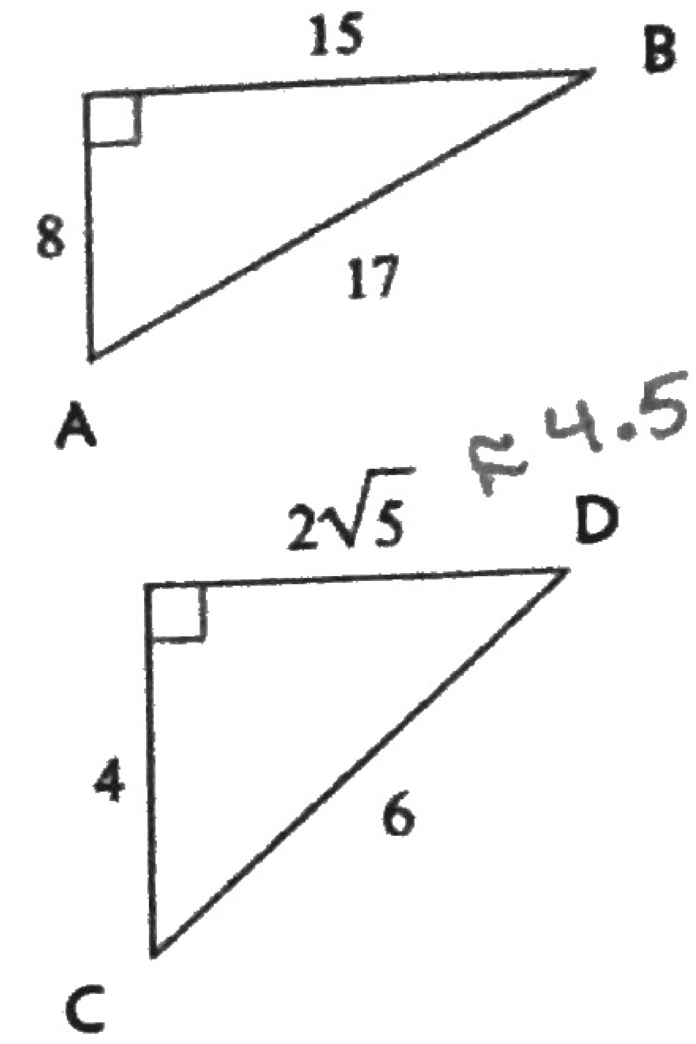
Name: Answer key Per: 3,7 Date: ____________
Serafino · Precalculus S2

2A2 Trig Ratios in the Coordinate Plane

Notes / Classwork



Evaluating Trig Ratios: (Geometry Review)
Use your knowledge of ratios to fill in the blanks. Then check.

sin B ≈0.471 cos A ≈0.471 tan B ≈.533

sin D ≈0.667 cos C ≈0.667 tan D ≈0.894

sin C ≈ 0.745 cos D ≈ 0.745 tan C ≈1.118

sin A ≈0.882 cos B ≈0.882 tan A ≈1.875

Cofunction Theorem: This demonstrates a convenient little shortcut about CO–FUNCTIONS (like sine and COsine)
The trig ratio of an angle is exactly the same as the co–function of its complement.

Examples: $\cos 70° = \sin 20°$; $\sin 40° = \cos 50°$; $\sin 36.8 = \cos 53.2°$; $\cos\theta = \sin(90-\theta)$

Using Inverses to Solve for Angles:
See if you can use your Trig Ratios table to estimate the angle before you use inverses to solve for it.

Using Chart:

m∠A ≈ 62° m∠B ≈ 28° m∠C ≈ 48° m∠D ≈ 42°

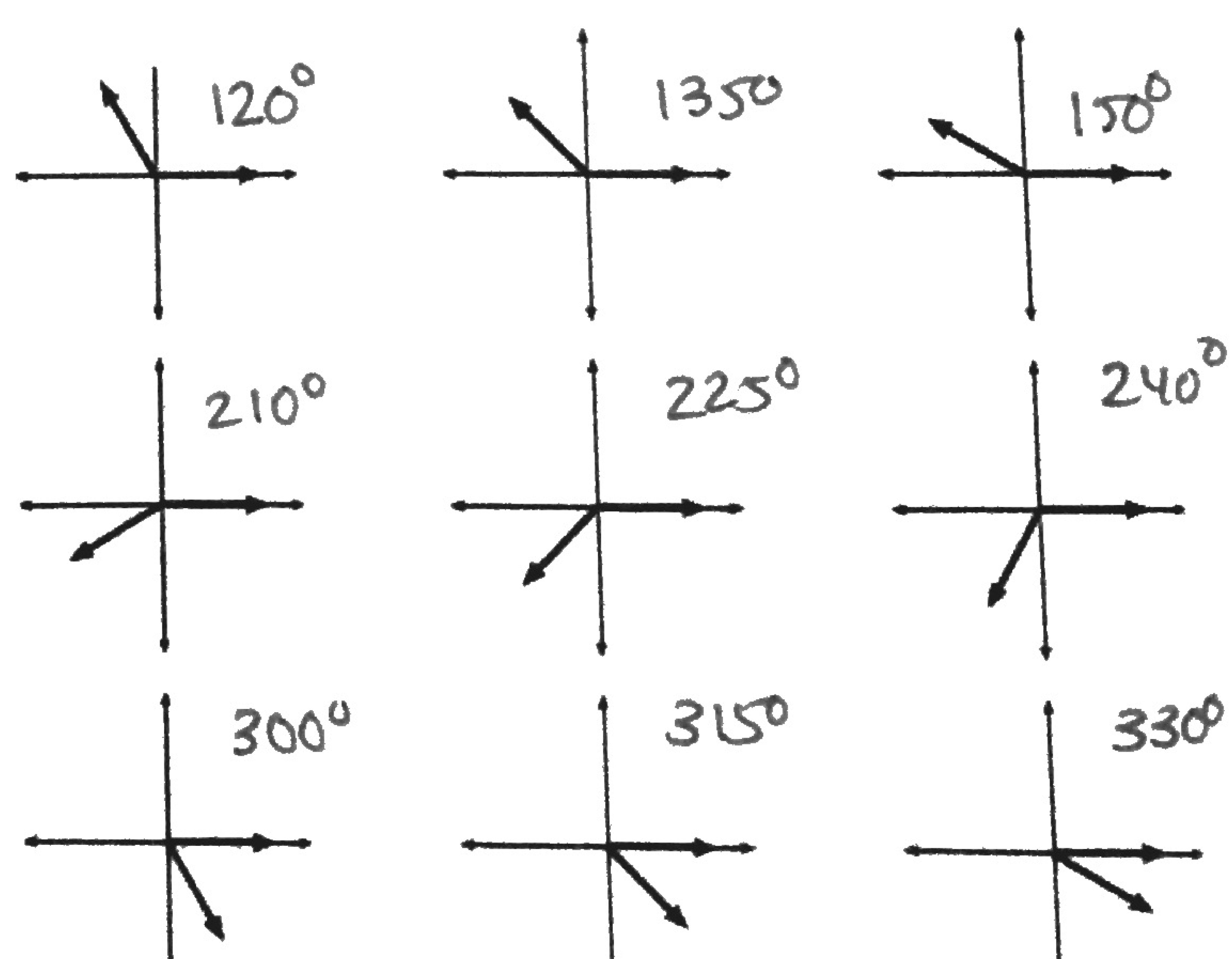
Using Inverses:

m∠A ≈ 61.9275° m∠B ≈ 28.0725° m∠C ≈ 48.1897° m∠D ≈ 41.8103°

Coterminal and Reference Angles: (9A1 Review)
Fill in the chart

Any θ Coterminal	Coterminal 0° ≤ θ < 360°	Quad	θ'
–130°	230°	III	50°
1,070°	350°	IV	10°
2720° –2320°	200°	III	20°
–1690°	110°	QII	70°
–3685° 4235°	275°	QIV	85°

Match each with the Special Angle: (9A1 Review)
150° 210° 330° 120° 240° 300° 135° 225° 315°

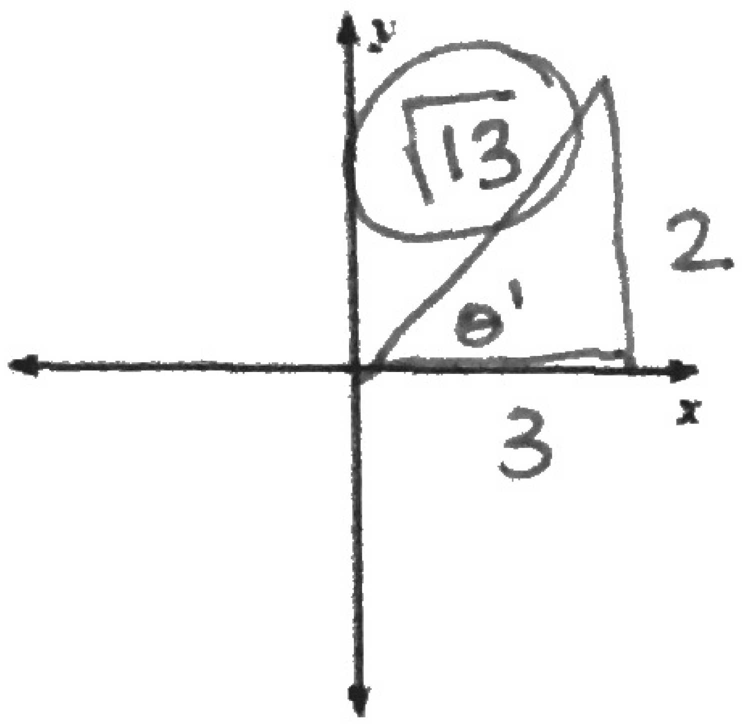


DRAWING A REFERENCE TRIANGE:

