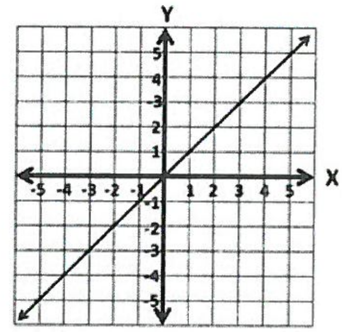
Name: Answer Key Per: 1 Date: ____________
Serafino · Algebra 2E M T W R F

2A

Linear (& Piecewise) Functions

Notes & Practice Packet



Parent Function: $f(x) = x$

Slope–Intercept Form: $y = mx + b$
Point–Slope Form: $y = m(x - x_1) + y_1$
Standard Form: $Ax + By = C$

In this packet, we will:
• Graph all 3 forms • Convert between forms • Write equations in all 3 forms given a situation • Write equations given graphic requirements • Solve equations in all 3 forms • Write equations of piecewise functions • Transform linear functions.

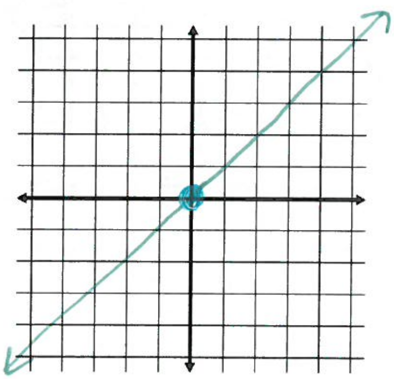
1. Graphing Linear Functions

rate of change

I. Slope –Intercept Form: $y = m\cancel{x} + b$ ← when x=0

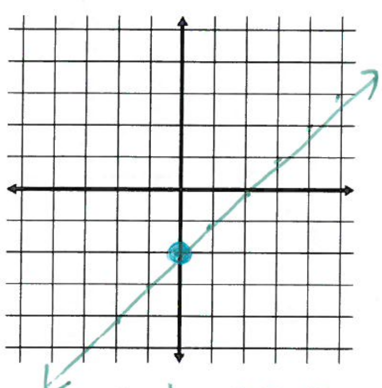
Graph each of the following. Then calculate the coordinates for the x–intercepts for each.

a) $y = x$



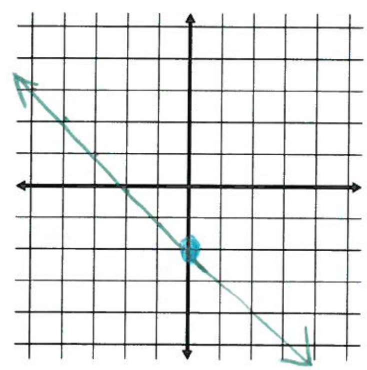
x int: (0,0)

b) $y = x - 2$



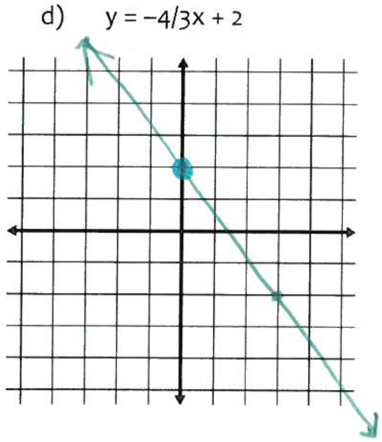
x int: (2,0)

c) $y = -x + 3$



x int: (−2,0)

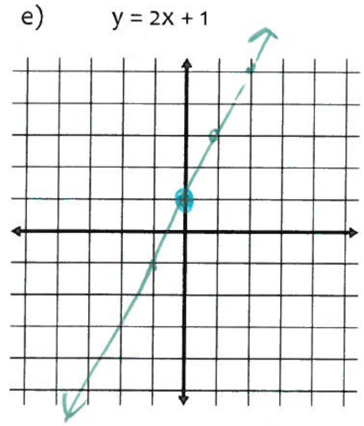
d) $y = -4/3x + 2$



$-\frac{4}{3}x + 2 = 0$ $x = (1.5, 0)$

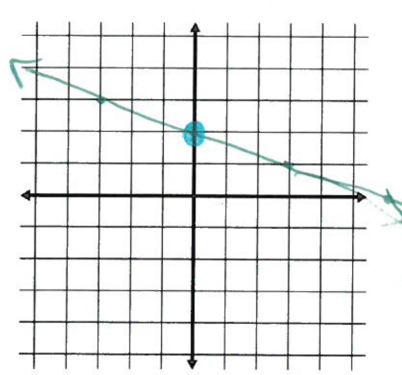
$\frac{3}{4} \cdot \frac{4}{3}x = 2 \cdot \frac{3}{4}$

e) $y = 2x + 1$



x-int $(-\frac{1}{2}, 0)$

f) $y = -1/3x + 2$



x-int: (6,0)

