Equation:

$y = 5(x+3)^2$

Domain: $x \in [-4,-2]$

Range: $y \in [0,5]$

4. $x = \cos^2 t - 1$ $\quad x+1 = \cos^2 t$
$y = \cos t - 4$ $\quad \pm\sqrt{x+1} = \cos t$



t	x	y
0	0	-3
90	-1	-4

Rectangular Equation:

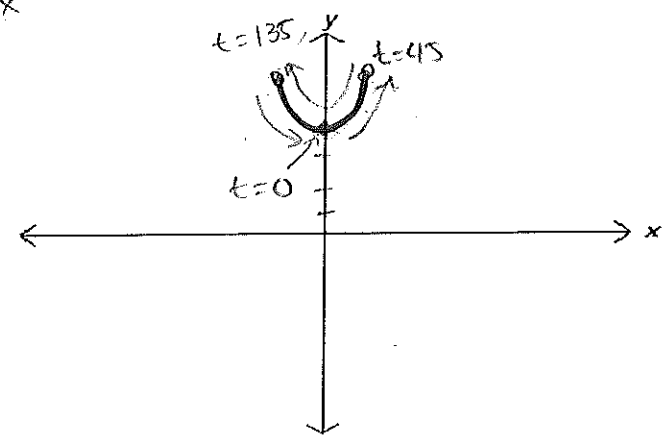
$y = \pm\sqrt{x+1} - 4$

Domain: $x \in [-1,0]$

Range: $y \in [-5,-3]$

I still want to use sin(90) so I use t=45

5. $x = \sin(2t)$ $\quad \sin(2t) = x$
$y = \sin^2(2t) + 4$



t	x	y
0	0	4
45	1	5

Rectangular Equation:

$y = x^2 + 4$

Domain: $x \in [-1,1]$

Range: $y \in [4,5]$

"moves" twice as fast b/c frequency = 2.

ELLIPSES & CIRCLES

$x = r_1 \cos(t) + h$ $\quad$ $x = r_1 \sin(t) + h$
$y = r_2 \sin(t) + k$ $\quad$ $y = r_2 \cos(t) + k$

* Identity used: $(\sin\theta)^2 + (\cos\theta)^2 = 1$

6. $x = 5\cos t + 3$
$y = 3\sin t - 2$

$C(3, -2)$

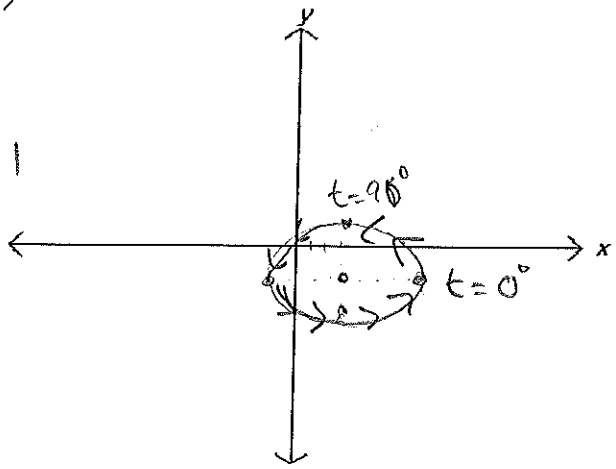
Rectangular Equation:

$\sin^2 + \cos^2 = 1$

$\left(\frac{y+2}{3}\right)^2 + \left(\frac{x-3}{5}\right)^2 = 1$

$\frac{y+2}{3} = \frac{3\sin t}{3}$

$\frac{(x-3)^2}{25} + \frac{(y+2)^2}{9} = 1$



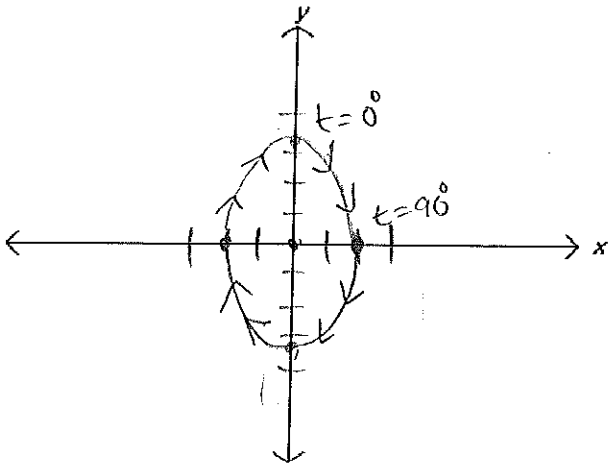
Domain: $x \in [-2, 8]$

Range: $y \in [-5, 1]$

7. $x = 2\sin t$
$y = \sqrt{10}\cos t$

Rectangular Equation:

