CHAPTER 2 Right Triangle Trigonometry

Problem Set 2.1

1.

$a=\sqrt{c^2-b^2}$ Pythagorean Theorem

$=\sqrt{(5)^2-(3)^2}$ Substitute known values

$=\sqrt{25-9}$ Simplify

$=\sqrt{16}=4$

$\sin A=\frac{a}{c}=\frac{4}{5}$ $\cot A=\frac{b}{a}=\frac{3}{4}$

$\cos A=\frac{b}{c}=\frac{3}{5}$ $\sec A-\frac{c}{b}=\frac{5}{3}$

$\tan A=\frac{a}{b}=\frac{4}{3}$ $\csc A=\frac{c}{a}=\frac{5}{4}$

3.

$c=\sqrt{a^2+b^2}$ Pythagorean Theorem

$=\sqrt{(2)^2+(1)^2}$ Substitute known values

$=\sqrt{4+1}$ Simplify

$=\sqrt{5}$

$\sin A=\frac{a}{c}=\frac{2}{\sqrt{5}}$ $\cot A=\frac{b}{a}=\frac{1}{2}$

$\cos A=\frac{b}{c}=\frac{1}{\sqrt{5}}$ $\sec A=\frac{c}{b}=\frac{\sqrt{5}}{1}=\sqrt{5}$

$\tan A=\frac{a}{b}=\frac{2}{1}=2$ $\csc A=\frac{c}{a}=\frac{\sqrt{5}}{2}$

5.

$c=\sqrt{a^2+b^2}$ Pythagorean Theorem

$=\sqrt{(2)^2+\left(\sqrt{5}\right)^2}$ Substitute known values

$=\sqrt{4+5}$ Simplify

$=\sqrt{9}=3$

$\sin A=\frac{a}{c}=\frac{2}{3}$ $\cot A=\frac{b}{a}=\frac{\sqrt{5}}{2}$

$\cos A=\frac{b}{c}=\frac{\sqrt{5}}{3}$ $\sec A=\frac{c}{b}=\frac{3}{\sqrt{5}}$

$\tan A=\frac{a}{b}=\frac{2}{\sqrt{5}}$ $\csc A=\frac{c}{a}=\frac{3}{2}$

7.

$b=\sqrt{c^2-a^2}$ Pythagorean Theorem

$=\sqrt{(6)^2-(5)^2}$ Substitute known values

$=\sqrt{36-25}$ Simplify

$=\sqrt{11}$

$\sin A=\frac{a}{c}=\frac{5}{6}$ $\sin B=\frac{b}{c}=\frac{\sqrt{11}}{6}$

$\cos A=\frac{b}{c}=\frac{\sqrt{11}}{6}$ $\cos B=\frac{a}{c}=\frac{5}{6}$

$\tan A=\frac{a}{b}=\frac{5}{\sqrt{11}}$ $\tan B=\frac{b}{a}=\frac{\sqrt{11}}{5}$

9.

$c=\sqrt{a^2+b^2}$ Pythagorean Theorem

$=\sqrt{(1)^2+(1)^2}$ Substitute known values

$=\sqrt{1+1}$ Simplify

$=\sqrt{2}$

$\sin A=\frac{a}{c}=\frac{1}{\sqrt{2}}$ $\sin B=\frac{b}{c}=\frac{1}{\sqrt{2}}$

$\cos A=\frac{b}{c}=\frac{1}{\sqrt{2}}$ $\cos B=\frac{a}{c}=\frac{1}{\sqrt{2}}$

$\tan A=\frac{a}{b}=\frac{1}{1}=1$ $\tan B=\frac{b}{a}=\frac{1}{1}=1$

11. $b=\sqrt{c^2-a^2}$ Pythagorean Theorem

$=\sqrt{10^2-6^2}$ Substitute known values

$=\sqrt{100-36}$ Simplify

$=\sqrt{64}=8$

$\sin A=\frac{a}{c}=\frac{6}{10}=\frac{3}{5}$ $\sin B=\frac{b}{c}=\frac{8}{10}=\frac{4}{5}$

$\cos A=\frac{b}{c}=\frac{8}{10}=\frac{4}{5}$ $\cos B=\frac{a}{c}=\frac{6}{10}=\frac{3}{5}$

$\tan A=\frac{a}{b}=\frac{6}{8}=\frac{3}{4}$ $\tan B=\frac{b}{a}=\frac{8}{6}=\frac{4}{3}$

13. $a=\sqrt{c^2-b^2}$ Pythagorean Theorem

$=\sqrt{(2x)^2-(x)^2}$ Substitute known values

$=\sqrt{4x^2-x^2}$ Simplify

$=\sqrt{3x^2}$

$=x\sqrt{3}$

$\sin A=\frac{a}{c}=\frac{x\sqrt{3}}{2x}=\frac{\sqrt{3}}{2}$ $\sin B=\frac{b}{c}=\frac{x}{2x}=\frac{1}{2}$

