Name: ______________________ Per: ________ Date: ______________

Serafino ▪ Precalculus S2 M T W R F

1B:2 The Ambiguous Case of the LOS (**ASS**–UME shady info)

Notes & Practice problems

When given ∠A a b: When asked to solve for a triangle given the information about an angle, the opposite (corresponding) side, and another side, you have "ASS" or "SSA" information … and you have to think about the number of triangles you could make with that info.

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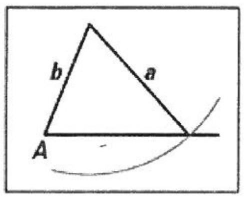

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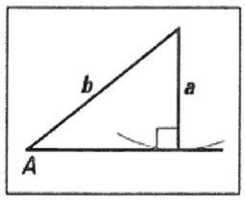

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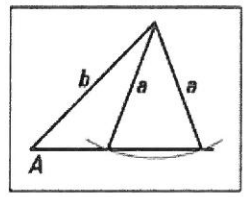

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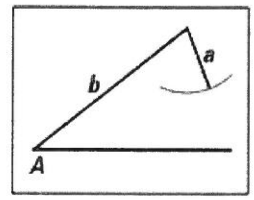


1. $a = h$ scenario ________ # △s possible: ________

   Why: ____

2. $h < b \le a$ scenario ________ # △s possible: ________

   Why: ____

3. $a < h$ scenario ________ # △s possible: ________

   Why: ____

4. $h < a < b$ scenario ________ # △s possible: ________

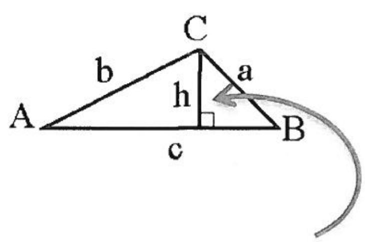
   Why: ____

☆ Two super, amazing, error–reducing, mind–blowing, time–saving fun facts:

#1: Because the height (h) of the triangle is critical, we have a shortcut to get it.

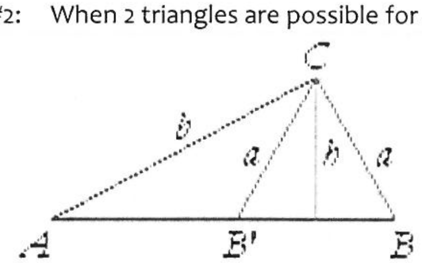
$\sin A = \frac{h}{b}$

so….



Height of the △ is : ______________________

#2: When 2 triangles are possible for △ ABC…



∠B and ∠B' are always ______________________

Do not attempt to "memorize" the rules on the front page. Math isn't about memorizing; it's about understanding.

1. **How many triangles?** Draw and determine how many triangles are possible from the info. Explain your reasoning.

   a. $A = 115°$, $a = 20$ and $b = 11$ #△s ______

   Why: ______________________________

   c. $A = 115°$, $a = 11$ and $b = 20$ #△s ________

   Why: ______________________________

   b. $A = 40°$, $a = 13$, and $b = 16$ #△s ________

   Why: ______________________________

   d. $A = 51°$, $a = 3.5$, and $b = 5$ #△s ________

   Why: ______________________________

2. **Solve any possible triangle(s) from #1:** Solve the triangles when one or two were possible, above.

   a. Problem ______ ; △ABC:

   A = ________ a = ________

   B = ________ b = ________

   C = ________ c = ________

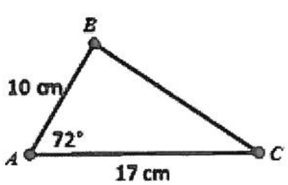
   b. Problem _______ ; △ABC and △AB'C'

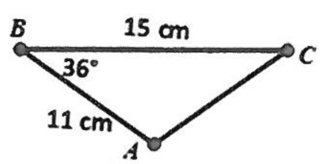
   A = ________ a = ________ A = ________ a = ________

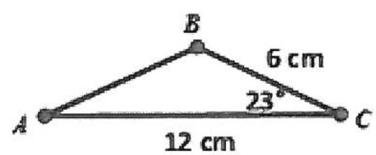
   B = ________ b = ________ B' = ________ b = ________

   C = ________ c = ________ C' = ________ c' = ________

3. **Find the area of the triangles.** You pick the "base" – calculating the height is easy now that you have a shortcut.







   a. ________________ b. ________________ c. ________________

☆ Making formulas is easy! Use what you did above to write a general formula to find the area of any oblique triangle ☺

4. **On a separate paper, solve the triangle(s) using the given information.** Remember, only SSA info is ambiguous. Otherwise, you may use the Law of Sines with confidence and reckless abandon ☺.