$\frac{x^2}{4} + \frac{y^2}{10} = 1$



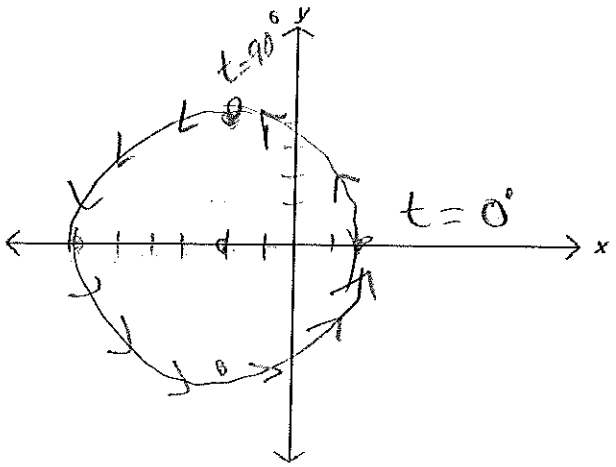
Domain: $x \in [-2, 2]$

Range: $y \in [-\sqrt{10}, \sqrt{10}]$

8. $x = 4\cos t - 2$
$y = 4\sin t$

Rectangular Equation:

$(x+2)^2 + y^2 = 16$



Domain: $x \in [-6, 2]$

Range: $y \in [-4, 4]$

9. $x = \sqrt{3}\sin t + 4$
$y = \sqrt{3}\cos t + 3$
$t \in [0, 3\pi/2)$

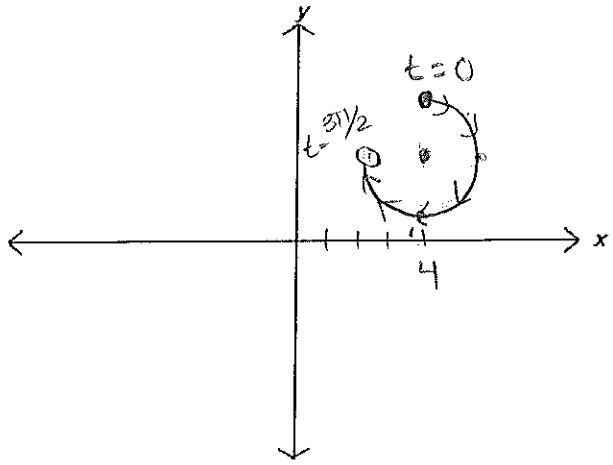
careful!

Rectangular Equation:

$(x-4)^2 + (y-3)^2 = 3$

Domain: $x \in (4-\sqrt{3}, 4+\sqrt{3}]$

Range: $y \in [3-\sqrt{3}, 3+\sqrt{3}]$



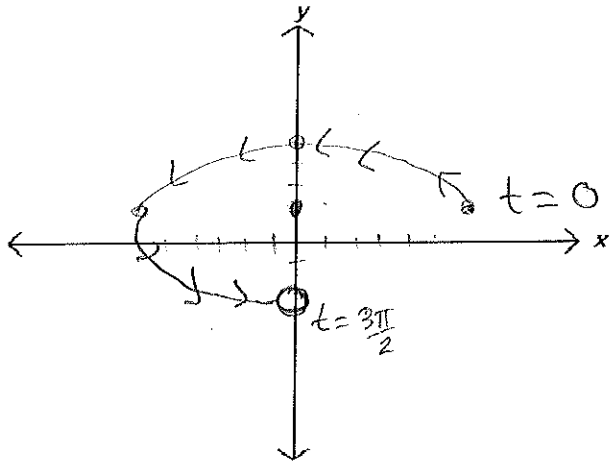
10. $x = 6\cos t$
$y = 3\sin t + 1$
$t \in [0, 3\pi/2)$

Rectangular Equation:

