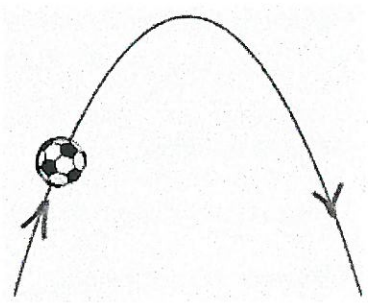
Name: Answer Key Per: 6 Date: _____
Serafino · Algebra 2E

3A2 Equations & Applications of Vertex Form

Notes Packet



PART 1: SOLVING FOR ROOTS

For each of the following quadratics:
a. Determine how many real roots it will have before solving
b. Solve for them using square roots
c. Classify them as rational, irrational, or imaginary
d. Check to see the AOS is the same # that you are ± from
e. If it's irrational, approximate the decimal to two places.

1. $f(x) = x^2 - 9$ (2)

$x^2 - 9 = 0$
$\sqrt{x^2} = \sqrt{9}$
$x = \pm 3$
rational

2. $f(x) = 4x^2 + 108$ (0)

$4x^2 + 108 = 0$
$\frac{4x^2}{4} = \frac{-108}{4}$
$x^2 = \sqrt{-27}$
$x = \pm 3i\sqrt{3}$
imaginary

3. $f(x) = 3(x-3)^2 - 42$ (2)

$3(x-3)^2 - 42 = 0$
$3(x-3)^2 = 42$
$\sqrt{(x-3)^2} = \sqrt{14}$
$x = 3 \pm \sqrt{14}$
irrational

≈ -0.7416
≈ 6.74

4. $f(x) = -\frac{1}{2}(x-1)^2 + 2$ (2)

$0 = -\frac{1}{2}(x-1)^2 + 2$ $x = 1 \pm 2$
$-2 = -\frac{1}{2}(x-1)^2$ (-2)
$4 = (x-1)^2$
$x - 1 = \pm 2$
$x = -1$ or 3
rational

5. $f(x) = 3(x+2)^2$ (1)

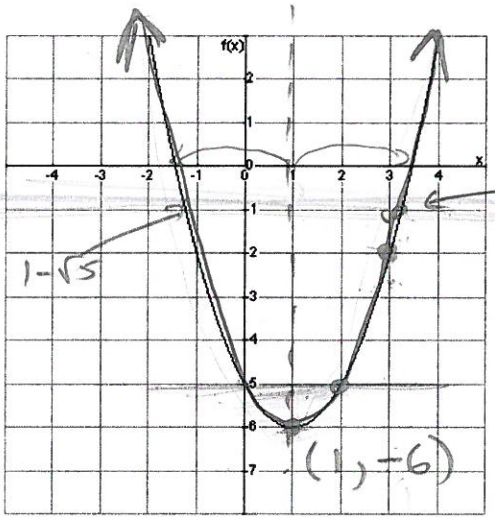
$3(x+2)^2 = 0$
$(x+2)^2 = 0$
$x + 2 = 0$
$x = -2$
rational

6. $f(x) = \frac{1}{2}(3x+5)^2 + 24$ (0)

$\frac{1}{2}(3x-5)^2 + 24 = 0$
$(3x-5)^2 + 48 = 0$
$\sqrt{(3x+5)^2} = \sqrt{-48}$
$3x + 5 = \pm 4i\sqrt{3}$
$\frac{3x}{3} = \frac{-5 \pm 4i\sqrt{3}}{3}$
$x = \frac{-5 \pm 4i\sqrt{3}}{3}$
imaginary

PART 2: ANALYZING FUNCTIONS

7. Use the graph of the function to the right:



a. Write the equation of this function:

$y = 1(x-1)^2 - 6$

b. What is the AOS? $x = 1$

c. What is this function's max or min?

d. Where is f(x) decreasing? $x < 1$ or $x \in (-\infty, 1)$

e. Where is f(x) increasing? $x > 1$ or $x \in (1, \infty)$

f. f(x) = 0 $x = 1 \pm \sqrt{6}$

$(x-1)^2 - 6 = 0$

$\sqrt{(x-1)^2} = \sqrt{6}$

$x - 1 = \pm\sqrt{6}$

7. f(x) = – 6 $x = 1$

$-6 = (x-1)^2 - 6$

8. f(x) = –9 no real solution!