$-\frac{1}{3}x + 2 = 0$

$-\frac{1}{3} = -2$

$x = 6$

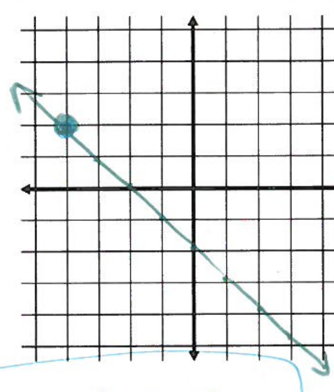
(h, k) is a point

$y = m(x - h) + k$

II. Point–Slope Form

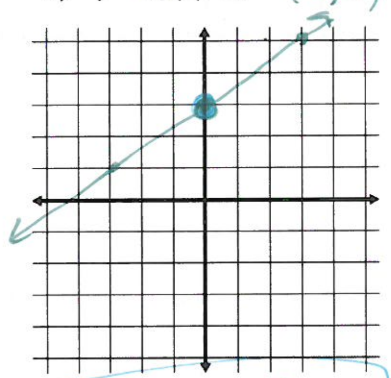
Graph each of the following. Then calculate the x-intercept and y-intercept for each:

a) $y = -(x + 4) + 2$ (−4, 2)



x: (−2, 0)
y = (0, −2)

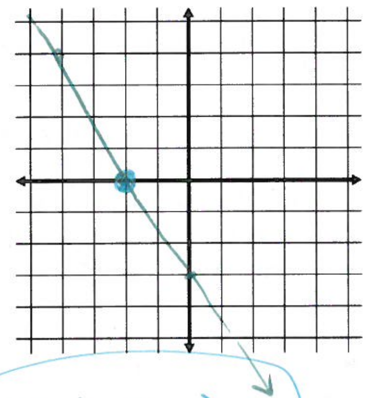
b) $y = 2/3(x) + 3$ (0, 3)



x: (−9/2, 0)
y: (0, 3)

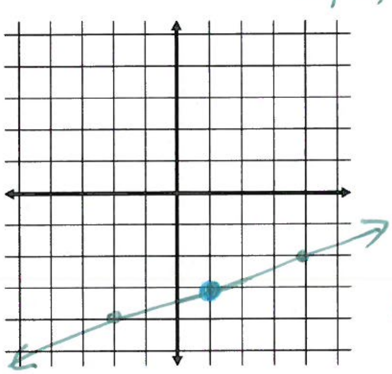
$0 = \frac{2}{3}x + 3 \quad \frac{3}{2} \cdot -3 = \frac{\cancel{2}}{\cancel{3}}x \cdot \frac{\cancel{3}}{\cancel{2}}$

c) $y = -3/2(x + 2)$ (−2, 0)



x: (−2, 0)
y: (0, −3)

d) $y = 1/3(x - 1) - 3$ (1, −3)



$y = \frac{1}{3}(0-1) - 3$

$\frac{1}{3}(-1) - 3\frac{3}{3}$

$-\frac{1}{3} - \frac{9}{3} = -\frac{10}{3}$

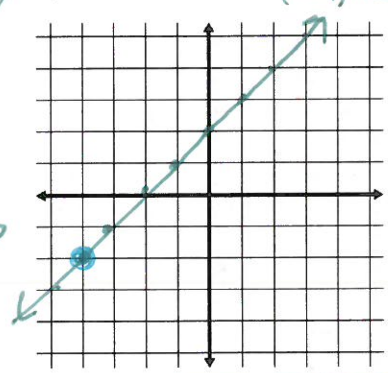
$\boxed{y = (0, -10/3) \quad x = (10, 0)}$

$0 = \frac{1}{3}(x-1) - 3$

$3 = \frac{1}{3}(x-1)$

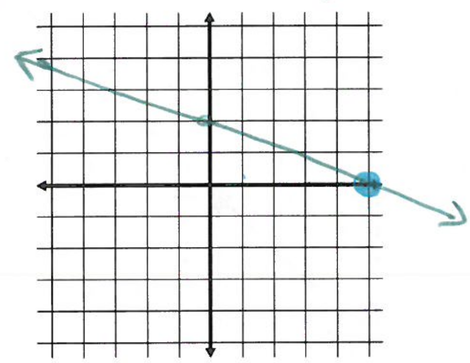
$9 = (x-1)$

e) $y + 2 = x + 4$ (−4, −2)



x = (−2, 0)
y = (0, 2)

f) $y = -2/5(x - 5)$ (5, 0)