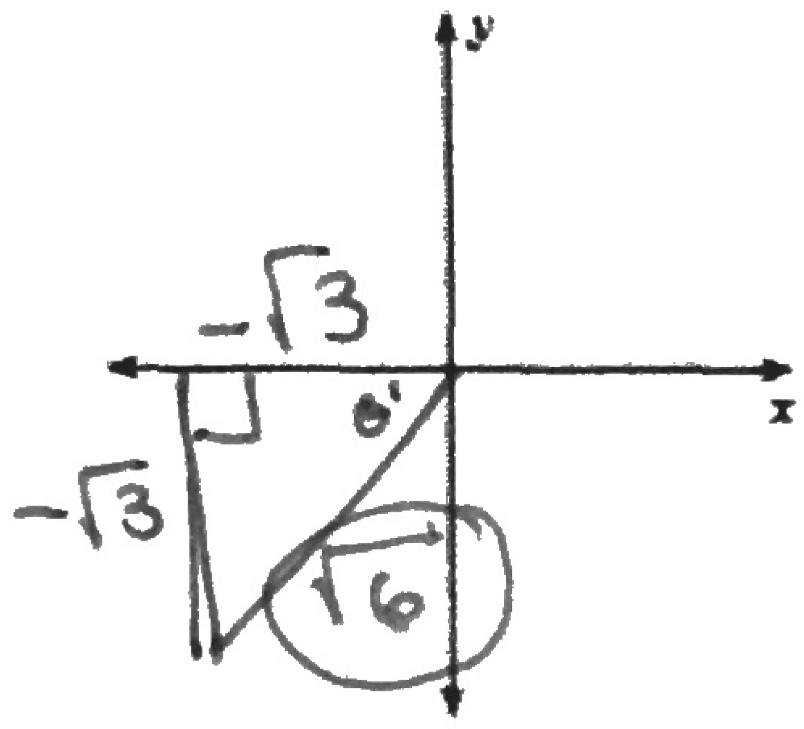
When you have an angle, θ, in standard position, the terminal side is going to intersect a bunch of points: (x, y)
If you pick a point and drop down a line perpendicular to the nearest x–axis, you will make a right triangle.
<u>X will be its adjacent side. Y will be its opposite side. The angle in the your triangle will be the reference angle, θ'</u>

For each point intersected by θ, draw the reference triangle containing θ' and find its hypotenuse/distance.

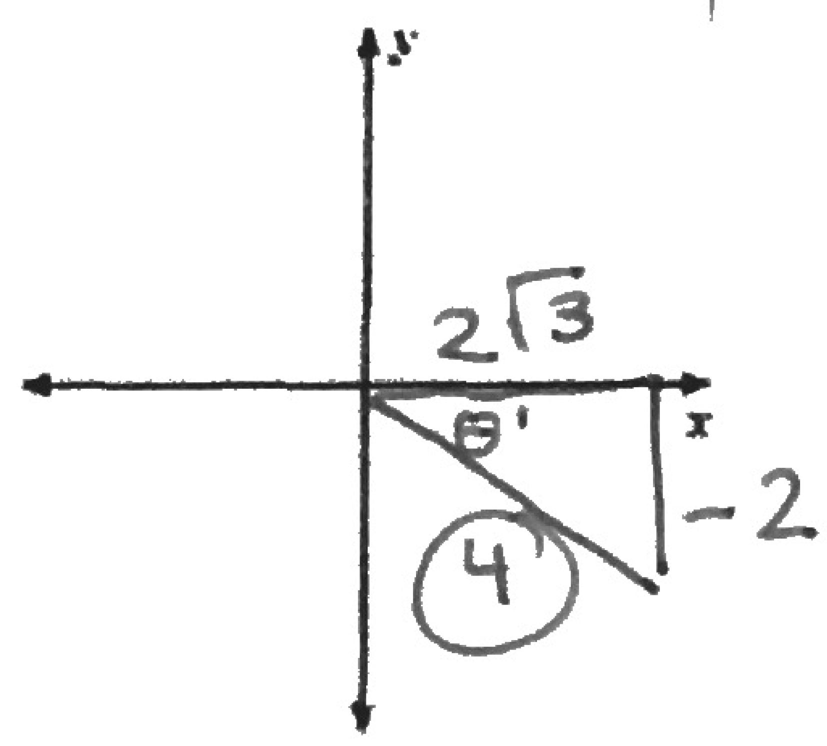
(3, 2)



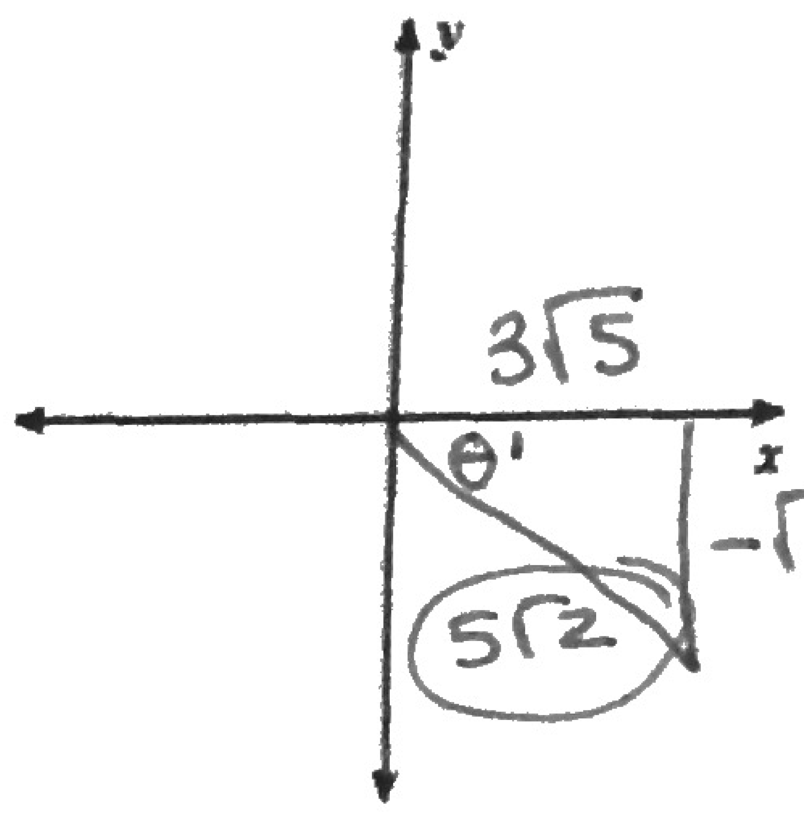
(−√3, −√3)



(2√3, −2)

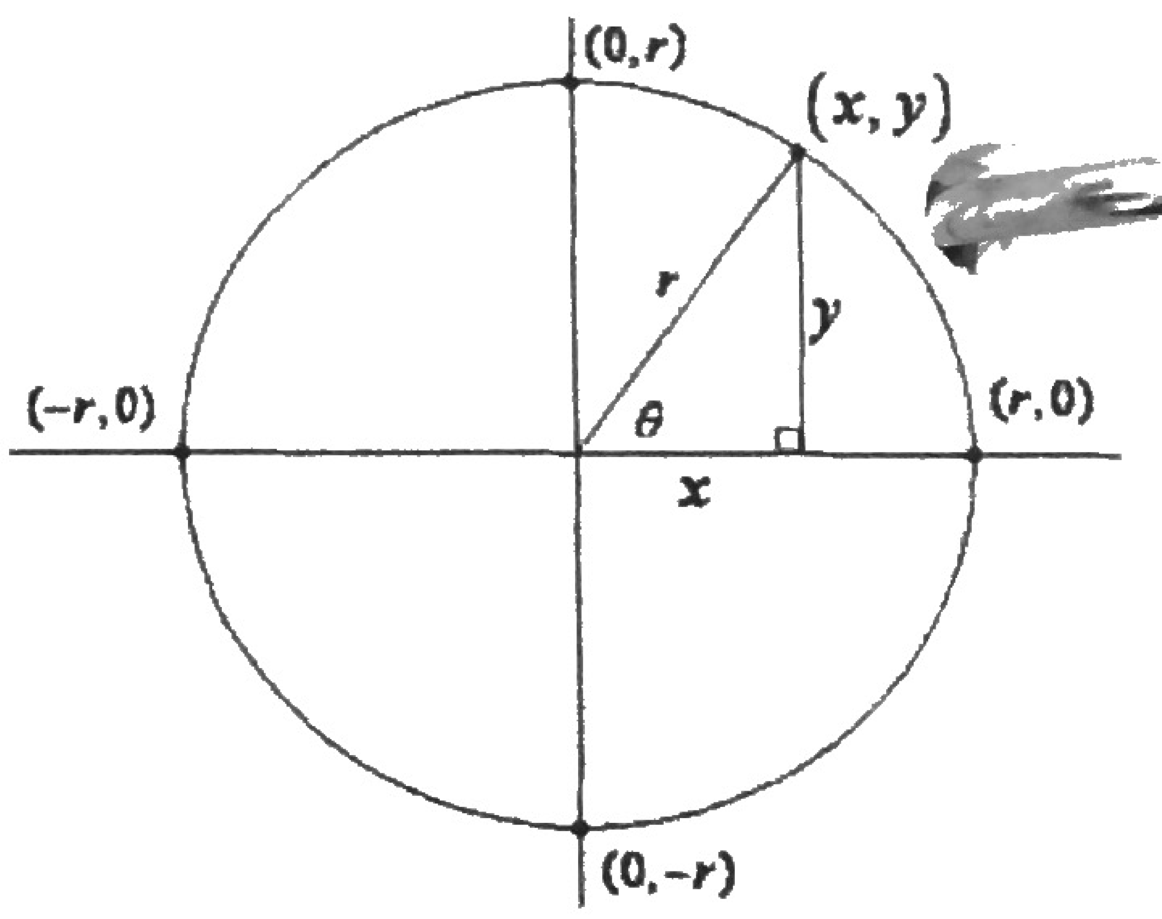


(3√5, −√5) $\sqrt{45+5}$



You can start to see that regardless of where θ terminates, to do trig, the only angle we need is θ'

Noticing connections...
This is the point in your math careers where you need to make connections between ALL the math courses you've taken, not just connections between topics in the same course.