$x = 1 \pm i\sqrt{3}$

9. f(x) < –5 $0 < x < 2$

$(x-1)^2 - 6 = -5$

$\sqrt{(x-1)^2} = \sqrt{1}$

$x - 1 = \pm 1$

1 ± 1

10. f(x) < –1 $1-\sqrt{5} < x < 1+\sqrt{5}$

11. f(x) ≥ 115 $x \leq -10$ or $x \geq 12$

12. f(x) > –10 All real #'s $x \in \mathbb{R}$

13. Determine the max/min if f(x) was compressed by a factor of 1/3.

min @ y = -2

14. Determine the y–intercept if f(x) was stretched by a factor of 2 and shifted 3 to the left?

$f(x) = (x-1)^2 - 6$ $(0, -4)$

$2(x-1)^2 - 12$

$+3$

$g(x) = 2(x+2)^2 - 12$

15. Determine the x–intercepts if f(x) was reflected over the x–axis.

$y = (x-1)^2 - 6$

vs $y = -(x-1)^2 + 6$

$-(x-1)^2 + 6 = 0$ $x = 1 \pm \sqrt{6}$

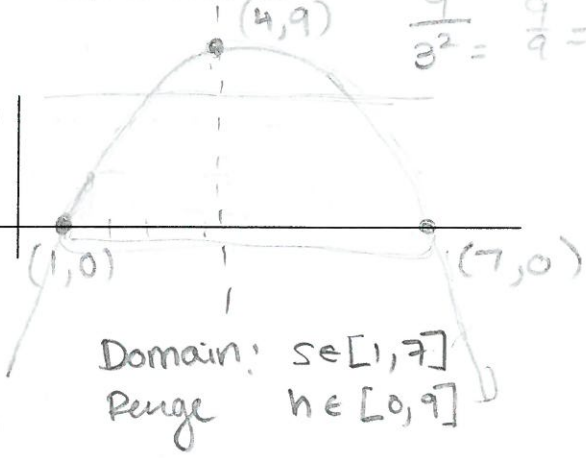
they're the same

PART 3: APPLICATIONS

8. Billy is playing soccer. The whistle blows... and he takes a running start to kick the ball. He kicks the ball 1 second after the whistle blows and it reaches a maximum height of 9 feet, after 4 seconds.

a. Write the equation of the path of the ball:

$h(s) = -(s-4)^2 + 9$



$\frac{9}{3^2} = \frac{9}{9} = 1$

Domain: $s \in [1,7]$
Range $h \in [0,9]$

b. How long is the ball in the air?

$-(s-4)^2 + 9 = 0$
$+\sqrt{(s-4)^2} = \pm\sqrt{9}$
$s - 4 = \pm 3$
$s = 4 \pm 3$
$s = 1$ or 7

6 seconds

c. Billy is 5 feet tall. When is the ball eye level?

2pts

$-(s-4)^2 + 9 = 5$
$-(s-4)^2 = -4$
$\sqrt{(s-4)^2} = \sqrt{4}$
$s - 4 = \pm 2$
4 ± 2

at 2 seconds or 6 seconds

d. When is the ball higher than 8 feet in the air?

$-(x-4)^2 + 9 > 8$
$-(x-4)^2 > -1$
$(x-4)^2 < 1$
$x - 4^2 < 1$
4 ± 1

$3 < x < 5$

b/w 3 and 5 seconds

e. When is the ball 6 inches off the ground?

$-(x-4)^2 + 9 = .5$
$-(x-4)^2 = -8.5$
$\sqrt{(x-4)^2} = \sqrt{8.5}$
$x - 4 = \pm 2.9155$

$x \approx 1.0845$ seconds or 6.9155 sec

f. What equation would model the path of the ball if Billy kicked it 10 seconds later?

shift 10 right, $f(x-10)$

$h(s) = -(x-14)^2 + 9$

g. Jumping Jennifer tried to catch the ball mid–air. She jumps off the ground at 6 seconds, reaches a maximum height of 3 feet one second after that.