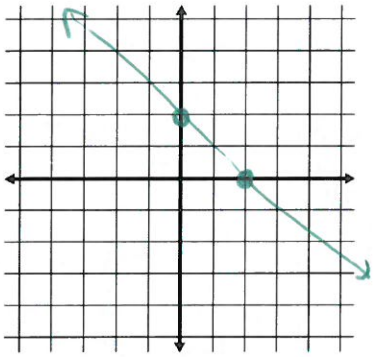


x = (5, 0)
y = (0, 2)

III. Standard Form: $Ax + By = C$

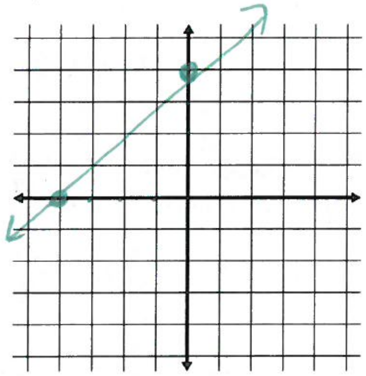
Graph each of the following. Then calculate the slope for each.

a) $x + y = 2$



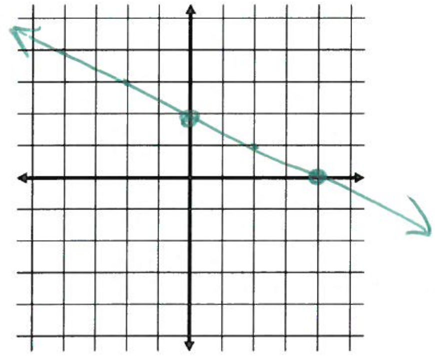
$m = -1$

b) $x - y = -4$



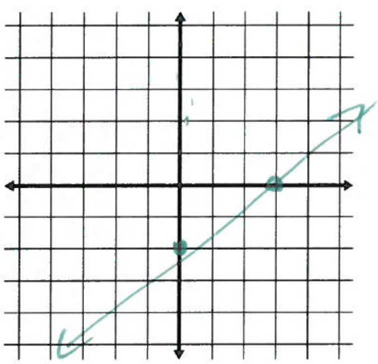
$m = 1$

c) $2x + 4y = 8 \rightarrow x + 2y = 4$



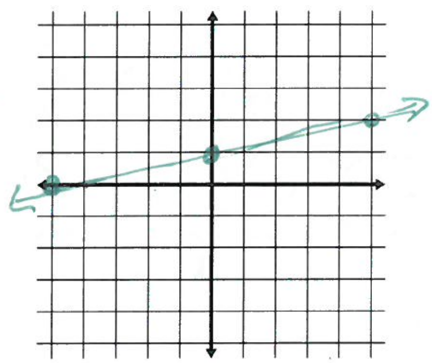
$m = -\frac{1}{2}$

d) $2x - 3y = 6$



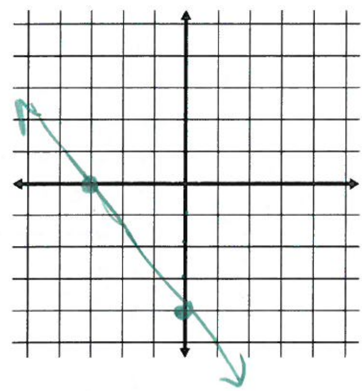
$m = \frac{2}{3}$

e) $x - 5y = -5$



$m = 1/5$

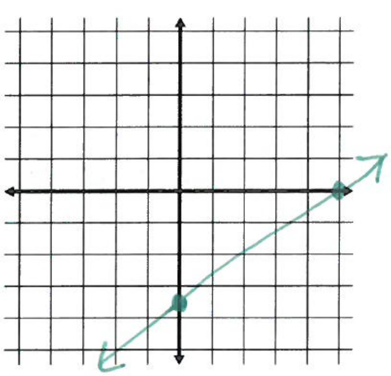
f) $4x + 3y = -12$



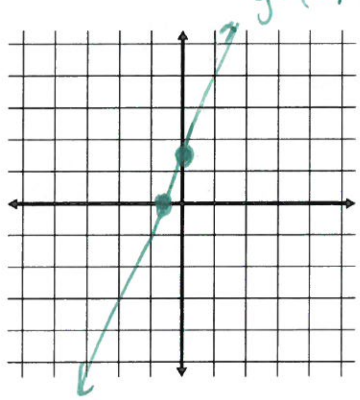
$m = -\frac{4}{3}$

These are not as "pretty".... But they can still be graphed using the same principle ((just plot intercepts between the correct boxes)

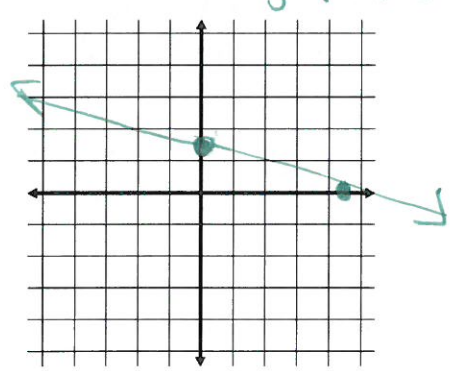
g) $2x - 3y = 10$ x: $(5, 0)$ y: $(0, -10/3)$



h) $6x - 2y = -3$ x: $(-1/2, 0$ y: $(0, 3/2)$



i) $4x + 15y = 17$ x: $(\frac{17}{4}, 0)$ ≈ 4..) y: $(0, 17/15)$



so many options!

