Name: Key Per: 1, 5 Date: _____

Serafino • Precalculus S2

↑only!

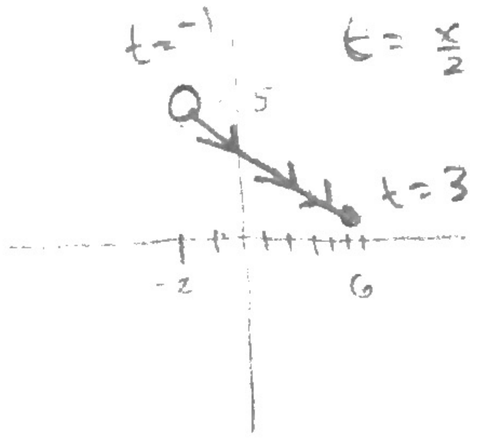
11–R Parametric Equations

Review

1. **Graphing Parametric Equations:** Graph the following parametric equations for the given domain of t (*label starting point and arrows for orientation).* Then, transform into Cartesian equations and state domain & range.

a. $x = 2t$, $y = -t + 4$, $t \in (-1, 3]$

t	-1	0	1	2	3
x	-2	0	2	4	6
y	5	4	3	2	1



$t = \frac{x}{2}$

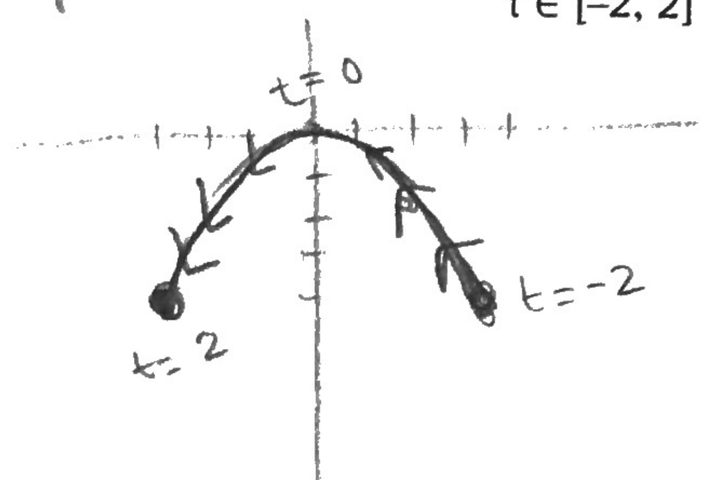
$y = -\frac{1}{2}x + 4$

$x \in (-2, 6]$

$y \in [1, 5)$

b. $x = -2t$, $y = -t^2$, $t \in [-2, 2]$

t	-2	-1	0	1	2
x	4	2	0	-2	-4
y	-4	-1	0	-1	-4



$t = -\frac{1}{2}x$

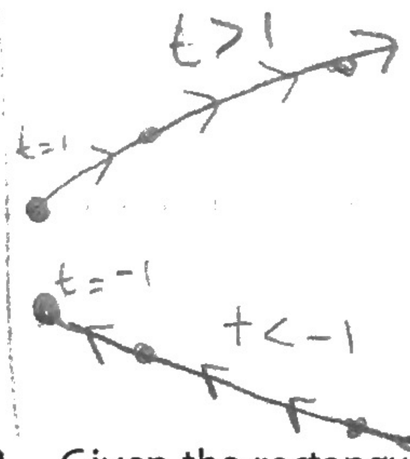
$y = -\left(-\frac{1}{2}x\right)^2$

$y = -\frac{1}{4}x^2$

$x \in [-4, 4]$ $y \in [-4, 0]$

c. $x = t^2$, $y = 2t - 2$, $t \le -1$ or $t \ge 1$

t	-3	-2	-1	0	1	2	3
x	9	4	1	0	1	4	9
y	-8	-6	-4	-2	0	2	4



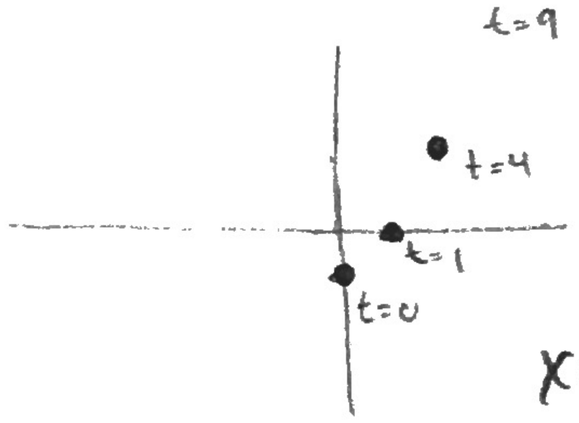
$t = \pm\sqrt{x}$

$y = \pm 2\sqrt{x} - 2$

$x \ge 1$ $y \le -4$ or $y \ge 0$

d. $x = \sqrt{t}$, $y = t - 1$, $t \in \{0, 1, 4, 9\}$

$t = x^2$	0	1	4	9
	0	1	2	3
t=9	-1	0	3	8



$y = x^2 - 1$

$x \in \{0, 1, 2, 3\}$

$y \in \{-1, 0, 3, 8\}$

2. Given the rectangular equation and the parameter, provide a set of parametric equations

.a. $y = 3x^2 + 4x - 2$

Parameter: $t = 2x$

$x = \frac{t}{2}$

$y = \frac{3}{4}t^2 + 2t - 2$