(6, 0) (7, 3)

$\frac{3}{1^2} = 3$

What equation is modeled by the Jennifer's path? $j(s) = -3(s-7)^2 + 3$

Graph the two functions on your calculator. Use your calculator's "calculate intersection" function to determine if the jumper catches the ball. If so, after at many seconds after the whistle was blown and how high up in the air?

yes! at 6.56 seconds, 2.43ft in the air

h. What is the soccer ball's average speed (rate of change) between...

mad ups.

1 and 1.5 seconds?

(1, 0) (1.5, 2.75) $\frac{\Delta y}{\Delta x}$ $\frac{2.75}{0.5}$

5.5 ft/sec

1 and 3 seconds?

(1,0) (3,8) $\frac{8}{2}$

4 ft/sec

2 and 4 seconds?

(2, 5) (4, 9) $\frac{4}{2}$

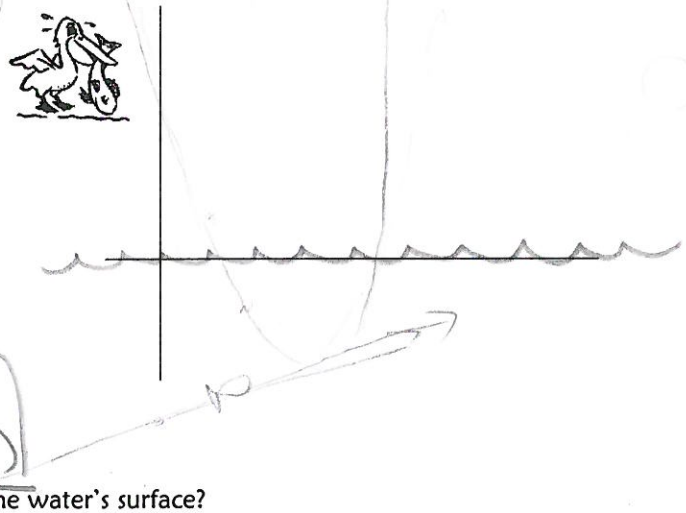
2 ft/sec

3 and 5 seconds?

(3,8) (5,8)

0 ft/sec

9. A pelican flying above the ocean, sees Nutritious Nemo in the waters below and dives to get him. The pelican's location is modeled by $p(t) = 1.8\,(t-5)^2 - 4.7$, where t represents seconds after he sees Nemo. Nemo's location is modeled by the function $n(t) = 0.58t - 7.8$.



a. How high up in the air was the pelican when he first saw Nemo?

$p(0) =$ **40.3 ft**

b. How far below the surface was Nemo when the pelican first saw him?

$h(0) = -7.8$ **-7.8 feet (below surface)**

c. How many seconds does it take for the pelican to get to the water's surface?

$1.8(x-5)^2 - 4.7 = 0$

$1.8(x-5)^2 = 4.7$

$(x-5)^2 = 2.611$

$x - 5 = \pm 1.61589$

$x = 5 \pm 1.61589$

$\approx 3.384, \approx 6.615$

≈ 3.384 sec

d. Graph both functions to see if the graphs intersect. Does the pelican eat Nemo at the lowest point of his dive? If yes, say after how many seconds and at what depth. If no, by how much does the pelican miss catching Nemo?

$p(5) = -4.7$

$n(5) = -4.9$

The functions do not intersect, so no, he doesn't get him (he misses him by 0.2 ft or 2.4 inches.

e. How long is the pelican holding his breath before he flies back out of the water?

Enters water 3.284
Leaves at 6.615

holds breath for 3.231 sec

f. How high in the air will the pelican be when Nemo surfaces to see what just happened?

$n(t) = 0 \leftarrow t = 13.448$ sec

$0.58t - 7.8 = 0$

$0.58t = 7.8$

$p(13.448) =$ **123.764 ft in air**

g. What is the pelican's average speed between...

0 and 1 seconds?

(0, 40.3) (1, 24.1) $\frac{16.2}{1}$

+16.2 ft/sec

2 and 3 seconds?

(2, 11.5) (3, 2.5) $\frac{9}{1}$

-9 ft/sec

3 and 4 seconds?

(3, 2.5) (4, -2.9)

-5.4 ft/sec