2. **Converting between forms – AND MAKING CONNECTIONS!! (Do all your work on a separate paper.)**

Info	Point–Slope Form $y = m(x - h) + k$; (h, k) is point	Slope–Intercept Form $y = mx + b$; b is y–int	Standard Form $Ax + By = C$; $A \in \mathbb{N}, B \in \mathbb{Z}$
ⓐ Slope: ¾ y–int: 2	$\boxed{y=\frac{3}{4}\left(x+\frac{8}{3}\right)}$ or $\boxed{y=\frac{3}{4}(x-4)+5}$	$\boxed{y=\frac{3}{4}x+2}$ Factor this for x-int. add slope to y-int for another point	$-\frac{3}{4}x+y=2$ $\boxed{3x-4y=-8}$
ⓑ $m=\frac{4}{-9}$ (4, –3) (–5, 1)	$\boxed{y=-\frac{4}{9}(x-4)-3}$ or $\boxed{y=-\frac{4}{9}(x+5)+1}$	$y=-\frac{4}{9}x-\frac{20}{9}+\frac{9}{9}$ $\boxed{y=-\frac{4}{9}x-\frac{11}{9}}$	$\frac{4}{9}x+y=-\frac{11}{9}$ $\boxed{4x+9y=-11}$
ⓒ	$\boxed{y = \frac{3}{2}(x - 4) - 2}$ $y=\frac{3}{2}x-6-2$ →	$\boxed{y=\frac{3}{2}x-8}$	$\frac{3}{2}x-y=8$ $\boxed{3x-2y=16}$
ⓓ	$\boxed{y=\frac{1}{3}(x-6)}$ or $\boxed{y=\frac{1}{3}(x-3)-1}$	$\frac{-3y}{-3}=\frac{-x}{-3}+\frac{6}{-3}$ $\boxed{y=\frac{1}{3}x-2}$	$\boxed{2x - 6y = 12}$ yikes! reduce! $\boxed{x-3y=6}$
ⓔ	$\boxed{y=2(x+3)}$ or $\boxed{y=2(x-1)+8}$	$\boxed{y = 2x + 6}$	$\boxed{2x-y=-6}$
ⓕ Slope: 0 (3, 5)	constant function!	← $\boxed{y=5}$ →	
ⓖ (3, 7) (3, –2)	NOT a function at all!!!	← $\boxed{x=3}$ →	
ⓕ Slope: –2 x–int: 4	$\boxed{y=-2(x-4)}$	$\boxed{y=-2x+8}$	$\boxed{2x+y=8}$

3. Writing Equations from Scenarios:

YOU MUST DEFINTE YOUR VARIABLES OR YOUR FUNCTION WILL NOT MAKE SENSE!

Slope Intercept Form: All you need is a rate of change and when $x=0$

a. You buy a cactus when it is 4 inches tall and it grows 1 in per year. Write a function that models the height of your cactus per year.

h= height in inches
t= time in years since purchase

$\boxed{h(t) = t + 4}$

b. Tony weighs 185 lbs and wants to lose 3 lbs every 2 weeks. Write a function that models how much Tony will weigh during the weeks of his diet.

w= weight in lbs
t= time in weeks from now

$\boxed{w(t) = -\frac{3}{2}t + 185}$

c. I open a bank account with \$2000 in and put in \$300 every 6 months. Write a function that models how much money you'll have each year.

m= dollars in account
t= time in years since opening

$\boxed{m(t) = 600t + 2000}$

Point–Slope Form: you need a rate & any point

d. The temperature has been decreasing about 2 degrees every week and there is a prediction that in 5 weeks, it's going to be 51°

t= temp in degrees F
w= # weeks from NOW

$\boxed{t(w) = -2(t-5)+51}$

e. Three weeks ago, I had 20 markers. I go through about 2 markers per week.