   a. △WHO: $W = 50°$, $w = 2.8$, $h = 4$
   b. △LVS: $L = 36°$, $l = 9$, $v = 12$
   c. △INA: $I = 51°$, $N = 44°$, $a = 11$
   d. △PYN: $Y = 105°$, $y = 13$, $p = 6$
   e. △APL: $A = 76.4°$, $a = 176$, $p = 189$
   f. △UND: $U = 48.2°$, $u = 15$, $n = 20$
   g. △DER: $D = 20°$, $d = 10$, $e = 11$
   h. △THE: $H = 95°$, $h = 8$, $t = 9$
   i. △SEA: $S = 14°$, $E = 50°$, $s = 10$

Name: Answer key Per: 3, 7 Date: 10/8 M T W R F

Serafino ▪ Precalculus S2

1B:2 The Ambiguous Case of the LOS (**ASS**–UME shady info)

Notes & Practice problems

When given ∠A a b: When asked to solve for a triangle given the information about an angle, the opposite (corresponding) side, and another side, you have "ASS" or "SSA" information … and you have to think about the number of triangles you could make with that info.

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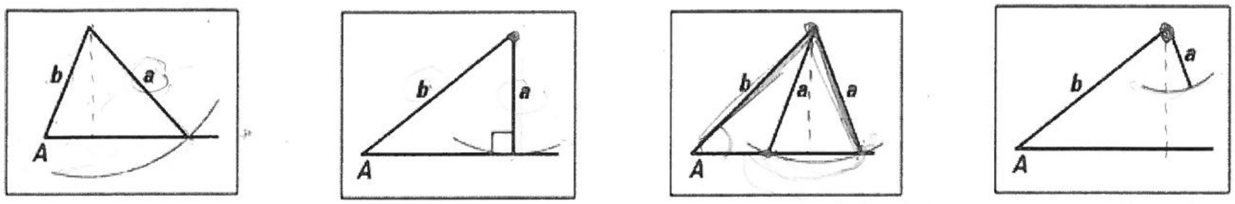
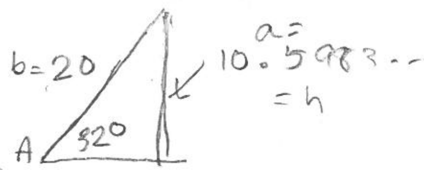


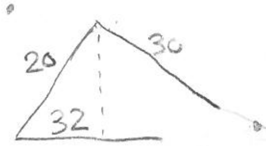
1. a = h scenario B # △s possible: 1

   Why: It "snaps" into place, can't swing or it won't reach base.



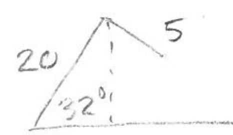
2. h < b ≤ a scenario A # △s possible: 1

   Why: too big to swing in



3. a < h scenario D # △s possible: 0

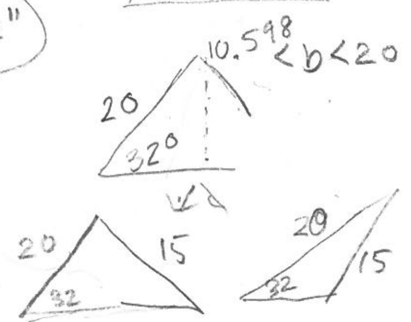
   Why: too short! Dangles! "Do or do not… there is no tri-angle"



4. h < a < b scenario C # △s possible: 2

   a > h and a < b

   Why: It's long enough to reach just the base, and short enough to tuck in next to ∠A.

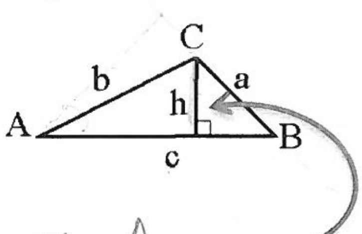


☆ Two super, amazing, error–reducing, mind–blowing, time–saving fun facts:

#1: Because the height (h) of the triangle is critical, we have a shortcut to get it.

$\sin A = \frac{h}{b}$

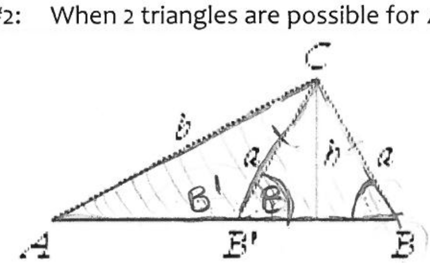
so….



Height of the △ is: b sin A

a sin B

#2: When 2 triangles are possible for △ ABC…



∠B and ∠B' are always supplementary!

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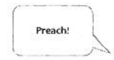


1. **How many triangles?** Draw and determine how many triangles are possible from the info. Explain your reasoning.