$\frac{x^2}{36} + \frac{(y-1)^2}{9} = 1$

Domain: $x \in [-6, 6]$

Range: $y \in (-2, 4]$



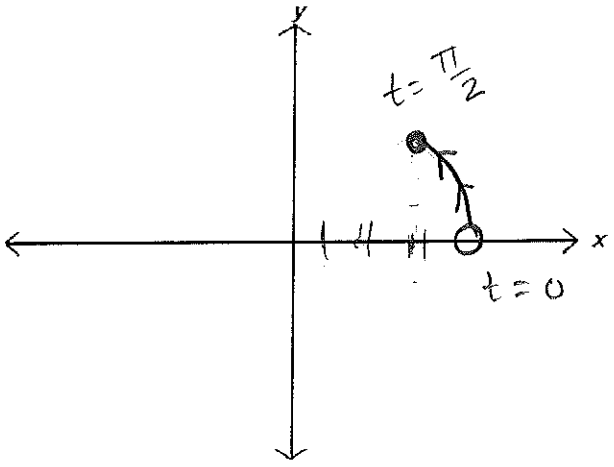
11. $x = \cos t + 3$
$y = 2\sin t$
$t \in (0, \pi/2]$

Rectangular Equation:

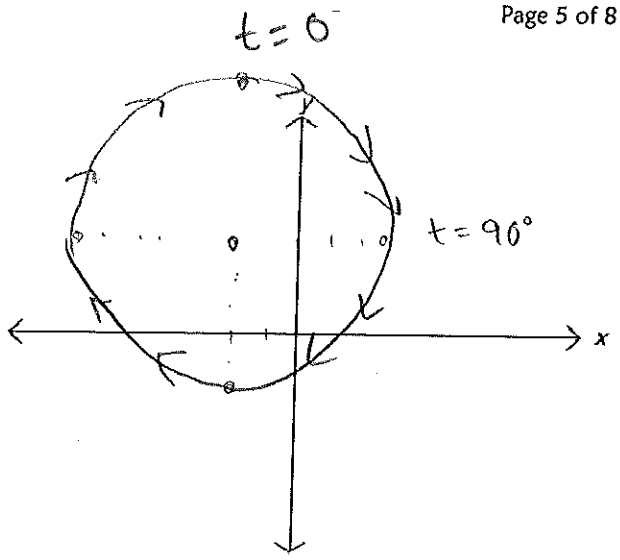
$(x-3)^2 + \frac{y^2}{4} = 1$

Domain: $x \in [3, 4)$

Range: $y \in (0, 2]$



12. $x = 5 \sin t - 2$
$y = 5 \cos t + 3$



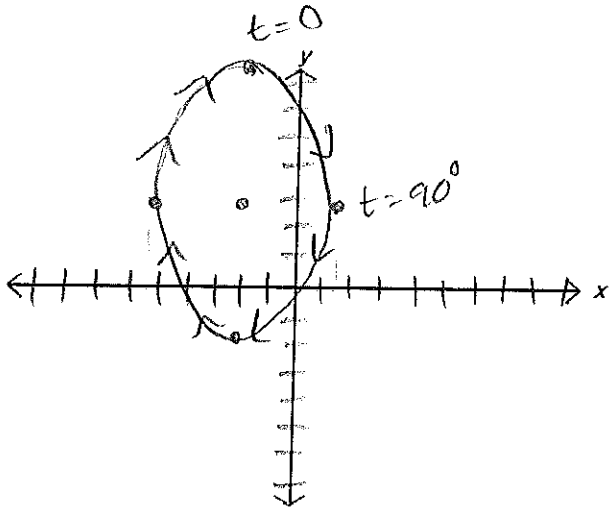
Rectangular Equation:

$$(x+2)^2 + (y-3)^2 = 25$$

Domain: $x \in [-7, 3]$

Range: $y \in [-2, 8]$

13. $x = 2\sqrt{3} \sin t - 2$
$y = 3\sqrt{2} \cos t + 3$



Rectangular Equation:

$$\frac{(x+2)^2}{12} + \frac{(y-3)^2}{18} = 1$$

Domain: $x \in [-2 - 2\sqrt{3}, -2 + 2\sqrt{3}]$

Range: $y \in [3 - 3\sqrt{2}, 3 + 3\sqrt{2}]$

14. Write two different sets of parametric equations for a circle with equation $(x - 2)^2 + (y + 4)^2 = 9$. Determine which one will move clockwise, and which will move counterclockwise.