(−3, 20)
m= # markers
w= weeks from now

$\boxed{m(w) = 2(w+3)+20}$

f. When my cat was 9, she weighed 10 lbs. She is now 12 and weighs 12 lbs. (9,10) (12,12) $m = \frac{2}{3}$

w= weight of cat in lbs
a= age of cat in years

$w(a) = \frac{2}{3}(a-9)+10$

$w(a) = \frac{2}{3}(a-12)+12$

Standard Form: Two vars. work together

g. Your wallet contains \$100, but only in \$10 and \$5 bills. Write a function that models how many of each bill you could have in your wallet.

t= # \$10 bills
f= # \$5 bills

$\boxed{10t + 5f = 100}$

h. You have to seat 150 guests at tables that can either fit 10 or 12 people. Write a function that models how many of each table you need.

t= # 10-per tables
w= # 12-per tables

$\boxed{10t + 12w = 150}$

i. The Maroons scored 38 points by only scoring Touchdowns and Two–Point conversions. What function models how many of each they could have?

t= # touchdowns
p= # 2-pt conv.

$\boxed{6t + 2p = 38}$

4. Slope: Rate of Change… it's △y / △x … which is the same thing as the change in y per 1 unit of input!
Ex) If I'm going up 3, over 2, my slope is $\frac{3}{2}$… which means y increases 3 units when x increases 2 units… but also, it means that y increases 1.5 units when x increases 1 unit!

Find the slope (rate of change) the two points

a. (4, 10) (6, 12) $\boxed{1}$
$2/2$

b. (3,5) and (−1,3) $\boxed{\frac{1}{2}}$
$-2/-4$

c. (−3, 5) (6, 2) $\boxed{-\frac{1}{3}}$
$-3/9$

d. (2, 5) (0, 5) $\boxed{0}$

e. (−6, 7) and (3, 4) $\boxed{-\frac{1}{3}}$
$-3/9$

f. (3, 6) (−2, −2) $\boxed{\frac{8}{5}}$
$-8/-5$

g. (7, −29) (−3, −4) $\boxed{-\frac{5}{2}}$
$25/-10$

h. (4, −8) (4, 11) $\boxed{\text{und.}}$

Parallel lines: $a/b \parallel a/b$ Perpendicular Lines: $a/b \perp -b/a$

5. Write all possible versions of the equations of the line with the given requirements. Then provide the x– and y–intercepts (if they exist)

a) slope: 2, (4, 6)

$y = 2(x-4)+6$
$y = 2x-2$
$2(x-1)$
$2x - y = 2$
x: (1,0)
y: (0,−2)

b) slope 3/2, (1, 5)

$\frac{3}{2}x - \frac{3}{2} + 5\frac{(2)}{(2)}$ 10

$y = \frac{3}{2}(x-1)+5$
$y = \frac{3}{2}x + \frac{7}{2}$
$3x - 2y = -7$
x: (−7/3, 0)
y: (0, 7/2)

c) slope of zero, (4, −3)

$y = -3$
x: none
y: (0,−3)

d) Perpendicular to y = −2x + 6 though (−4, −1) $m = 1/2$

$y = \frac{1}{2}(x+4)-1$
$y = \frac{1}{2}x + 1$
$x - 2y = -2$
x: (−2,0)
y: (0, 1)

b) (3, 6) (7, 2) $m = -4/4 = -1$

$-x+3+6$

$y = -(x-3)+6$
$y = -x+9$
$x + y = 9$
x: (9,0)
y: (0,9)

b) (−2, −6) (1, −4) $m = 2/3$

$\frac{2}{3}x - \frac{2}{3} - 4\frac{12}{3}$ $\frac{}{3}$

$y = \frac{2}{3}(x-1)-4$
$y = \frac{2}{3}x - \frac{14}{3}$
$2x - 3y = 14$
x: (7,0)
y: (0, 14/3)

c) (−4, 5) (2, 5)

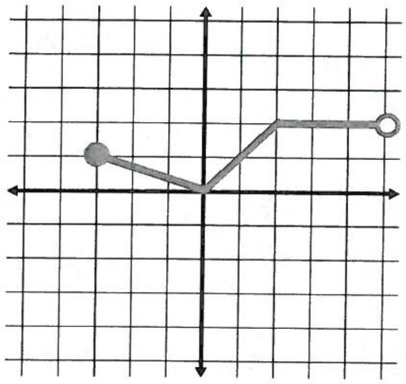
$y = 5$
x: none
y: (0,5)

d) (−4, 6) (−4, 2)

$x = -4$
x: (−4,0)
y: none

6. **Writing Piecewise Linear Functions (In class notes)** *State domain & range, too!