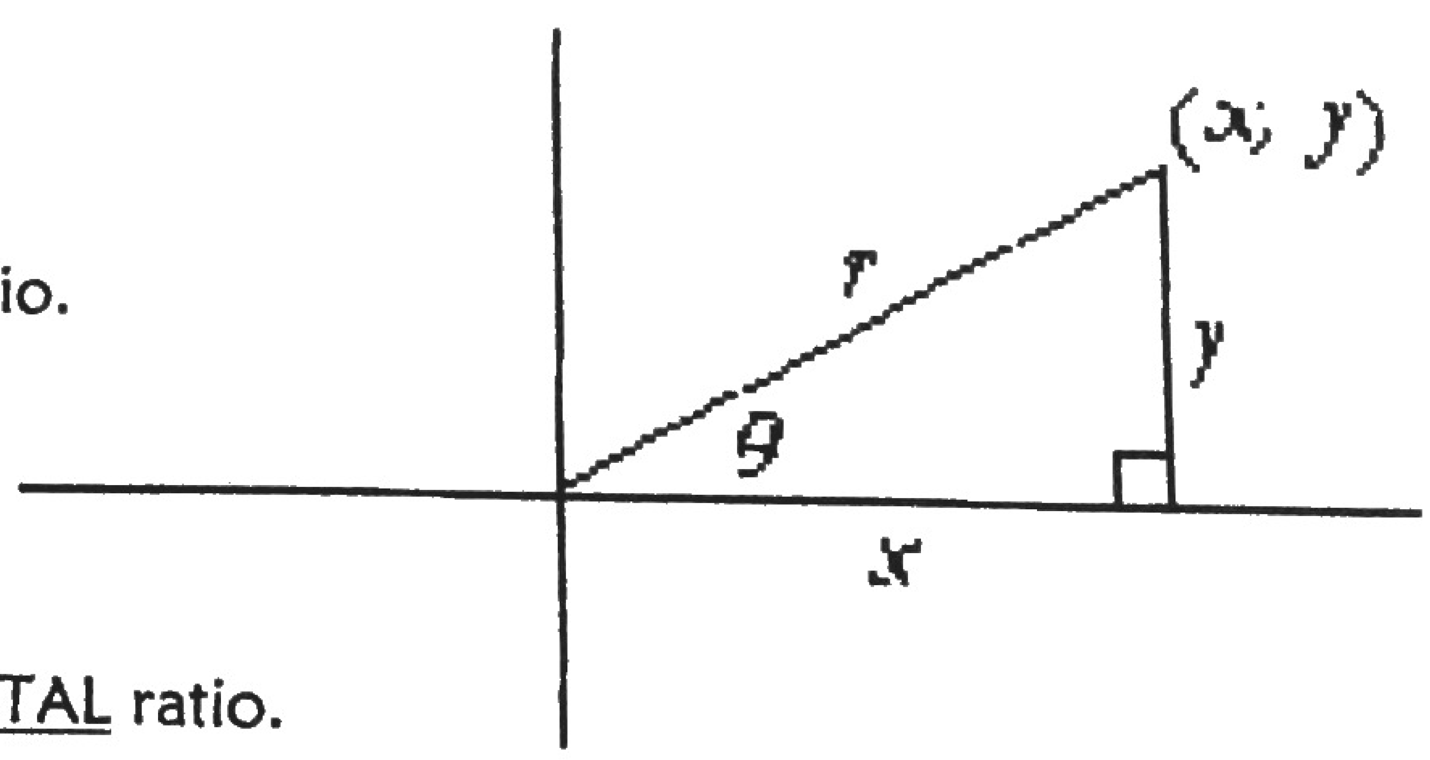
Geometry: <u>Pythagorean Theorem</u> is: $a^2 + b^2 = c^2$

Geometry: <u>Equation of a circle</u> with a center at the origin is: $x^2 + y^2 = r^2$
All radii of the circle are hypotenuses of all possible right triangles with legs x and y.
The length of the hypotenuse is its distance from the origin.

Algebra 1: The <u>distance formula</u> is $d = \sqrt{(\Delta x)^2 + (\Delta y)^2}$
Square both sides... you'll see it.

THE THREE TRIG RATIOS:

If θ intersects a point (x, y), write the trig ratios as ratios of what side lengths / distances they compare:



$\sin\theta = \frac{y}{r} = \frac{\text{vertical height}}{\text{distance from the origin}}$

Y goes with **SINE**, because it is <u>OPPOSITE</u> of θ'. It is a <u>VERTICAL</u> ratio.

$\cos\theta = \frac{x}{r} = \frac{\text{horizontal distance}}{\text{distance from the origin}}$

X goes with COSINE, because it is <u>ADJACENT</u> to θ'. It's a <u>HORIZONTAL</u> ratio.

$\tan\theta = \frac{y}{x} = \frac{\text{vertical height}}{\text{horizontal distance}} = \frac{\text{RISE}}{\text{RUN}}$

Y/X is TANGENT. In a way, a measurement of SLOPE of the line that contains θ (whoa!)

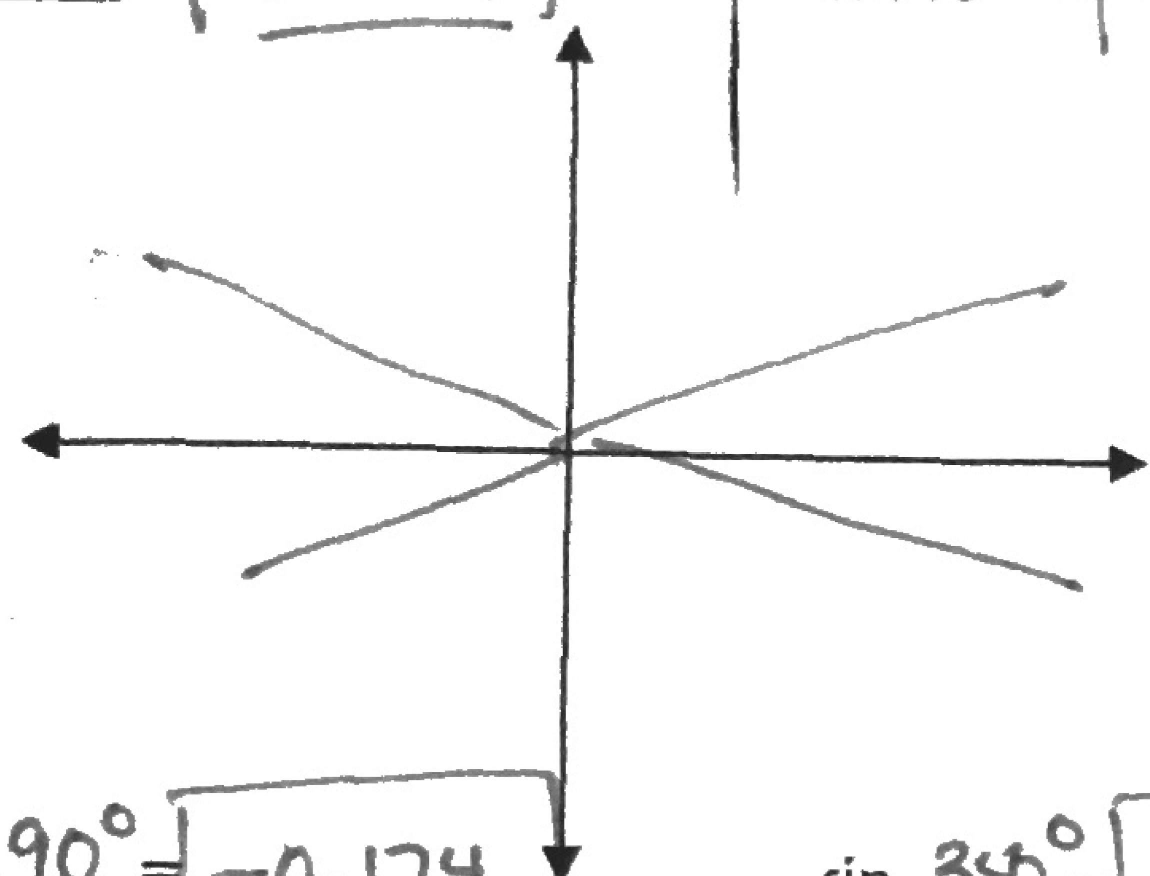
In another, more important way, TANGENT θ is actually SLOPE

Determining the Signs of the Trig Ratios:

Draw the same reference angle in each of the quadrants, then draw a reference triangles.
On the x and y axes, we use the sign (±) associated with that side of the axis. The point's distance is always positive.

SIN θ; θ' = 10°

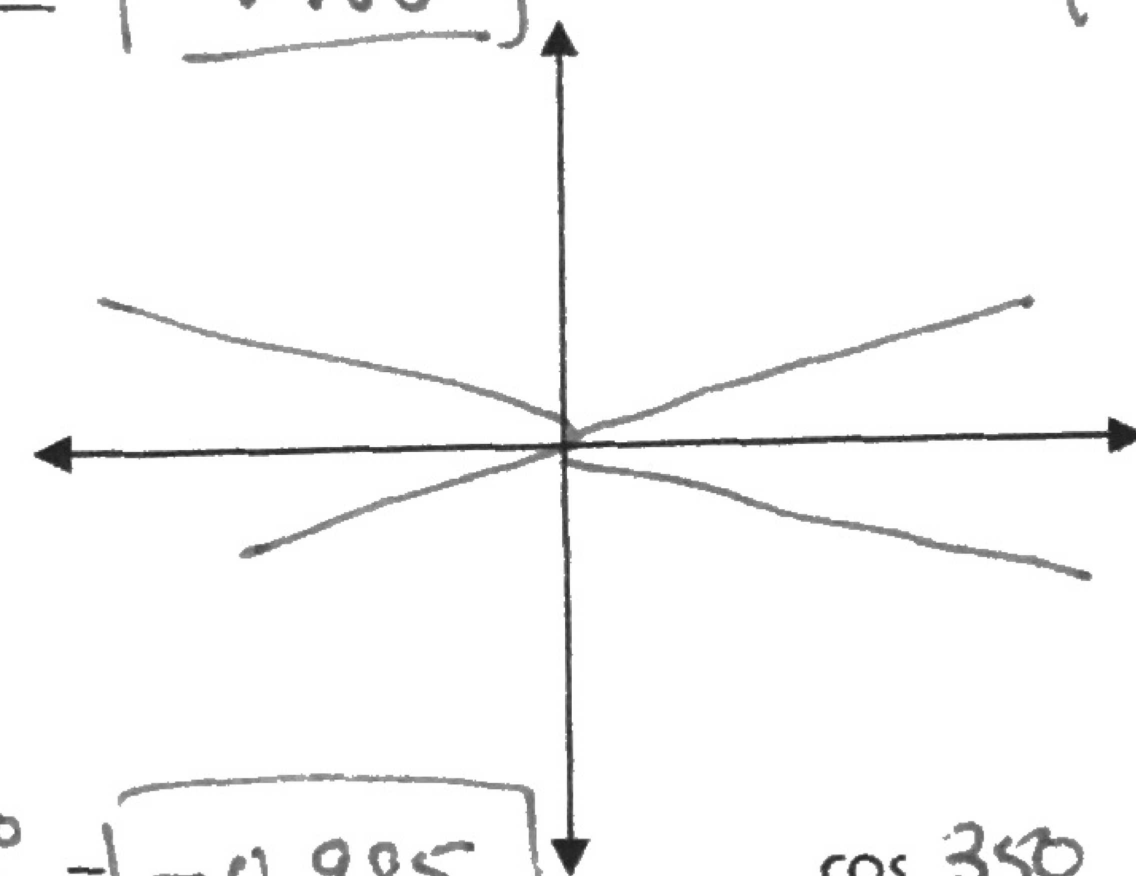
sin 170° = $\boxed{0.174}$ sin 10° = $\boxed{0.174}$