$x = 3 \sin t + 2$
$y = 3 \cos t - 4$
clockwise

$x = 3 \cos t + 2$
$y = 3 \sin t - 4$
counterclockwise

15. Write a parametric equation for $\frac{(x+5)^2}{8} + \frac{y^2}{12} = 1$ for which t will rotate clockwise

$x = 2\sqrt{2} \sin t - 5$
$y = 2\sqrt{3} \cos t$

16. Write a parametric equation for $\frac{(x-5)^2}{16} + \frac{(y+2)^2}{25} = 1$ for which t will rotate counterclockwise.

$x = 4 \cos t + 5$
$y = 5 \sin t - 2$

$\sin^2 + \cos^2 = 1$ $1 + \cot^2 = \csc^2$ $\tan^2 + 1 = \sec^2$

$\frac{\sin^2}{\cos^2} \quad \frac{\cos^2}{\cos^2} \quad \frac{\sin^2}{\cos^2}$ $\csc^2 - \cot^2 = 1$ $\sec^2 - \tan^2 = 1$

HYPERBOLAS

Horizontal		Vertical	
$x = r_1 \sec(t) + h$	$x = r_1 \csc(t) + h$	$x = r_1 \tan(t) + h$	$x = r_1 \cot(t) + h$
$y = r_2 \tan(t) + k$	$y = r_2 \cot(t) + k$	$y = r_2 \sec(t) + k$	$y = r_2 \csc(t) + k$

Identities: $\sec^2 - \tan^2 = 1$ or $\csc^2 - \cot^2 = 1$

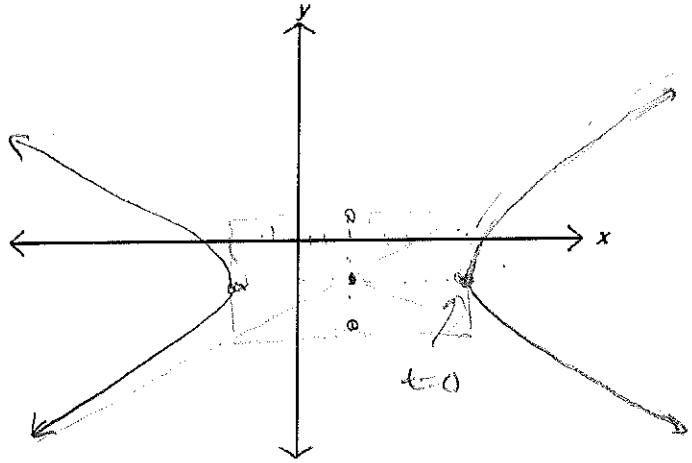
Note: Orientation/direction as t increases in hyperbolas is VERY funky. I will show you in class, but you will not be accountable for it on an assessment. It is worth seeing at least once, though, because it's kind of cool.

17. $x = 6 \sec t + 3$
$y = 3 \tan t - 2$

$\sec^2 - \tan^2 = 1$

Rectangular Equation:

$$\frac{(x-3)^2}{36} - \frac{(y+2)^2}{9} = 1$$



Domain: $x \leq -3$ or $x \geq 9$

Range: $y \in \mathbb{R}$

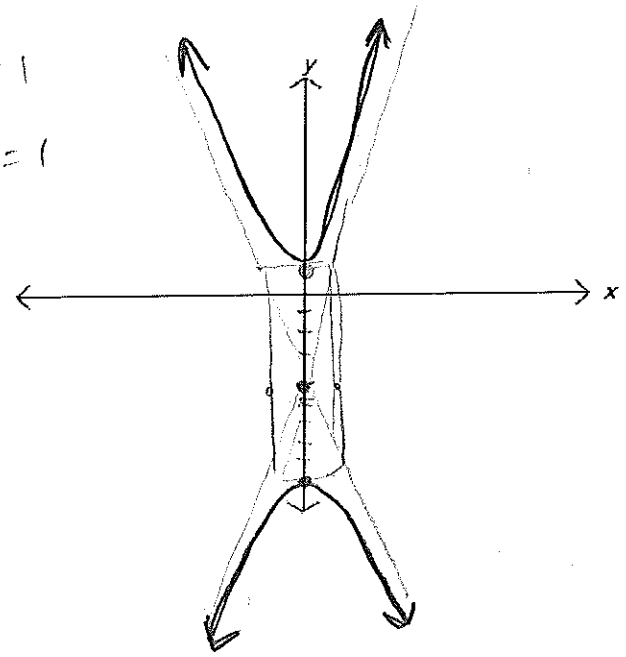
18. $x = \sqrt{3} \tan t$
$y = 5 \sec t - 4$

