Name: Answer Key Per: ______ Date: ______________

Serafino ▪ Precalculus

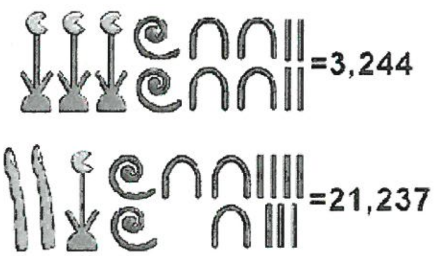
## 3B Radians

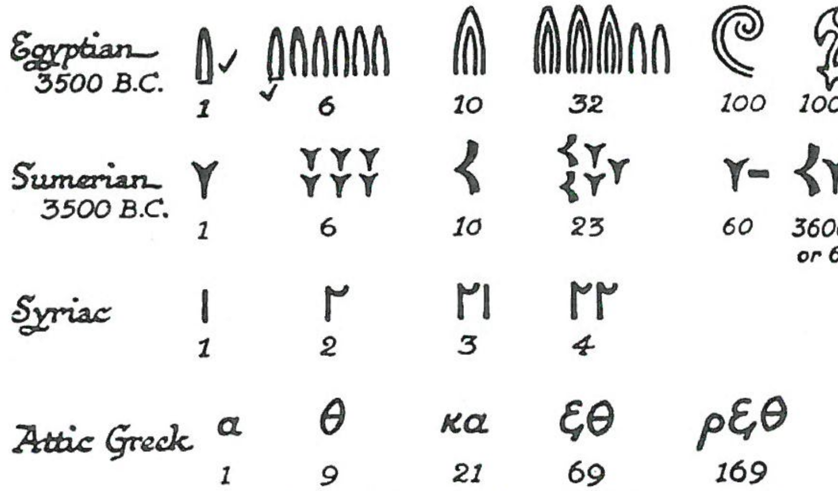
### 3.2 Notes / Homework

**First, some history ...**

People have been doing math for a long time. Throughout history, different cultures thought of numbers differently. They used different symbols and different numbers to measure and represent the world around them. Even now, the way we write dates, times, and measure things isn't universal. (That map shows the countries that don't use the metric system).

Ancient Egyptian Numerals:



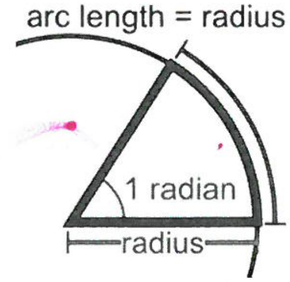

(Reproduced from "Mathematics for the Million," by L. Hogben.)
Ancient Number Scripts.

We have to remind ourselves that when working with numbers, someone at some point, made those numbers up. The number of hours in a day, months in a year, inches in a foot, and even degrees in a triangle or circle... all based on how people thought the world worked or how they could best make sense of it.

The Ancient Babylonians in Mesopotamia ($\approx$ 5000 BC) used "6" and "60" in most of their calculations. Their whole system of mathematics was Base 60 (ours is base 10) because it has so many factors, making it easier to work with fractions. In fact, to get the circumference of a circle, the Babylonians simply multiplied the radius by 6. Thanks to them, there are 60 seconds in a minutes, 60 minutes in an hour, and 360 degrees in a circle.

**Radians are more practical...**

A radian is a way to measure an angle that doesn't rely on an arbitrary number, like 360. It relies only on the length of the radius of the circle.