sin 190° = $\boxed{-0.174}$ sin 350° = $\boxed{-0.174}$

COS θ; θ' = 10°

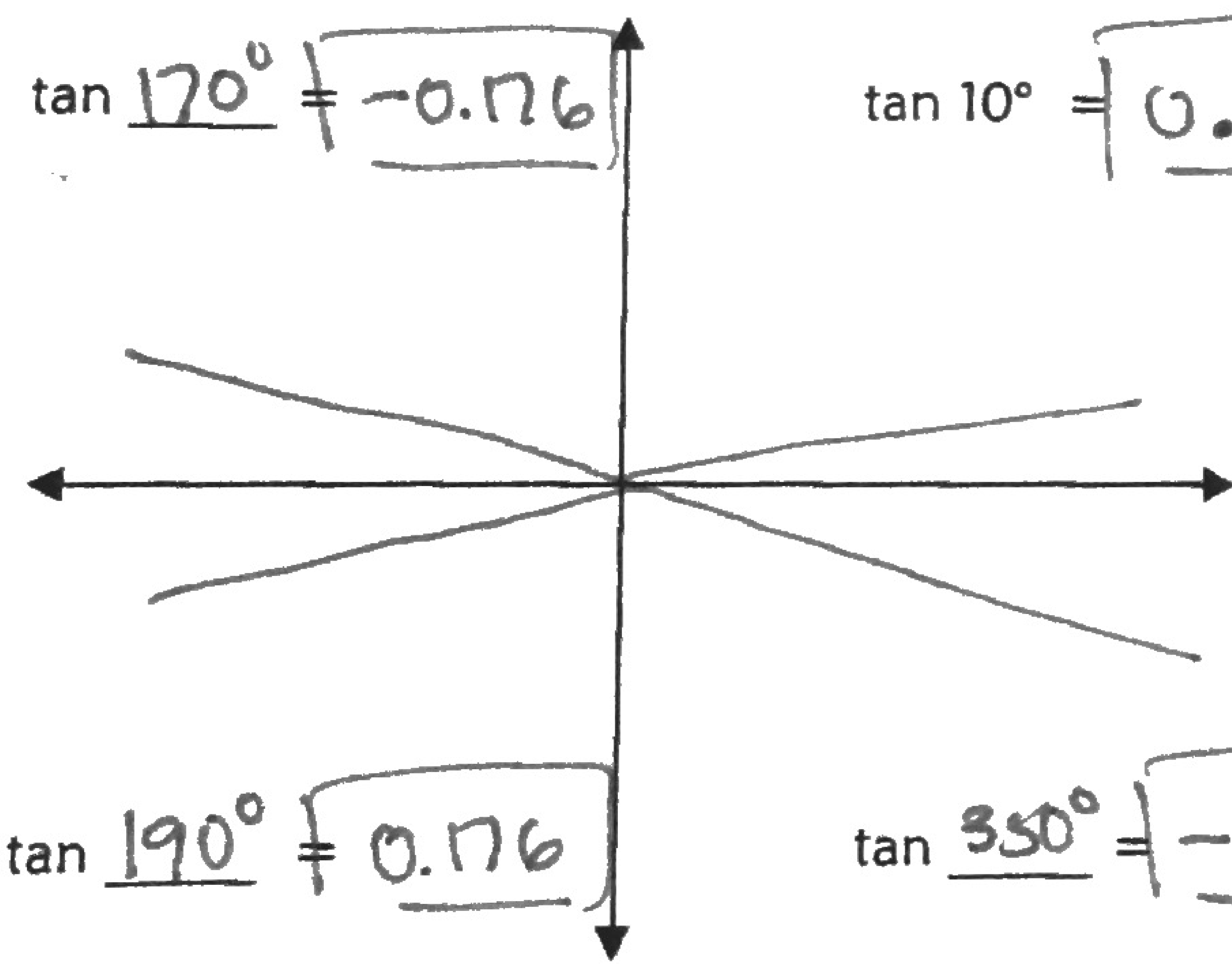
cos 170° = $\boxed{-.985}$ cos 10° = $\boxed{0.985}$



cos 190° = $\boxed{-0.985}$ cos 350 = $\boxed{0.985}$

TAN θ; θ' = 10°

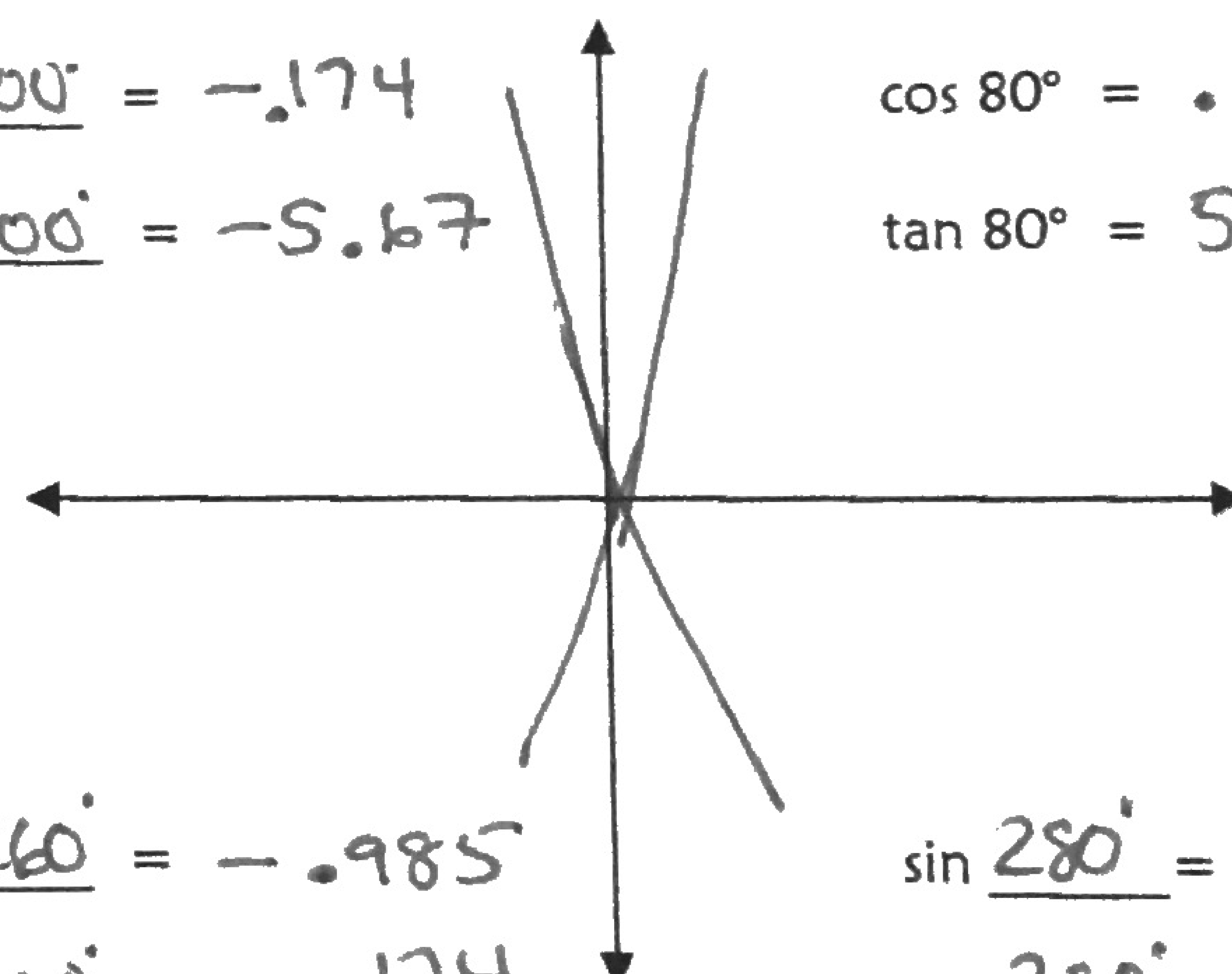
tan 170° = $\boxed{-0.176}$ tan 10° = $\boxed{0.176}$



tan 190° = $\boxed{0.176}$ tan 350° = $\boxed{-0.176}$

ALL THREE ☺ θ' = 80°

sin 100° = .985   sin 80° = .985
cos 100° = -.174   cos 80° = .174
tan 100° = -5.67   tan 80° = 5.67



sin 260° = -.985   sin 280° = -.985
sin 260° = -.174   cos 280° = .174
tan 260° = 5.67   tan 280° = -5.67