$\sec^2 - \tan^2 = 1$

$$\left(\frac{y+4}{5}\right)^2 - \left(\frac{x}{\sqrt{3}}\right)^2 = 1$$

Rectangular Equation:

$$\frac{(y+4)^2}{25} - \frac{x^2}{3} = 1$$



Domain: $x \in \mathbb{R}$

Range: $y \leq -9$ or $y \geq 1$

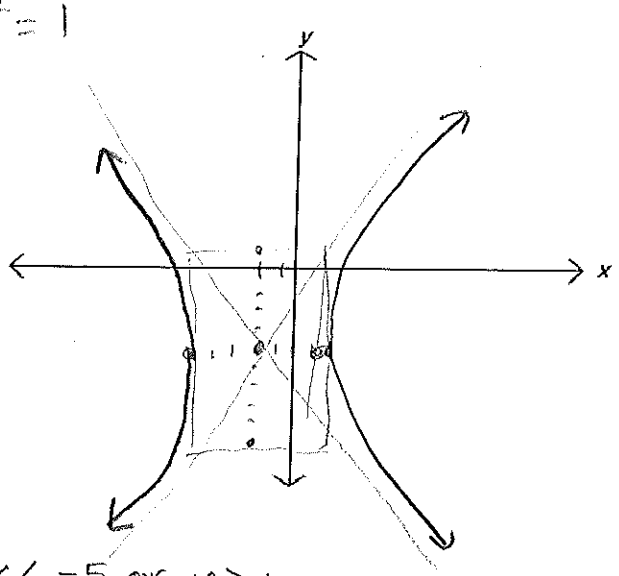
To check on Desmos ex: #17

○ $(6\sec t + 3, 3\tan t - 2)$
$0 \leq t \leq 360$

○ $\frac{(x-3)^2}{36} - \frac{(y+2)^2}{9} = 1$

$\csc^2 - \cot^2 = 1$

19. $x = 3\csc t - 2$
$y = 5\cot t - 4$



Rectangular Equation:

$$\frac{(x+2)^2}{9} - \frac{(y+4)^2}{25} = 1$$

Domain: $x \notin (-5, 1)$

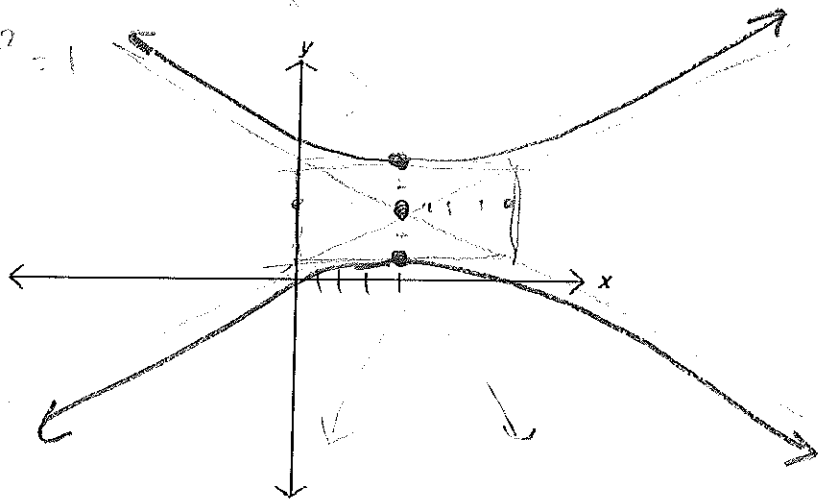
Range: $y \in \mathbb{R}$

aka: $x \leq -5$ or $x \geq 1$

$x \in (-\infty, -5] \cup [1, \infty)$ (all same thing)

20. $x = 4\cot t + 4$
$y = 2\csc t + 3$

$\csc^2 - \cot^2 = 1$



Rectangular Equation:

$$\frac{(y-3)^2}{4} - \frac{(x-4)^2}{16} = 1$$

Domain: $x \in \mathbb{R}$

Range: $y \leq 1$ or $y \geq 5$

21. Write a parametric equation for $\frac{(y-8)^2}{36} - \frac{(x+1)^2}{49} = 1$
(you could write two)

$\sec^2 - \tan^2 = 1$
$\csc^2 - \cot^2 = 1$

$\begin{cases} x = 7\tan t - 1 \\ y = 6\sec t + 8 \end{cases}$ or $\begin{cases} x = 7\cot t - 1 \\ y = 6\csc t + 8 \end{cases}$