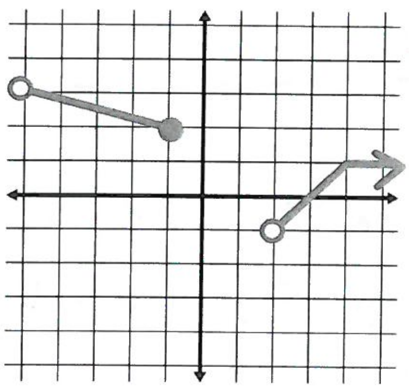
A.



$$f(x)=\begin{cases}-\frac{1}{3}x, & -3\le x\le 0\\ x, & 0\le x\le 2\\ 2, & 2\le x<5\end{cases}$$

*D: $x\in[-3,5)$ R: $y\in[0,2]$

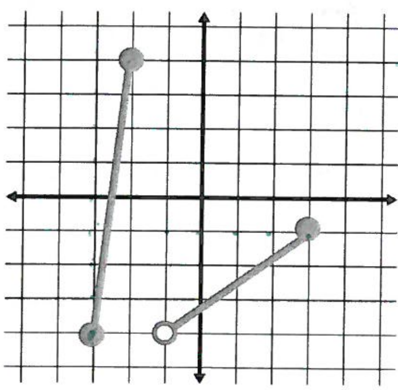
B.



$$f(x)=\begin{cases}-\frac{1}{4}(x+1)+2, & -5<x\le 1\\ (x-2)-1, & 2<x\le 4\\ 1, & x\ge 4\end{cases}$$

*D: $x\in(-5,-1)\cup(2,\infty)$ R: $y\in(-1,1]\cup[2,3)$

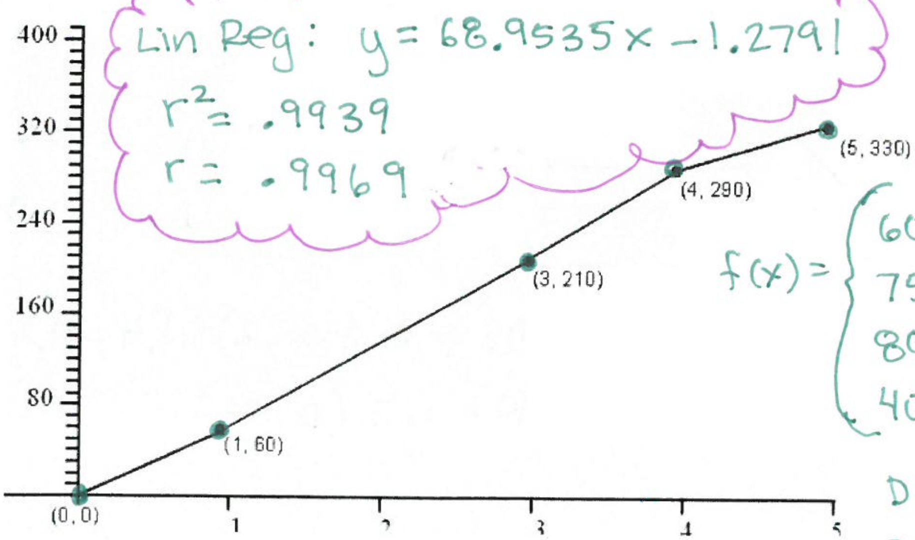
C.



$$f(x)=\begin{cases}8(x+3)-4, & -3\le x\le -2\\ \frac{3}{4}(x+1)-1, & -1<x\le 3\end{cases}$$

*D: $x\in[-3,-2]\cup(-1,3]$ R: $y\in[-4,4]$

H.



Lin Reg: $y=68.9535x-1.2791$

$r^2=.9939$

$r=.9969$

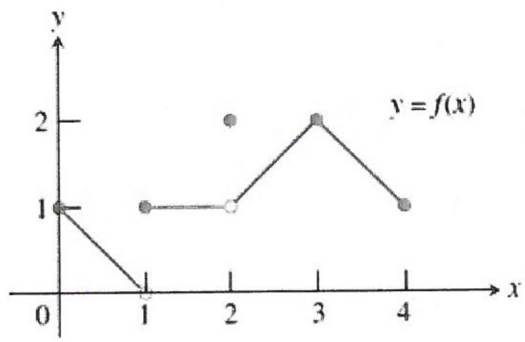
$$f(x)=\begin{cases}60x, & 0\le x\le 1\\ 75(x-1)+60, & 1\le x\le 3\\ 80(x-3)+210, & 3\le x\le 4\\ 40(x-4)+290, & 4\le x\le 5\end{cases}$$

D: $x\in[0,5]$

R: $y\in[0,330]$

3. Writing Piecewise Linear Functions (In class notes)

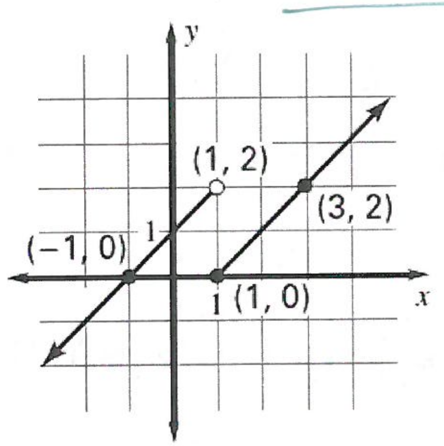
E.