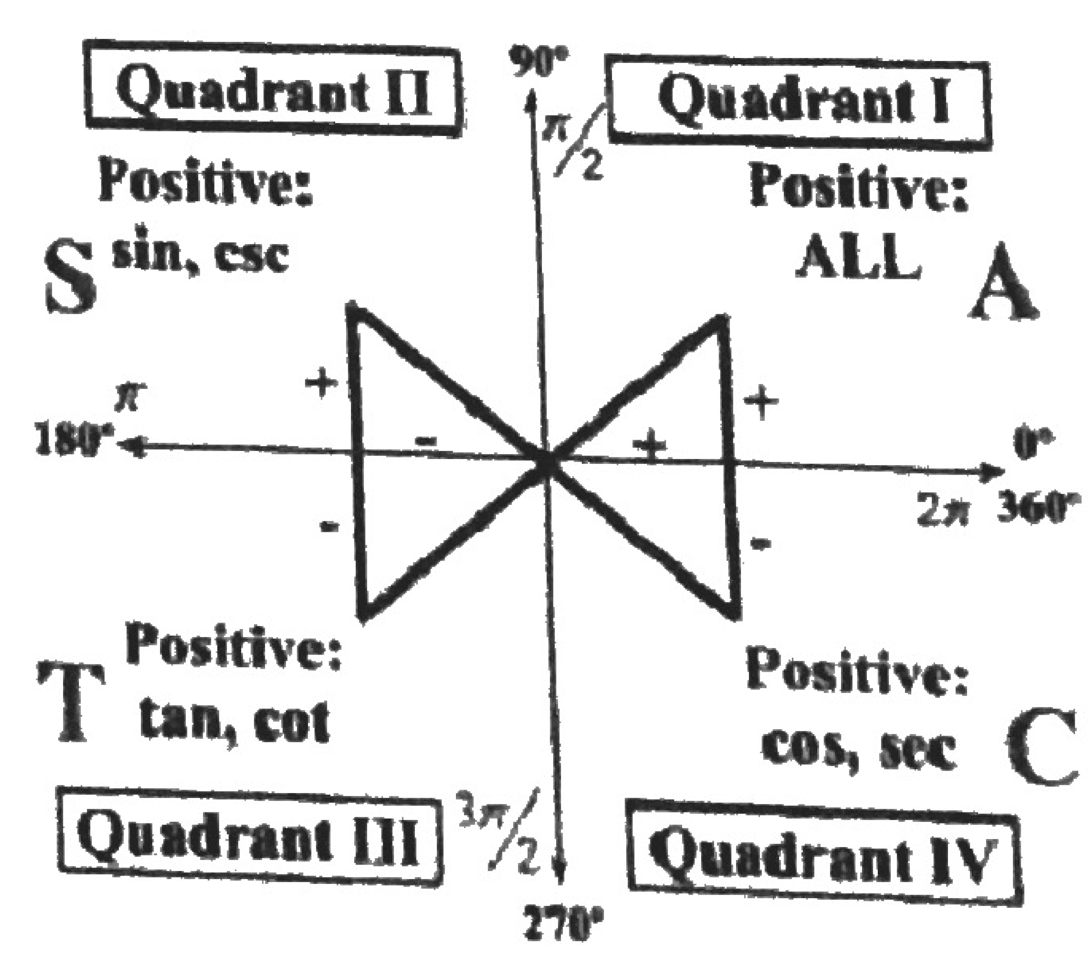
HUGE DEAL: SAME REFERENCE ANGLE = SAME RATIOS, DIFFERENT SIGNS

The X values (COS) are POSITIVE (+) in Quadrants I and IV

The X values (COS) are NEGATIVE (–) in Quadrants II and III

The Y values (SIN) are POSITIVE (+) in Quadrants I and II

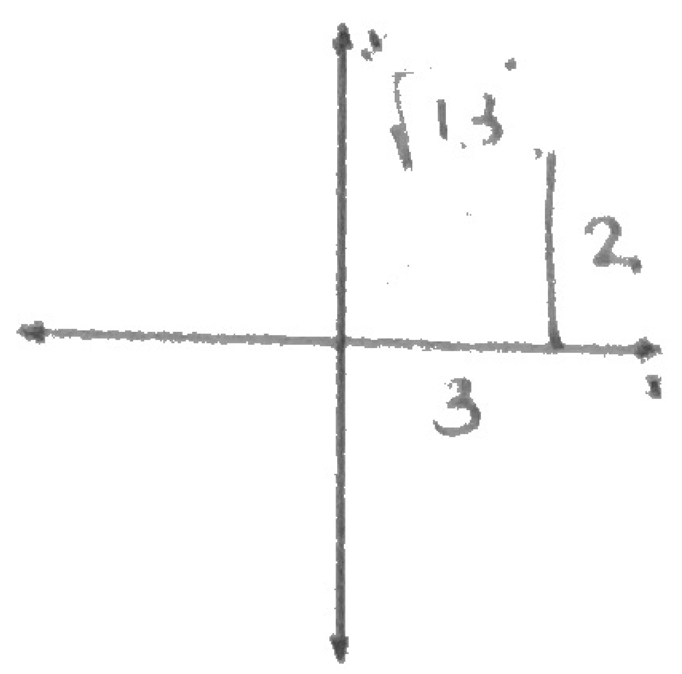
The Y values (SIN) are NEGATIVE (–) in Quadrants III and IV



Who is positive where? Above, circle the positive trig functions in each quadrant.
I REALLY don't advise memorizing "ASTC" – but it's there if you need it.

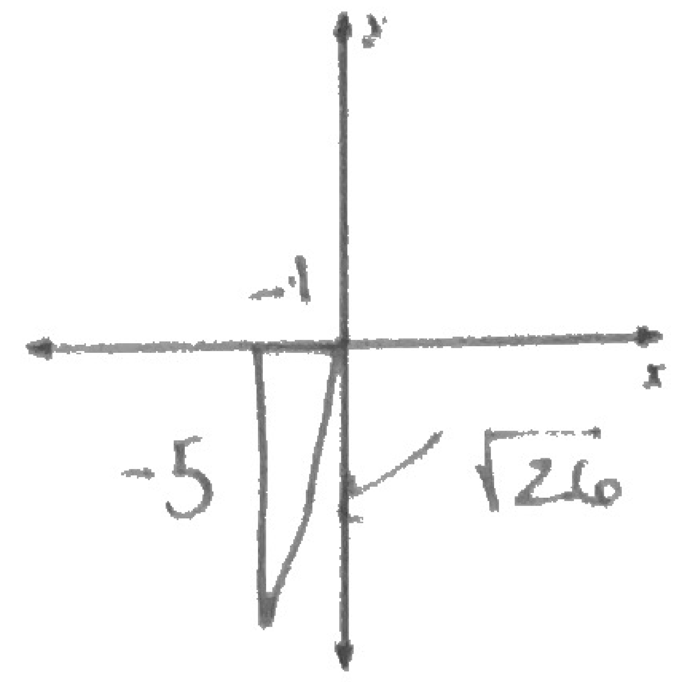
Trig Ratios in the Coordinate Plane: We can now find the trig function of any angle. We'll start when θ intersects a point (x, y). Find the exact ratio.

Point: (3, 2)



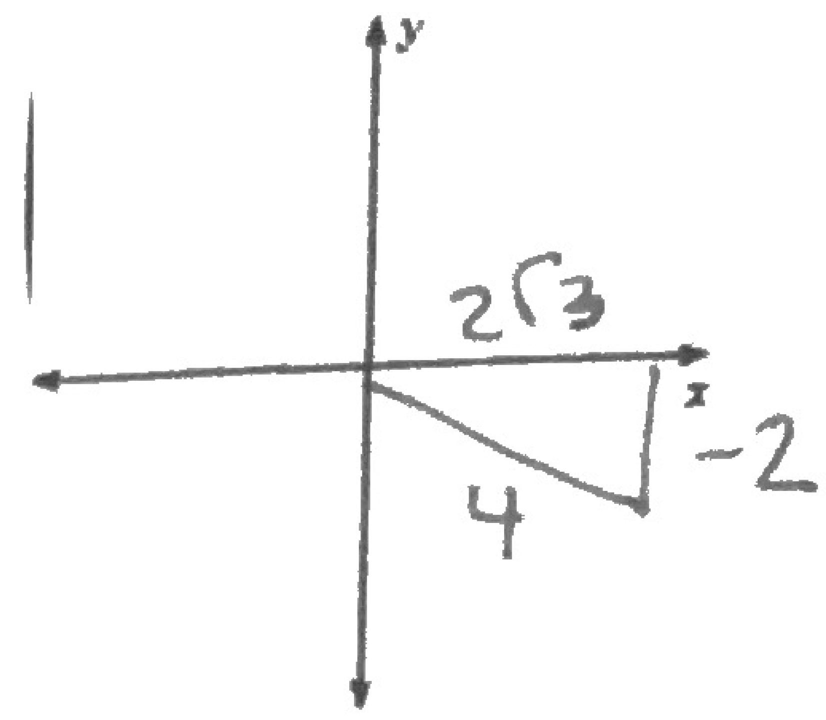
$\sin\theta = \frac{2\sqrt{13}}{13}$	$\csc\theta = \frac{\sqrt{13}}{2}$
$\cos\theta = \frac{3\sqrt{13}}{13}$	$\sec\theta = \frac{\sqrt{13}}{3}$
$\tan\theta = \frac{2}{3}$	$\cot\theta = \frac{3}{2}$

Point (–1, –5)



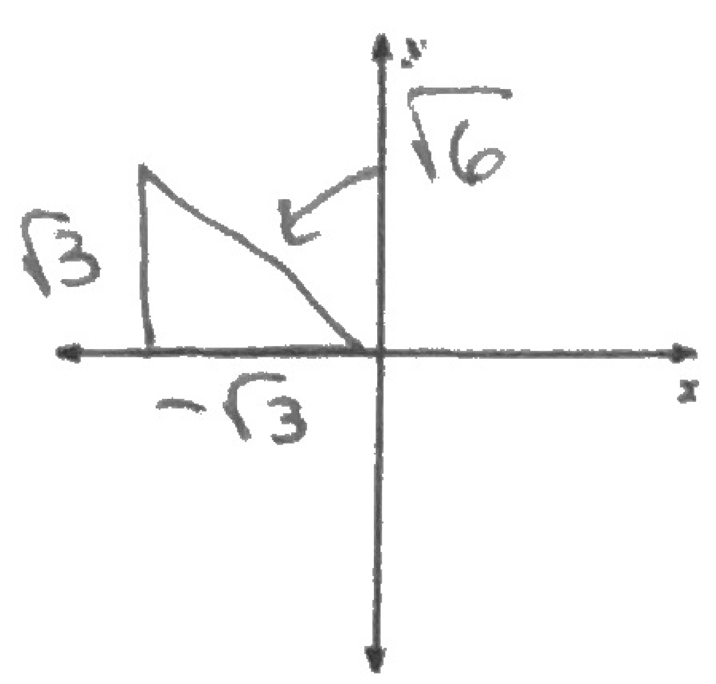
$\sin\theta = \frac{-5\sqrt{26}}{26}$	$\csc\theta = \frac{-\sqrt{26}}{5}$
$\cos\theta = \frac{-\sqrt{26}}{26}$	$\sec\theta = -\sqrt{26}$
$\tan\theta = 5$	$\cot\theta = \frac{1}{5}$