22. Write a parametric equation for $\frac{x^2}{20} - \frac{(y+2)^2}{12} = 1$
(you could write two)

$\begin{cases} x = 2\sqrt{5}\csc t \\ y = 2\sqrt{3}\cot t - 2 \end{cases}$

or

$\begin{cases} x = 2\sqrt{5}\sec t \\ y = 2\sqrt{3}\tan t - 2 \end{cases}$

* parabolas, circles, ellipses
sin ↔ cos

* hyperbolas
sec ↔ csc, tan ↔ cot

Put it all together:

1. Classify the conic with direction of opening, and write a set of parametric equations for each.

a. $(x+3)^2+(y-1)^2=16$
circle
$x=4\cos t-3$
$y=4\sin t+1$

b. $\frac{(x+5)^2}{8}+\frac{y^2}{12}=1$
V. Ellipse
$x=2\sqrt{2}\sin t-5$
$y=2\sqrt{3}\cos t$

c. $\frac{x^2}{20}-\frac{(y+2)^2}{12}=1$
H. Hyperbola
$x=2\sqrt{5}\sec t$
$y=2\sqrt{3}\tan t-2$

d. $(x-1)^2=(y+5)$
R. Parabola
$x=\sin t+1$
$y=\sin^2 t-5$

e. $\frac{(x-5)^2}{16}+\frac{(y+2)^2}{25}=1$
V. Ellipse
$x=4\cos t+5$
$y=5\sin t-2$

f. $\frac{(y-8)^2}{36}-\frac{(x+1)^2}{49}=1$
V. Hyperbola
$x=7\tan t-1$
$y=6\sec t+8$

g. $x^2+y^2-6x-2y-10=0$
Circle
(3,1) $r=2\sqrt{5}$
$x=2\sqrt{5}\cos t+3$
$y=2\sqrt{5}\sin t+1$

h. $y^2=(x+2)$
Up. Parabola
$x=\cos^2 t-2$
$y=\cos t$

2. Classify the conic with direction of opening, and write a rectangular equation for each.

a. $\begin{cases}x=\cos(t)+2\\ y=\cos^2(t)-3\end{cases}$
Up parabola
$y=(x-2)^2-3$

b. $\begin{cases}x=3\cos(t)+2\\ y=5\sin(t)-3\end{cases}$
V. Ellipse
$\frac{(x-2)^2}{9}+\frac{(y+3)^2}{25}=1$

c. $\begin{cases}x=\sqrt{5}\sin(t)-4\\ y=3\cos(t)+8\end{cases}$
V. Ellipse
$\frac{(x+4)^2}{5}+\frac{(y-8)^2}{9}=1$

d. $\begin{cases}x=3\tan(2t)\\ y=5\sec(2t)\end{cases}$
V. Hyperbola
$\frac{y^2}{25}-\frac{x^2}{9}=1$

e. $\begin{cases}x=3\sec(t)+9\\ y=5\tan(t)-8\end{cases}$
H. Hyperbola
$\frac{(x-9)^2}{9}-\frac{(y+8)^2}{25}=1$

f. $\begin{cases}x=\sin^2(t)-2\\ y=\sin(t)\end{cases}$
R. Parabola
$y=\pm\sqrt{x+2}$

g. $\begin{cases}x=12\cos(5t)+4\\ y=12\sin(5t)-1\end{cases}$
Circle
$(x-4)^2+(y+1)^2=144$

h. $\begin{cases}x=3\csc(t)+2\\ y=8\cot(t)-3\end{cases}$
H. Hyperbola
$\frac{(x-2)^2}{9}-\frac{(y+3)^2}{64}=1$

i. $\begin{cases}x=4+3\cot(t)\\ y=4+5\csc(t)\end{cases}$
V. Hyperbola
$\frac{(y-4)^2}{25}-\frac{(x-4)^2}{9}=1$

j. $\begin{cases}x=5\cos(t)+4\\ y=5\sin(t)+4\end{cases}$
Circle
$(x-4)^2+(y-4)^2=25$

* Extra credit on unit Quest: identify / calculate the "eccentricity" of a given conic *