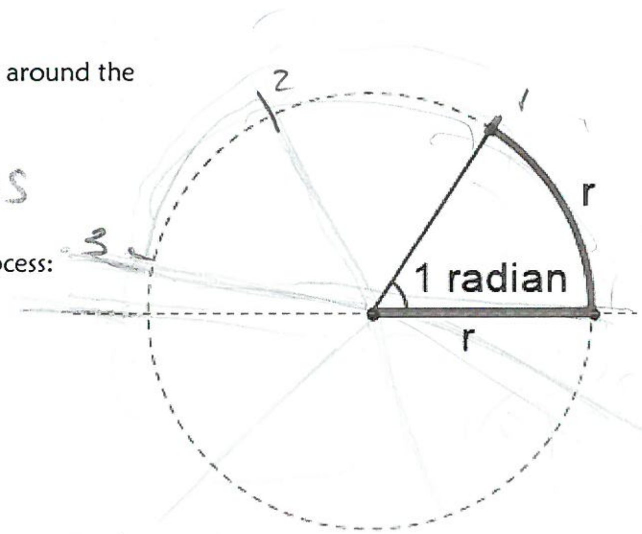

**One radian is the measure of the central angle when the arc length is equal to the radius length .**

Basically, if I took one radius and wrapped it around the circumference, the distance it would create and angle of 1 radian. In that case, $\theta = 1$
If I put two radii along the arc, $\theta = 2$, etc...

We still note angles as Θ, but if you don't put the degree symbol, it is assumed you're measuring your angle in radians.

How many radians are in a circle?
Well, see if you can estimate how many times you could wrap the radius around the
It may help to see how many times you can go around half of the circle.

a little more than 3 times




Unnecessarily and painfully slow animation of the "radius wrapping" process:
https://www.youtube.com/watch?v=7QhgYX8cRE4

**Calculating Radians:**
To calculate exactly how many times a radian fits inside a circle, let's do some simple
Another way of asking "how many radians" is asking
"How many times does that radius fit inside the given arc?"
So let's call the radius "r" and length of the arc, "s".

$$\theta = \frac{arc\ length}{radius} = \frac{s}{r}$$

A quarter of a circle:
Find the arc length in a circle with a 90 ° central angle and a radius of 8.
Then, let's divide by the radius to see how many times it fits inside.

$\frac{1}{4} 16\pi$

Half a circle:
Find the arc length in a circle with a 180 ° central angle and a radius of 4.
Again, divide by the radius to figure out how many radii fit.

$\frac{180}{30} \cdot$ $\frac{1}{2} \cdot 8\pi$

The whole circle:
Find the arc length in a circle with a 360° central angle and a radius of 6
Now let's divide by the radius to see how many times it fits inside.

$1 \cdot 12\pi$

| s | θ |
|---|---|
| $4\pi$ | $\frac{4\pi}{8} = \boxed{\frac{\pi}{2}}$ |
| $4\pi$ | $\frac{4\pi}{4}$ $\boxed{\pi}$ |
| $12\pi$ | $\frac{12\pi}{6} = \boxed{2\pi}$ |

$360° \rightarrow 2\pi$

**Converting:**

Degrees → Radians: $\cdot \frac{\pi}{180}$

Radians → Degrees: $\cdot \frac{180}{\pi}$

So how many radians are in 360°?