Point ($2\sqrt{3}$, –2)



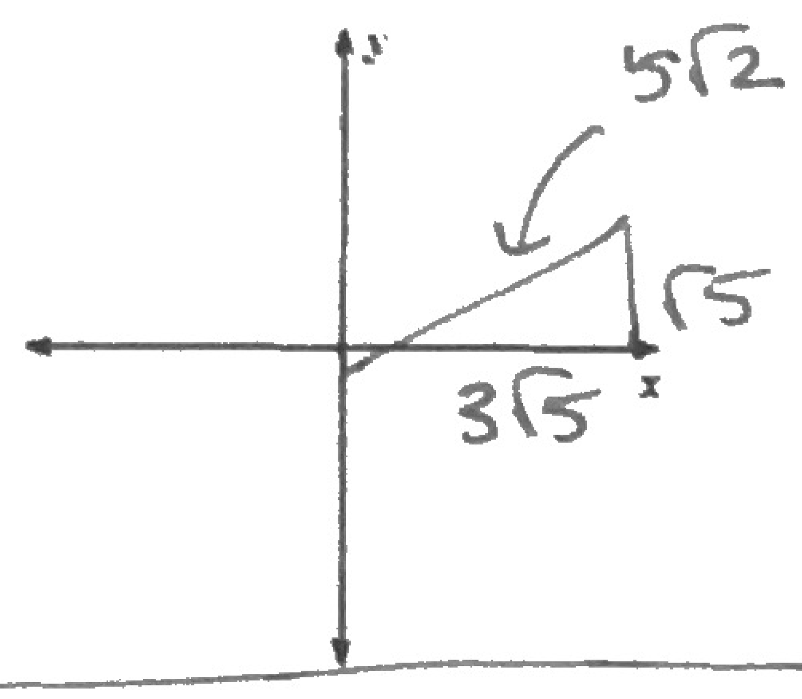
$\sin\theta = -\frac{1}{2}$	$\csc\theta = -2$
$\cos\theta = \frac{\sqrt{3}}{2}$	$\sec\theta = \frac{2\sqrt{3}}{3}$
$\tan\theta = \frac{-\sqrt{3}}{3}$	$\cot\theta = -\sqrt{3}$

Point ($-\sqrt{3}$, $\sqrt{3}$)



$\sin\theta = \frac{\sqrt{2}}{2}$	$\csc\theta = \sqrt{2}$
$\cos\theta = \frac{-\sqrt{2}}{2}$	$\sec\theta = -\sqrt{2}$
$\tan\theta = -1$	$\cot\theta = -1$

Point ($3\sqrt{5}$, $\sqrt{5}$)

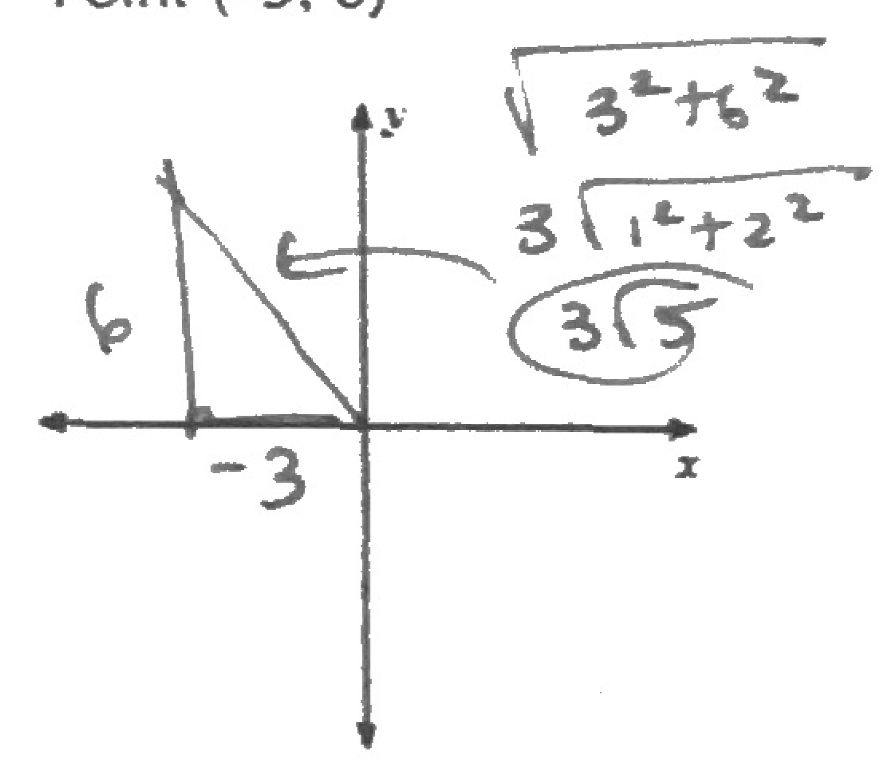


$\frac{5\sqrt{2}\sqrt{5}}{3\sqrt{5}\sqrt{5}}$

$\frac{3\sqrt{5}\sqrt{2}}{5\sqrt{2}\sqrt{2}}$

$\sin\theta = \frac{\sqrt{10}}{10}$	$\csc\theta = \sqrt{10}$
$\cos\theta = \frac{3\sqrt{10}}{10}$	$\sec\theta = \frac{\sqrt{10}}{3}$
$\tan\theta = \frac{1}{3}$	$\cot\theta = 3$

Point (–3, 6)

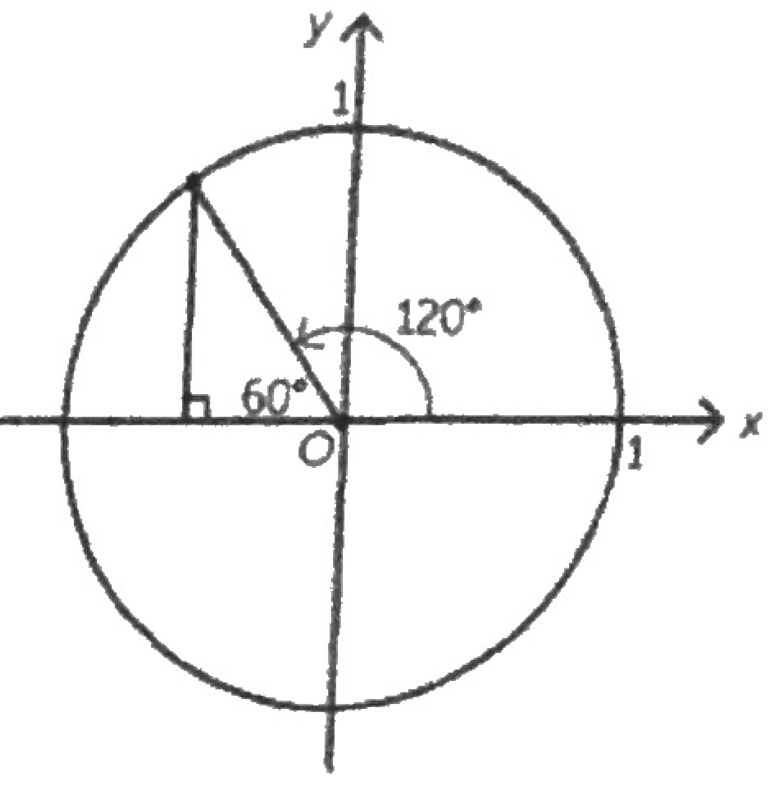


$\sin\theta = \frac{2\sqrt{5}}{5}$	$\csc\theta = -\frac{\sqrt{5}}{2}$
$\cos\theta = \frac{-\sqrt{5}}{5}$	$\sec\theta = -\sqrt{5}$
$\tan\theta = -2$	$\cot\theta = -\frac{1}{2}$

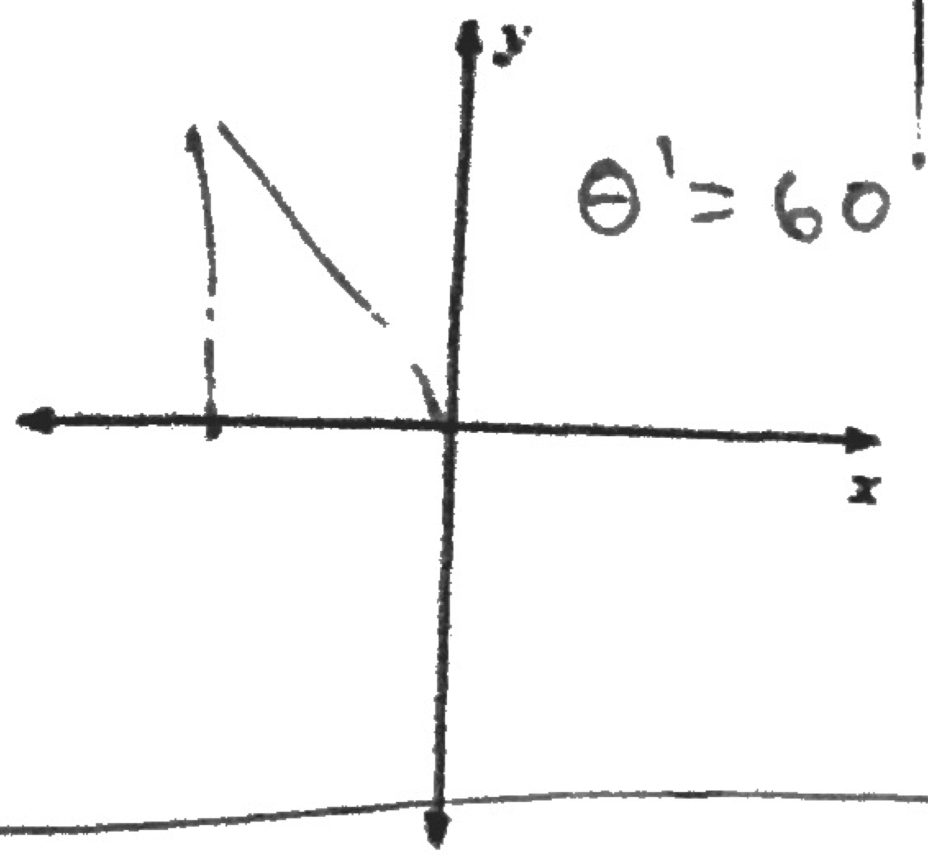
When you're done, approximate the decimals. Ensure the decimals make sense, based on the location of the point (is it higher up vertically or further away horizontally?)