$$f(x) = \begin{cases} -x+1, & 0 \le x < 1 \\ 1, & 1 \le x < 2 \\ 2, & x = 2 \\ (x-3)+2, & 2 < x \le 3 \\ -(x-3)+2, & 3 \le x \le 4 \end{cases}$$

D: $x \in [0,4]$ R: $y \in (0,2]$

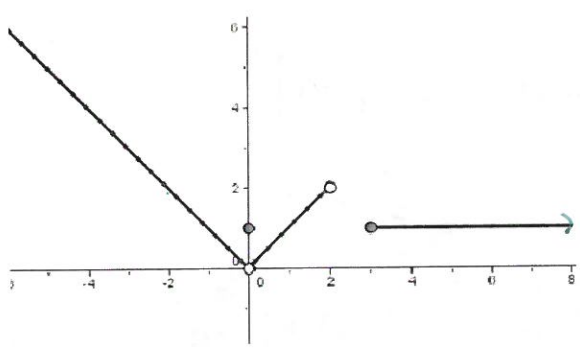
F.



$$f(x) = \begin{cases} x+1, & x < 1 \\ (x-1), & x \ge 1 \end{cases}$$

D: $x \in \mathbb{R}$, R: $y \in \mathbb{R}$

G



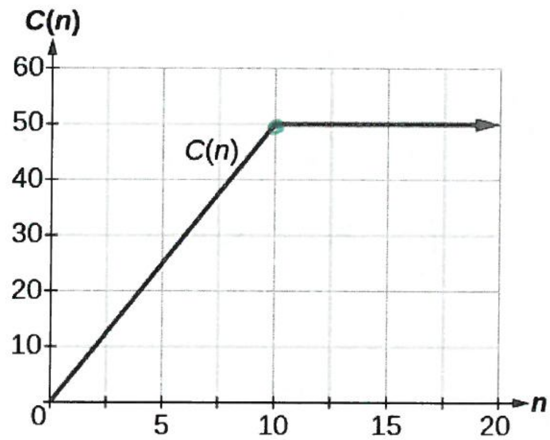
$$f(x) = \begin{cases} -x, & x < 0 \\ 1, & x = 0 \\ x, & 0 < x < 2 \\ 1, & x \ge 3 \end{cases}$$

D: $x \in (-\infty, 2) \cup [4, \infty)$

R: $y \in (0, \infty)$

Using Piecewise Functions to tell a story & make predictions:

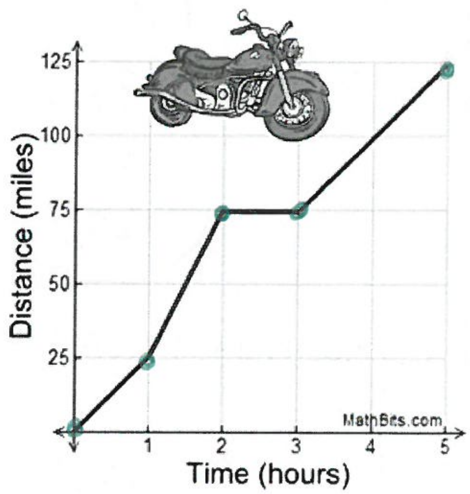
I.



$$f(x)=\begin{cases} 5x, & 0 \le x \le 10 \\ 50, & x \ge 10 \end{cases}$$

D: $x \ge 0$, R: $y \in [0, 50]$

J.



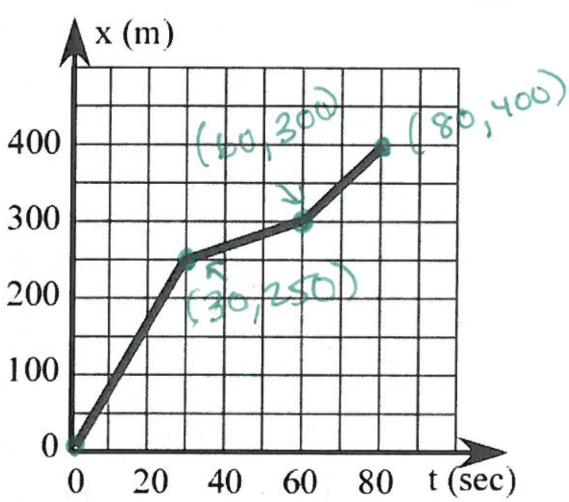
$$f(x)=\begin{cases} 25x, & 0 \le x \le 1 \\ 50(x-1)+25, & 1 < x \le 2 \\ 75(x-2)+75, & 2 \le x \le 3 \\ 25(x-3)+75, & 3 < x \le 5 \end{cases}$$

Lin Reg: $y = 24.6622 + 5.7432$

$r^2 = .9475$

$r = .9734$

K.