Exact: $\boxed{2\pi}$

Approximation: $\approx \boxed{6.2832}$

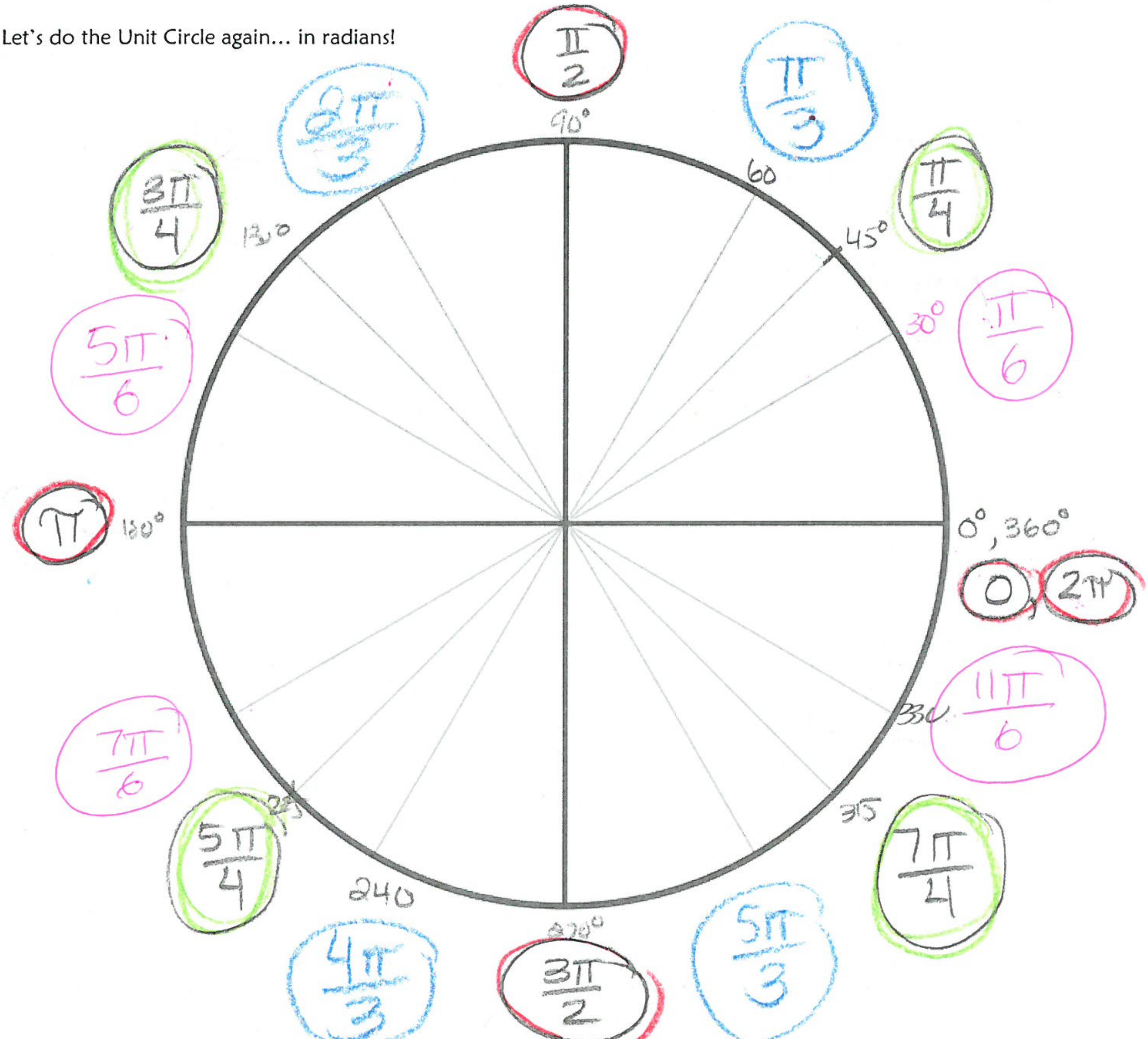
How many degrees are in 1 radian?

$1 \cdot \frac{180}{\pi} =$

Exact: $\boxed{\frac{180°}{\pi}}$

Approximation: $\boxed{\approx 57.296°}$

Let's do the Unit Circle again… in radians!




Let's see if we can find some patterns with the reference angles and the radian measures:

| Quadrantal angles | θ' = 30° | θ' = 45° | θ' = 60° |
|---|---|---|---|
| $\frac{\#}{1}$ or $\frac{\#}{2}$ | $\frac{\#}{6}$ | $\frac{\#}{4}$ | $\frac{\#}{3}$ |

1. Convert the following:

$120^\circ = \frac{120\cdot\pi}{180}\ \boxed{\frac{2\pi}{3}}$

$160^\circ = \boxed{\frac{8\pi}{9}}$

$265^\circ = \boxed{\frac{53\pi}{36}}$

$400^\circ = \boxed{\frac{20\pi}{9}}$

$\frac{\pi}{8} = \cdot\frac{180}{\pi}\ \boxed{\frac{45}{2}}^{\circ}$

$3\pi = \frac{180}{\pi}\ \boxed{540^\circ}$

$\frac{-7\pi}{12} = \boxed{-105^\circ}$

$5 = \frac{180}{\pi}\ \boxed{\frac{900}{\pi}}^{\circ}$

2. Fill in this chart

| $\theta$ | 0 | $\pi/6$ | $\pi/4$ | $\pi/3$ | $\pi/2$ |
|---|---|---|---|---|---|
| $\sin(\theta)$ | 0 | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1 |
| $\cos(\theta)$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | 0 |
| $\tan(\theta)$ | 0 | $\sqrt{3}/3$ | 1 | $\sqrt{3}$ | und |