$\cos A=\frac{b}{c}=\frac{x}{2x}=\frac{1}{2}$ $\cos B=\frac{a}{c}=\frac{x\sqrt{3}}{2x}=\frac{\sqrt{3}}{2}$

$\tan A=\frac{a}{b}=\frac{x\sqrt{3}}{x}=\sqrt{3}$ $\tan B=\frac{b}{a}=\frac{x}{x\sqrt{3}}=\frac{1}{\sqrt{3}}$

15. The coordinates of B are (4, 3).

$a=3,\quad b=4,\quad c=5$

$\sin A=\frac{a}{c}=\frac{3}{5}$

$\cos A=\frac{b}{c}=\frac{4}{5}$

$\tan A=\frac{a}{b}=\frac{3}{4}$

17. $\sin 10^\circ=\cos(90^\circ-10^\circ)=\cos 80^\circ$

19. $\tan 8^\circ=\cot(90^\circ-8^\circ)=\cot 82^\circ$

21. $\sin x^\circ=\cos(90^\circ-x^\circ)$

23. $\tan(90^\circ-x^\circ)=\cot x^\circ$

25. $\csc x=\frac{1}{\sin x}$

$\csc 0^\circ=\frac{1}{0}$ undefined

$\csc 30^\circ=\frac{1}{1/2}=2$

$\csc 45^\circ=\frac{1}{1/\sqrt{2}}=\sqrt{2}$

$\csc 60^\circ=\frac{1}{\sqrt{3}/2}=\frac{2}{\sqrt{3}}$

$\csc 90^\circ=\frac{1}{1}=1$

27. $4\sin 30^\circ=4\left(\frac{1}{2}\right)=2$

29. $\left(2\cos 30^\circ\right)^2 = \left[2\left(\frac{\sqrt{3}}{2}\right)\right]^2 = \left(\sqrt{3}\right)^2 = 3$

31.
$$\begin{aligned}\left(\sin 60^\circ + \cos 60^\circ\right)^2 &= \left(\frac{\sqrt{3}}{2}+\frac{1}{2}\right)^2\\ &= \left(\frac{\sqrt{3}+1}{2}\right)^2\\ &= \frac{\left(\sqrt{3}+1\right)\left(\sqrt{3}+1\right)}{4}\\ &= \frac{3+2\sqrt{3}+1}{4}\\ &= \frac{4+2\sqrt{3}}{4}\\ &= \frac{2\left(2+\sqrt{3}\right)}{4} = \frac{2+\sqrt{3}}{2}\end{aligned}$$

33.
$$\begin{aligned}\sin^2 45^\circ - 2\sin 45^\circ \cos 45^\circ + \cos^2 45^\circ &= \left(\frac{\sqrt{2}}{2}\right)^2 - 2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)^2\\ &= \frac{2}{4} - 2\left(\frac{2}{4}\right) + \frac{2}{4}\\ &= \frac{1}{2} - 1 + \frac{1}{2} = 0\end{aligned}$$

35.
$$\begin{aligned}\left(\tan 45^\circ + \tan 60^\circ\right)^2 &= \left(1+\sqrt{3}\right)^2\\ &= \left(1+\sqrt{3}\right)\left(1+\sqrt{3}\right)\\ &= 1+2\sqrt{3}+3\\ &= 4+2\sqrt{3}\end{aligned}$$

37.
$$\begin{aligned}2\sin 30^\circ &= 2\left(\frac{1}{2}\right)\\ &= 1\end{aligned}$$

39.
$$\begin{aligned}4\cos\left(z-30^\circ\right) &= 4\cos\left(60^\circ - 30^\circ\right)\\ &= 4\cos 30^\circ\\ &= 4\left(\frac{\sqrt{3}}{2}\right) = 2\sqrt{3}\end{aligned}$$

41. $-3\sin 2(30^\circ) = -3\sin 60^\circ$

$= -3\left(\frac{\sqrt{3}}{2}\right)$

$= -\frac{3\sqrt{3}}{2}$

43. $2\cos(3x - 45^\circ) = 2\cos(3 \cdot 30^\circ - 45^\circ)$

$= 2\cos(90^\circ - 45^\circ)$

$= 2\cos 45^\circ$

$= 2 \cdot \frac{\sqrt{2}}{2} = \sqrt{2}$

45.

$\sec 30^\circ = \frac{1}{\cos 30^\circ}$	Reciprocal identity
$= \frac{1}{\sqrt{3}/2}$	Substitute exact value from Table 1
$= \frac{2}{\sqrt{3}}$	Division of fractions

47. $\csc 60^\circ = \frac{1}{\sin 60^\circ}$

$= \frac{1}{\sqrt{3}/2}$

$= \frac{2}{\sqrt{3}}$

49.