$$f(x)=\begin{cases} \frac{25}{3}x, & 0 \le x \le 30 \\ \frac{5}{3}(x-30)+250, & 30 < x \le 60 \\ 5(x-60)+300, & 60 < x \le 80 \end{cases}$$

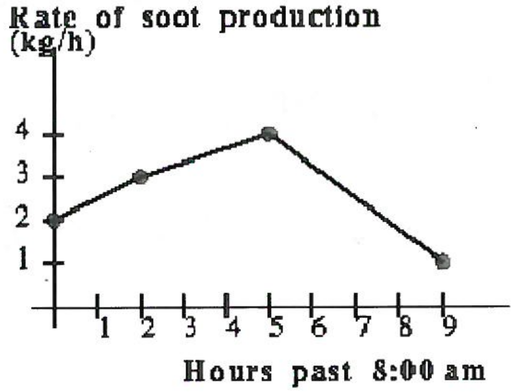
Lin Reg $y = 4.6599x + 39.4558$

$r^2 = .9186$

$r = .9584$

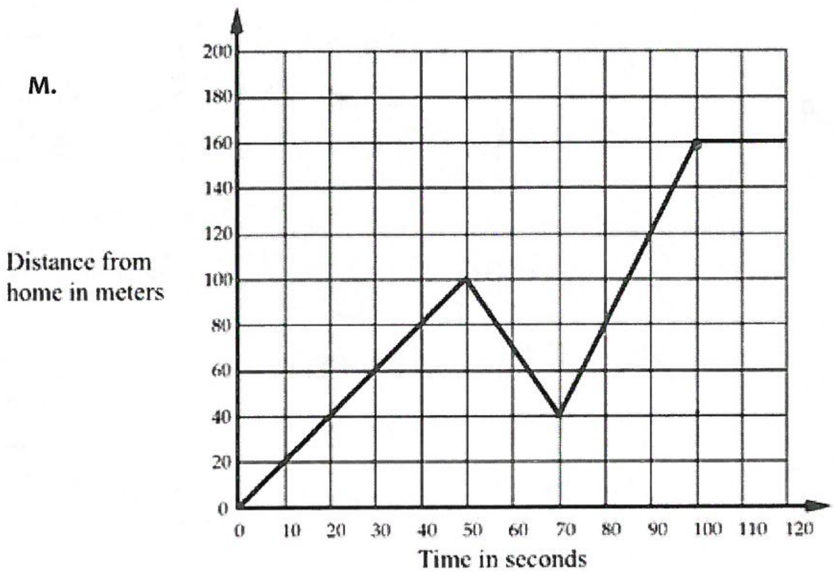
Using Piecewise Functions to tell a story & make predictions:

L.



$$f(x)=\begin{cases}\frac{1}{2}x+2, & 0\le x\le 2\\ \frac{1}{3}(x-2)+3, & 2\le x\le 5\\ -\frac{3}{4}(x-5)+4, & 5\le x\le 9\end{cases}$$

M.



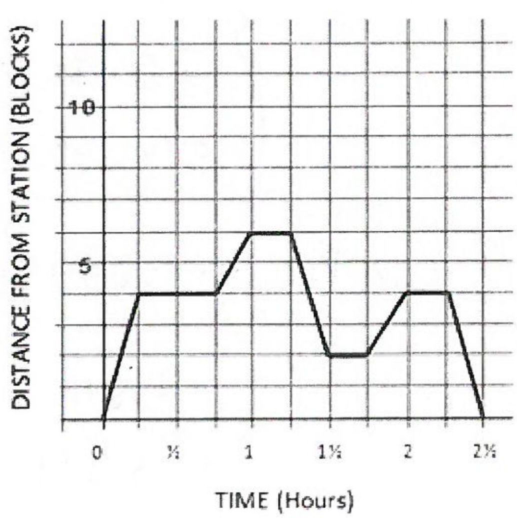
$$f(x)=\begin{cases}2x, & 0\le x\le 50\\ -3x, & 50\le x\le 70\\ 4x, & 70\le x\le 100\\ 160, & 100\le x\le 120\end{cases}$$

Lin Reg $y = 1.3502x + 0.1843$

$r^2 = .7727$

$r = .8792$

N.



$$f(x)=\begin{cases}16x, & 0\le x\le 0.25\\ 4, & .25\le x\le 0.75\\ 8(x-1)+6, & .75\le x\le 1\\ 6, & 1\le x\le 1.25\\ -12(x-1)+6, & 1.25\le x\le 1.5\\ 2(x-1.5)+2, & 1.5\le x\le 1.75\\ 8(x-2)+4, & 1.75\le x\le 2\\ 4, & 2\le x\le 2.25\\ -16(x-2)+4, & 2.25\le x\le 2.5\end{cases}$$