$x = \frac{t}{2}$

$3\left(\frac{t}{2}\right)^2 + 4\left(\frac{t}{2}\right) - 2$

b. $y = -2x + 3$

Parameter: $t = x + 1$

$x = t - 1$

$y = -2t + 5$

$x = t - 1$

$y = -2(t - 1) + 3$

3. A butterfly flies into the air from the top of a 6–inch flower with a horizontal speed of 0.8 ft/sec and vertical speed 0.6 ft/sec.

a. Write a set of parametric equations for the location of the butterfly:

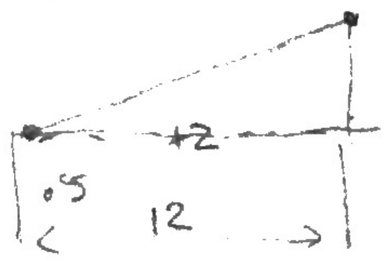
$x = .8t$

$y = .6t + .5$

b. What is the altitude of the butterfly after 40 seconds?

24.5 ft

c. How long would it take her to fly directly above another flower that is 12 feet away?



$x = .8t$

$12 = .8t$

15 seconds

4. a. Write a set of parametric equations for circle whose center is (4, 3) with a radius of 5 and rotates clockwise.

$x = 5\sin(t) + 4$

$y = 5\cos(t) + 3$

b. An ellipse with vertices (2, 10) & (2, 0) and minor axis length 6.

$a = 5$ $\quad b = 3$

$C: (2, 5)$

$x = 3\cos(t) + 2$ ✓ can switch sin & cos

$y = 5\sin(t) + 5$

("rotates" counter clockwise!)

c. A parabola that opens to the right that has a vertex at (5, –2)

$x = \sin^2(t) + 5$

$y = \sin(t) - 2$

5. For each problem, a. State what type of conic it is and its direction b. Sketch including orientation c. Give the equation in rectangular form d. state domain and range. (not for hyp.)

$t=0$ $t=90$ $t=180$ $t=270$

(3, -3) (5, -5) (3, -7) (1, -5)