* just do this in your calc... its gotta make sense to you.

Special Trig Ratios in the Coordinate Plane:
Now we can also find the trig ratios of any special angle!
It doesn't matter what the side lengths actually are...
what matters is the RATIO of the sides once the angle is locked in...

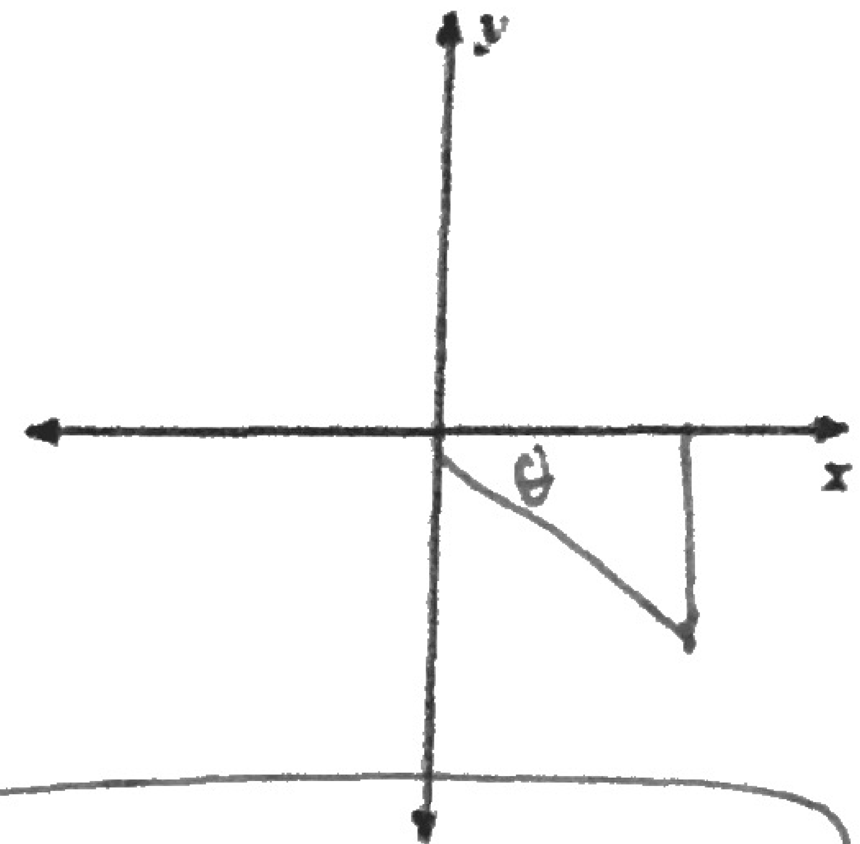


$\theta = 120°$



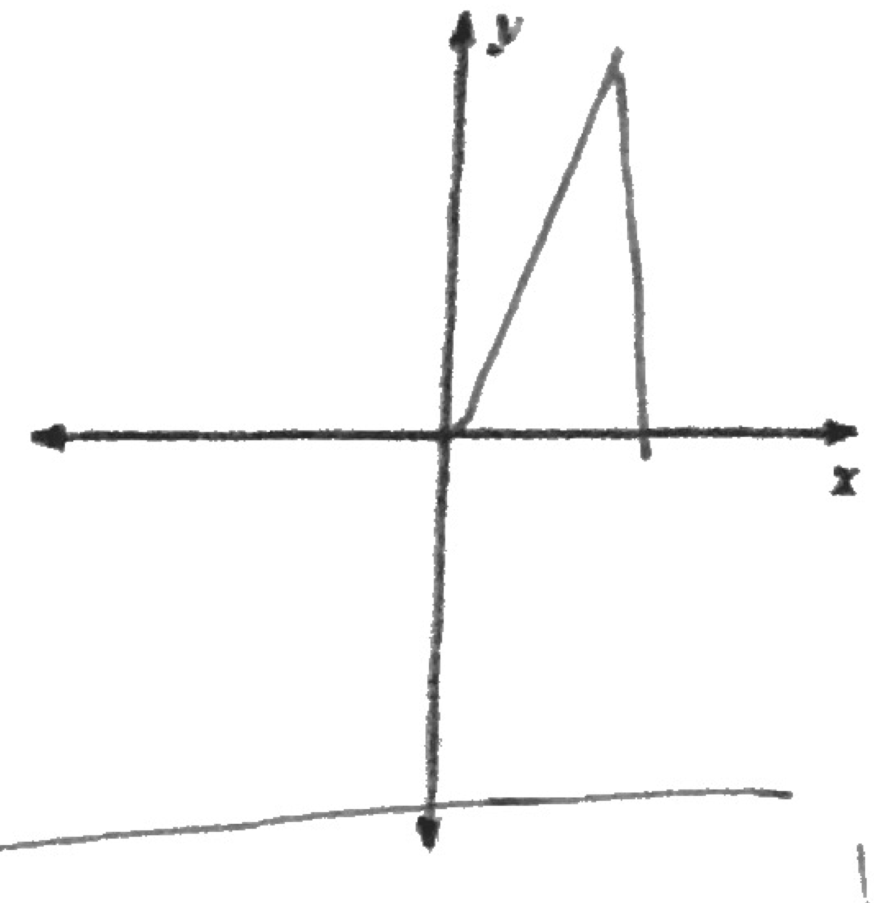
$\sin\theta =$	$\frac{\sqrt{3}}{2}$	$\csc\theta =$	$\frac{2\sqrt{3}}{3}$
$\cos\theta =$	$-\frac{1}{2}$	$\sec\theta =$	-2
$\tan\theta =$	$-\sqrt{3}$	$\cot\theta =$	$-\frac{\sqrt{3}}{3}$

$\theta = -45°$



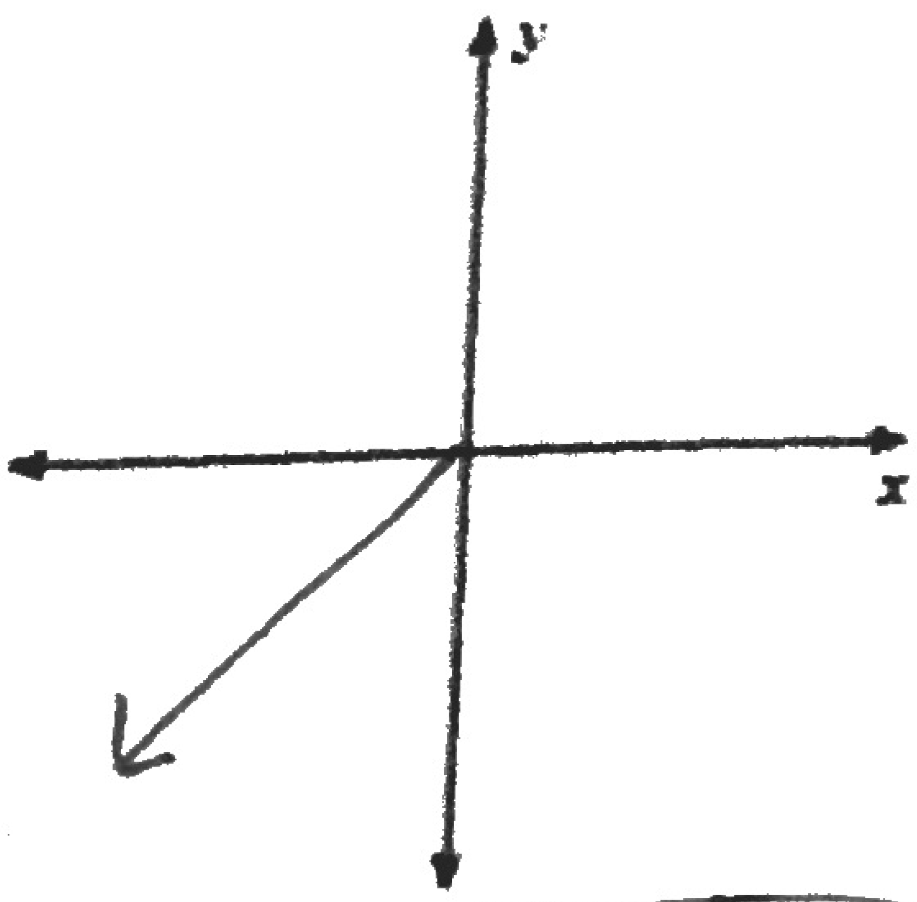
$\sin\theta =$	$-\sqrt{2}/2$	$\csc\theta =$	$-\sqrt{2}$
$\cos\theta =$	$\sqrt{2}/2$	$\sec\theta =$	$\sqrt{2}$
$\tan\theta =$	-1	$\cot\theta =$	-1

$\theta = 60°$



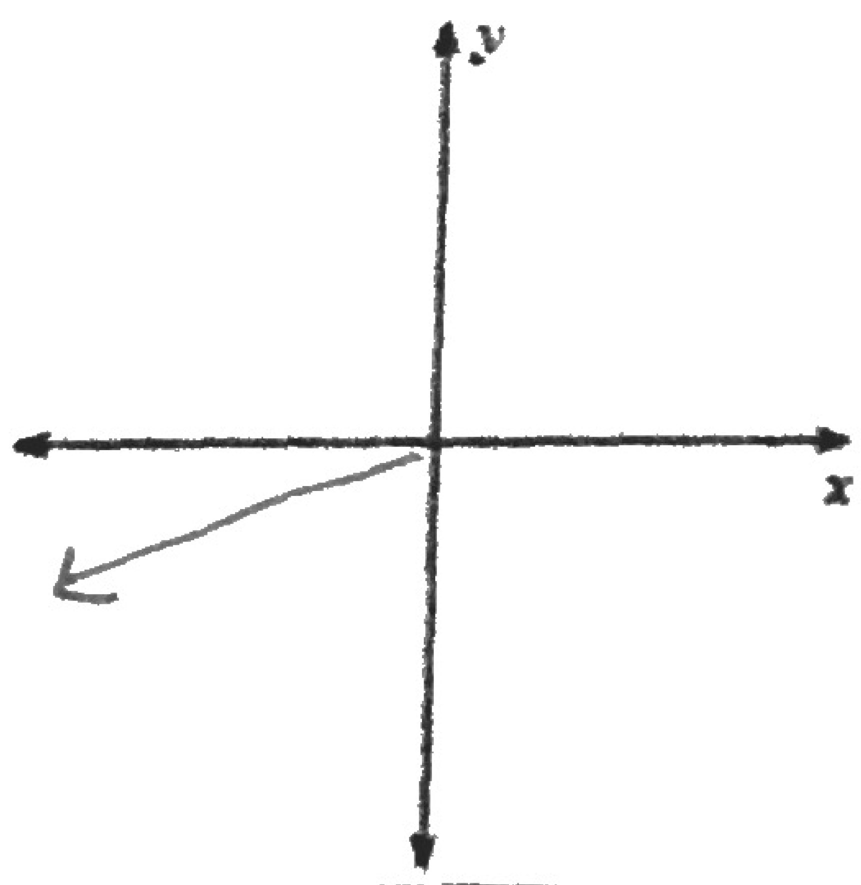
$\sin\theta =$	$\frac{\sqrt{3}}{2}$	$\csc\theta =$	$\frac{2\sqrt{3}}{3}$
$\cos\theta =$	$\frac{1}{2}$	$\sec\theta =$	2
$\tan\theta =$	$\sqrt{3}$	$\cot\theta =$	$\sqrt{3}/3$