$\cot 45^\circ = \frac{\cos 45^\circ}{\sin 45^\circ}$	Ratio identity
$= \frac{\sqrt{2}/2}{\sqrt{2}/2}$	Substitute values from Table 1
$= 1$	Simplify

51. $\sec 45^\circ = \frac{1}{\cos 45^\circ}$

$= \frac{1}{1/\sqrt{2}}$

$= \sqrt{2}$

53. $b = \sqrt{c^2 - a^2}$

$= \sqrt{(5.70)^2 - (3.42)^2}$

$= \sqrt{20.7936}$

$= 4.56$

$\sin A = \frac{a}{c} = \frac{3.42}{5.70} = 0.60$

$\cos A = \frac{b}{c} = \frac{4.56}{5.70} = 0.80$

$\sin B = \frac{b}{c} = \frac{4.56}{5.70} = 0.80$

$\cos B = \frac{a}{c} = \frac{3.42}{5.70} = 0.60$

55.

$$c=\sqrt{a^2+b^2}$$
$$=\sqrt{(19.44)^2+(5.67)^2}$$
$$=\sqrt{410.0625}$$
$$=20.25$$

$$\sin A=\frac{a}{c}=\frac{19.44}{20.25}=0.96$$
$$\cos A=\frac{b}{c}=\frac{5.67}{20.25}=0.28$$
$$\sin B=\frac{b}{c}=\frac{5.67}{20.25}=0.28$$
$$\cos B=\frac{a}{c}=\frac{19.44}{20.25}=0.96$$

57.

$$CH=\sqrt{(CD^2)+(DH)^2}$$
$$=\sqrt{5^2+5^2}$$
$$=\sqrt{25+25}$$
$$=\sqrt{50}=5\sqrt{2}$$

$$CF=\sqrt{(CH)^2+(FH)^2}$$
$$=\sqrt{(5\sqrt{2})^2+(5)^2}$$
$$=\sqrt{50+25}$$
$$=\sqrt{75}=5\sqrt{3}$$

$$\sin\theta=\frac{FH}{CF}$$
$$=\frac{5}{5\sqrt{3}}$$
$$=\frac{1}{\sqrt{3}}$$

$$\cos\theta=\frac{CH}{CF}$$
$$=\frac{5\sqrt{2}}{5\sqrt{3}}$$
$$=\frac{\sqrt{2}}{\sqrt{3}}\text{ or }\frac{\sqrt{6}}{3}$$

59.

$$CH=\sqrt{(CD^2)+(DH)^2}$$
$$=\sqrt{x^2+x^2}$$
$$=\sqrt{2x^2}$$
$$=x\sqrt{2}$$

$$CF=\sqrt{(CH)^2+(FH)^2}$$
$$=\sqrt{(x\sqrt{2})^2+x^2}$$
$$=\sqrt{2x^2+x^2}$$
$$=\sqrt{3x^2}=x\sqrt{3}$$

$$\sin\theta=\frac{FH}{CF}$$
$$=\frac{x}{x\sqrt{3}}$$
$$=\frac{1}{\sqrt{3}}$$

$$\cos\theta=\frac{CH}{CF}$$
$$=\frac{x\sqrt{2}}{x\sqrt{3}}$$
$$=\frac{\sqrt{2}}{\sqrt{3}}\text{ or }\frac{\sqrt{6}}{3}$$

61.

$$\begin{aligned} r &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} && \text{Distance formula} \\ &= \sqrt{(5-2)^2+(1-5)^2} && \text{Substitute known values} \\ &= \sqrt{(3)^2+(-4)^2} && \text{Simplify} \\ &= \sqrt{9+16} \\ &= \sqrt{25}=5 \end{aligned}$$

63.

$$\begin{aligned} r &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} && \text{Distance formula} \\ \sqrt{13} &= \sqrt{(x-1)^2+(2-5)^2} && \text{Substitute known values} \\ \sqrt{13} &= \sqrt{(x-1)^2+9} && \text{Simplify} \\ 13 &= (x-1)^2+9 && \text{Square both sides} \\ 4 &= (x-1)^2 && \text{Subtract 9 from both sides} \\ \pm 2 &= x-1 && \text{Solve using the Square Root Method} \\ x-1=2 &\text{ or } x-1=-2 \\ x=3 &\text{ or } x=-1 \end{aligned}$$

65. If $x=0$, then $y=2(0)-1$

$y=-1$

Therefore, the point $(0,-1)$ satisfies the equation.

If $x=2$, then $y=2(2)-1$

$y=4-1$

$y=3$

Therefore, the point (2, 3) satisfies the equation.

Plot the points $(0,-1)$ and (2, 3) and draw the line through these points.

67. The terminal side is the line $y=x$. Some points in quadrant I on the line $y=x$ are $(1,1)$, $(2,2)$, and $(3,3)$.

69. $-135^\circ+360^\circ=225^\circ$

71. $-300^\circ+360^\circ=60^\circ$