$\begin{cases} x = 5\cos(t) - 2 \\ y = 6\sin(t) \end{cases}$

Ellipse, vertical

$\frac{(x+2)^2}{25} + \frac{y^2}{36} = 1$

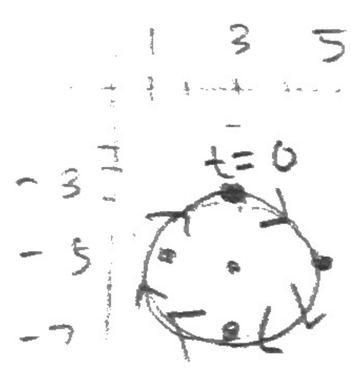
$x \in [-7, 3]$

$y \in [-6, 6]$

$\begin{cases} x = 3 - 2\sin(-t) \\ y = 2\cos(t) - 5 \end{cases}$

Circle

$(x-3)^2 + (y+5)^2 = 4$

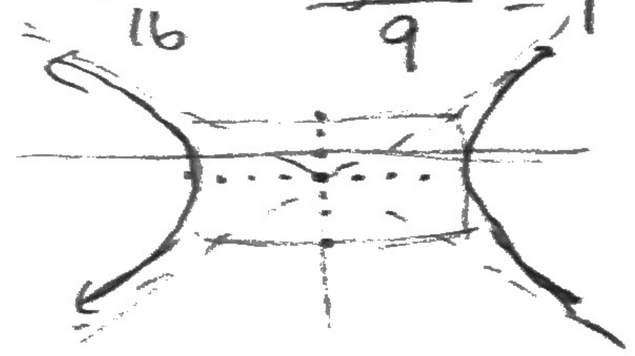


$x \in [1, 5]$ $\quad y \in [-7, -3]$

$\begin{cases} x = 4\sec(t) \\ y = 3\tan(t) - 1 \end{cases}$

Hyperbola, horizontal

$\frac{x^2}{16} - \frac{(y+1)^2}{9} = 1$



$x \leq 4$ or $x \geq 4$

$y \in \mathbb{R}$

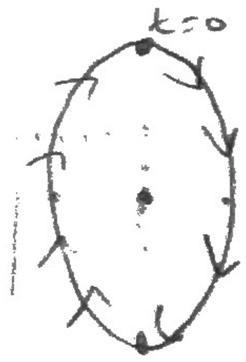
6. For each problem, a. State what type of conic it is,and its direction b. Sketch including orientation c. give the equation in parametric form (if it's a circle or ellipse state orientation). d. state domain and range. (not the hyp.)

Vert. Ellipse

$\frac{(x-5)^2}{16} + \frac{(y+2)^2}{18} = 1$

$x = 4\sin t + 5$

$y = 3\sqrt{2}\cos t - 2$



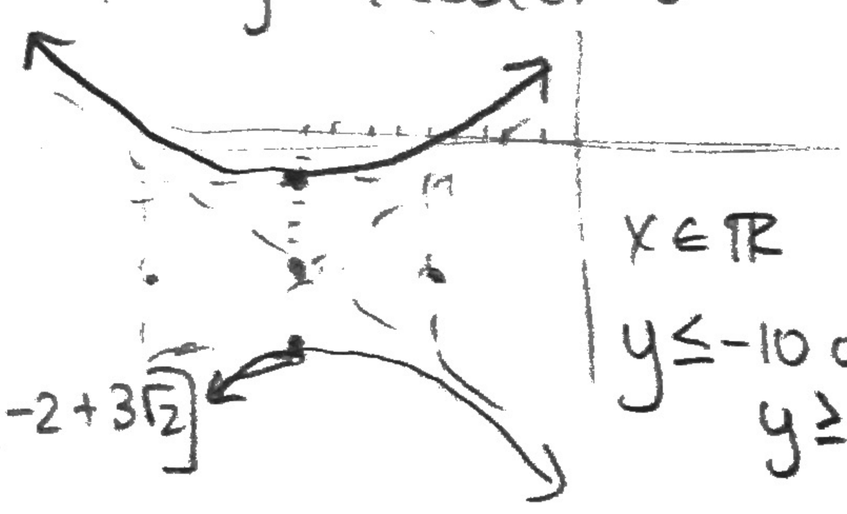
$x \in [1, 9]$ $\quad y \in [-2-3\sqrt{2}, -2+3\sqrt{2}]$

Vert. Hyperbola

$\frac{(y+6)^2}{16} - \frac{(x+9)^2}{25} = 1$

$x = 5\cot(t) - 9$

$y = 4\csc(t) - 6$



$x \in \mathbb{R}$

$y \leq -10$ or $y \geq -2$

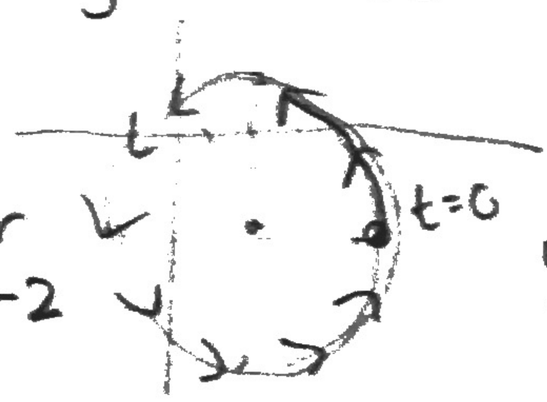
Circle

$x^2 + y^2 - 4x + 6y - 12 = 0$

$(x^2 - 4x + 4) + y^2 + 6y + 9) = 12 + 4 + 9$

$x = 5\cos t + 2$

$y = 5\sin(t) - 3$



$x \in [-3, 7]$

$y \in [-8, 2]$