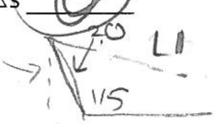
a. A = 115°, a = 20 and b = 11 #△s 1

h = 9.9694

Why: long enough to connect, too long to swing... plus A would change!

c. A = 115°, a = 11 and b = 20 #△s 0

18.1



Why: A is obtuse so a has to be largest

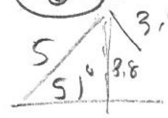
b. A = 40°, a = 13, and b = 16 #△s 2

h = 10.2846

Why: long enough to reach past, but can snap in two places

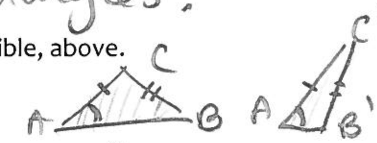
d. A = 51°, a = 3.5, and b = 5 #△s 0

h = 3.8



Why: too short! dangles!

2. **Solve any possible triangle(s) from #1:** Solve the triangles when one or two were possible, above.



a. Problem a ; △ABC:

use original into whenever possible

A = 115° a = 20

B = 29.6988° b = 11

C = 35.1012° c = 12.6893

$\frac{\sin 115}{20} = \frac{\sin B}{11}$ $\left(\frac{11\sin 115}{20}\right)$ $\sin^{-1}(.4984\ldots)$

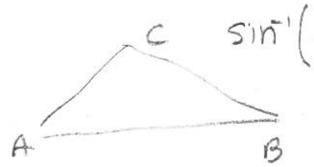
b. Problem b ; △ABC and △AB'C'

A = 40° a = 13

B = 52.2906 b = 16

C = 87.7094° c = 20.2083

$\sin^{-1}\left(\frac{16\sin 40}{13}\right)$



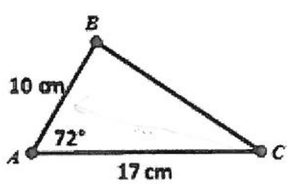
A = 40° a = 13

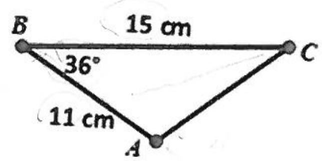
B' = 127.7094° b = 16

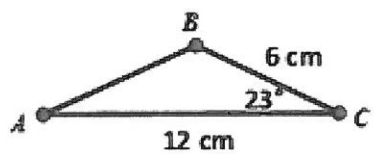
C' = 12.2906° c' = 4.3052

$\frac{\sin 40}{13} = \frac{\sin 12.2906}{c}$

3. **Find the area of the triangles.** You pick the "base" – calculating the height is easy now that you have a shortcut.







a. $\frac{1}{2}bh = \frac{1}{2}(17)(10\sin 72)$ $= 80.8398\text{ cm}^2$

b. $= \frac{1}{2}(15)(11\sin 36)$ $= 48.4923\text{ cm}^2$

c. $\frac{1}{2}(12)6\sin 23$ $= 14.066\text{ cm}^2$

☆ Making formulas is easy! Use what you did above to write a general formula to find the area of any oblique triangle ☺

$= \frac{1}{2}bc\sin A$ $= \frac{1}{2}ac\sin B$ $= \frac{1}{2}ba\sin C$

4. **On a separate paper, solve the triangle(s) using the given information.** Remember, only SSA info is ambiguous. Otherwise, you may use the Law of Sines with confidence and reckless abandon ☺.

a. △WHO: W = 50°, w = 2.8, h = 4
b. △LVS: L = 36°, l = 9, v = 12
c. △INA: I = 51°, N = 44°, a = 11
d. △PYN: Y = 105°, y = 13, p = 6
e. △APL: A = 76.4°, a = 176, p = 189
f. △UND: U = 48.2°, u = 15, n = 20
g. △DER: D = 20°, d = 10, e = 11
h. △THE: H = 95°, h = 8, t = 9
i. △SEA: S = 14°, E = 50°, s = 10

(see next page)

7B Worksheet : practice problems (#4)

a. △WHO : not possible (w is too short → dangles)

b. △LVS

L = 36° l = 9
V = 51.6019° v = 12
S = 92.3981° s = 15.29

△LV'S'

L = 36° l = 9
V' = 128.3981° v = 12
S' = 15.6019° s = 4.1181

c. △INA (not ambiguous case!)

I = 51° i = 8.5813
N = 44° n = 7.6704
A = 85° a = 11

d. △PYN

P = 26.4753° p = 6
Y = 105° y = 13
N = 48.5247° n = 10.0837

e. △APL : not possible (a is too short)

f. △UND :

U = 48.2° u = 15
N = 83.7037° n = 20
D = 48.0963° d = 14.9795

△UN'D'

U = 48.2° u = 15
N = 96.2963° n = 20
D = 35.5037° d = 11.6856

g. △DER

A D = 20° d = 10
B E = 22.0998° e = 11
C R = 137.9001° r = 19.6019

△DE'R'

D = 20° d = 10
E = 157.9° e = 11
R = 2.0999° r = 1.0713

h. $\triangle THE$ (not possible. B is obtuse so b is too short!)

i. $\triangle SEA$ (not ambiguous ☺)

S =	14°	s =	10
E =	50°	e =	37.1522
A =	116°	a =	131.6649