$\theta = 225°$



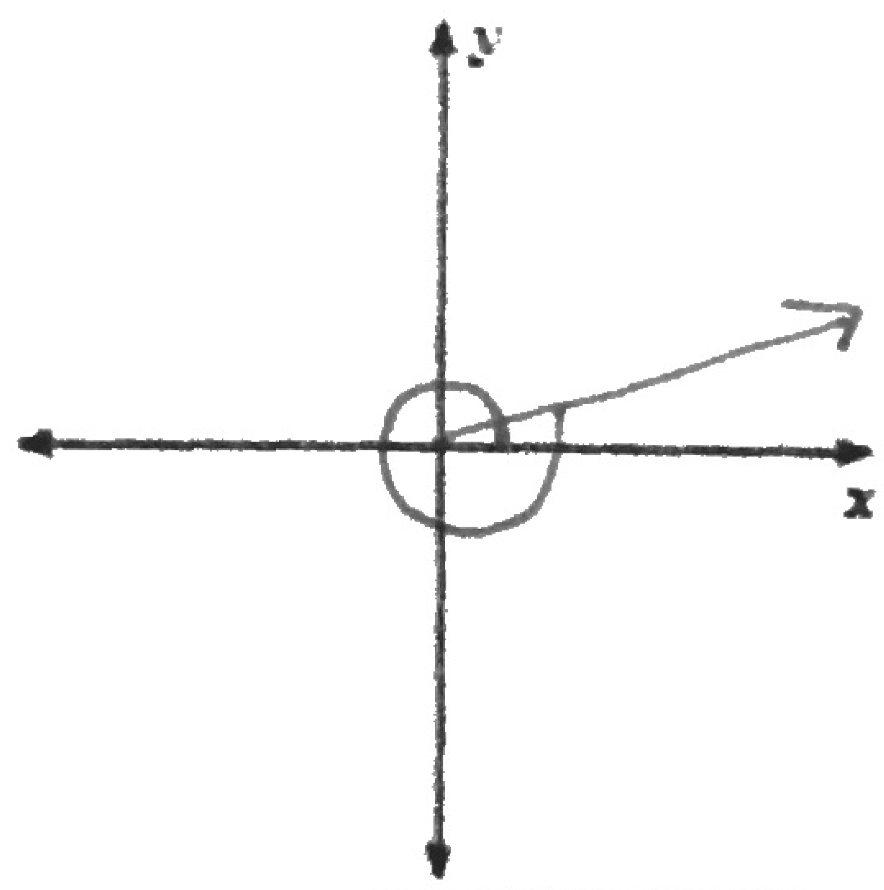
$\sin\theta =$	$-\frac{\sqrt{2}}{2}$	$\csc\theta =$	$-\sqrt{2}$
$\cos\theta =$	$-\frac{\sqrt{2}}{2}$	$\sec\theta =$	$-\sqrt{2}$
$\tan\theta =$	1	$\cot\theta =$	1

$\theta = -150°$



$\sin\theta =$	$-\frac{1}{2}$	$\csc\theta =$	-2
$\cos\theta =$	$-\frac{\sqrt{3}}{2}$	$\sec\theta =$	$\frac{2\sqrt{3}}{3}$
$\tan\theta =$	$\frac{\sqrt{3}}{3}$	$\cot\theta =$	$-\sqrt{3}$

$\theta = 390°$



$\sin\theta =$	$\frac{1}{2}$	$\csc\theta =$	2
$\cos\theta =$	$\frac{\sqrt{3}}{2}$	$\sec\theta =$	$\frac{2\sqrt{3}}{3}$
$\tan\theta =$	$\frac{\sqrt{3}}{3}$	$\cot\theta =$	$\sqrt{3}$

<u>Find the other two trigonometric ratios, given one ratio</u> .
Draw it out, solve for what's missing, then evaluate the other trig ratios. Watch your signs…

$\cos\theta = 2/7,\ \theta \in \text{QIV}$

$\sec\theta = \frac{7}{2}$

$\sin\theta = \frac{-3\sqrt{5}}{7}$ $\quad \csc\theta = \frac{-7\sqrt{5}}{15}$

$\tan\theta = \frac{-3\sqrt{5}}{2}$ $\quad \cot\theta = \frac{-2\sqrt{5}}{15}$

$\sin\theta = -4/5;\ \theta \in \text{QIII}$

$\csc\theta = -\frac{5}{4}$

$\cos\theta = -\frac{3}{5}$ $\quad \sec\theta = -\frac{5}{3}$

$\tan\theta = \frac{4}{3}$ $\quad \cot\theta = \frac{3}{4}$

$\cot\theta = -5;\ \theta \in \text{QII}$

$\tan\theta = -\frac{1}{5}$

$\sin\theta = \frac{\sqrt{26}}{26}$ $\quad \csc\theta = \sqrt{26}$

$\cos\theta = \frac{-5\sqrt{26}}{26}$ $\quad \sec\theta = \frac{-\sqrt{26}}{5}$

$\sec\theta = 2,\ \sin\theta < 0$ QIV

$\cos\theta = \frac{1}{2}$

$\sin\theta = -\frac{\sqrt{3}}{2}$ $\quad \csc\theta = \frac{-2\sqrt{3}}{3}$

$\tan\theta = -\sqrt{3}$ $\quad \cot\theta = -\frac{\sqrt{3}}{3}$

$\tan\theta = -1;\ \cos\theta < 0$ QII

$\cot\theta = -1$

$\sin\theta = \frac{\sqrt{2}}{2}$ $\quad \csc\theta = \sqrt{2}$

$\cos\theta = -\frac{\sqrt{2}}{2}$ $\quad \sec\theta = -\sqrt{2}$

$\tan\theta = \sqrt{3};\ \csc\theta > 0$ QI

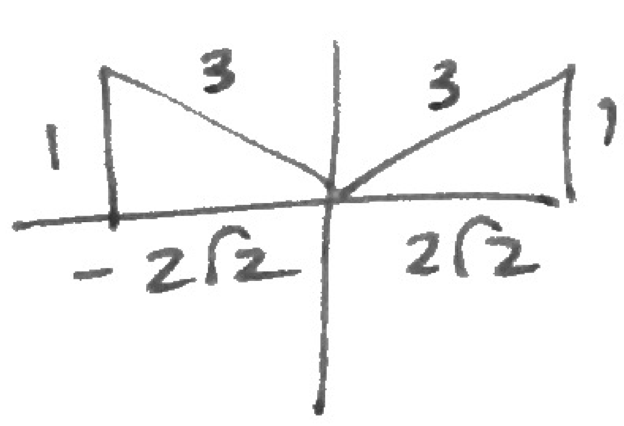
$\cot\theta = \sqrt{3}/3$

$\sin\theta = \frac{\sqrt{3}}{2}$ $\quad \csc\theta = \frac{2\sqrt{3}}{3}$

$\cos\theta = \frac{1}{2}$ $\quad \sec\theta = 2$

<u>IF NO QUADRANT IS SPECIFIED, TWO SCENARIOS ARE POSSIBLE</u> Since each trig function is positive or negative in two quadrants, you have to draw your reference triangles in BOTH quadrants and give both scenarios.

$\sin\theta = 1/3$



$\csc\theta = \frac{1}{3}$

in QII

$\cos\theta = \frac{-2\sqrt{2}}{3}$

$\sec\theta = \frac{-3\sqrt{2}}{4}$

$\tan\theta = \frac{-\sqrt{2}}{4}$

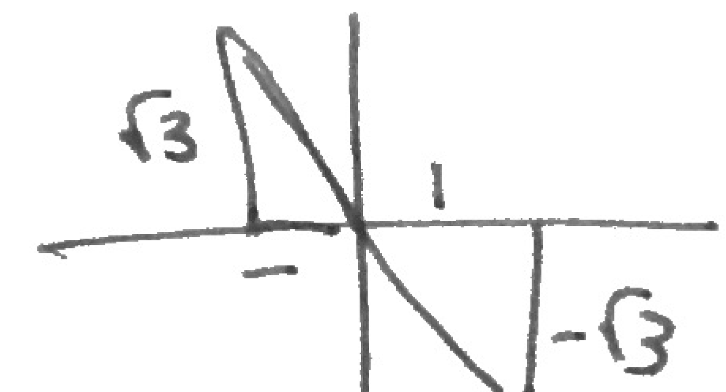
in QI

$\cos\theta = \frac{2\sqrt{2}}{3}$

$\sec\theta = \frac{3\sqrt{2}}{4}$

$\tan\theta = \frac{\sqrt{2}}{4}$

$\tan\theta = -\sqrt{3}$



$\cot\theta = -\sqrt{3}$

in QII

$\sin\theta = \frac{\sqrt{3}}{2}$

$\csc\theta = \frac{2\sqrt{3}}{3}$

$\cos\theta = -\frac{1}{2}$

$\sec\theta = -2$

in QIV

$\sin\theta = -\frac{\sqrt{3}}{2}$

$\csc\theta = -\frac{2\sqrt{3}}{3}$

$\cos\theta = \frac{1}{2}$

$\sec\theta = 2$