Name: Answer Key Per: 8 Date: _____
Serafino · Algebra 2E

3A2 Equations & Applications of Vertex Form

Notes Packet

PART 1: SOLVING FOR ROOTS

For each of the following quadratics:

a. Determine how many real roots it will have before solving
b. Solve for them using square roots
c. Classify them as rational, irrational, or imaginary
d. Check to see the AOS is the same # that you are ± from
e. Approximate the exact answer with a decimal rounded to 2 places.

1. $f(x) = x^2 - 9$ twice

$0 = x^2 - 9$
$\sqrt{9} = \sqrt{x^2}$
$\pm 3 = x$

$x = 3$ or -3
rational.

AOS
$x = 0$ ✓

2. $f(x) = 4x^2 + 108$ none

$0 = 4x^2 + 108$
$\frac{-108}{4} = \frac{4x^2}{4}$
$\sqrt{x^2} = \sqrt{-27}$
$\pm 3i\sqrt{3}$

$x = 3i\sqrt{3}$ or $-3i\sqrt{3}$
imaging

3. $f(x) = 3(x-3)^2 - 42$ twice

$0 = 3(x-3)^2 - 42$
$\frac{42}{3} = \frac{3(x-3)^2}{3}$
$\sqrt{14} = \sqrt{(x-3)^2}$
$x = 3 \pm \sqrt{14}$

$x = 3 + \sqrt{14}$ or $3 - \sqrt{14}$
$x \approx 6.74$
$\approx -.7416$
irrational

3. $f(x) = -\frac{1}{2}(x-1)^2 + 2$ twice

$0 = -\frac{1}{2}(x-1)^2 + 2$
$-2 = -\frac{1}{2}(x-1)^2$
$\pm\sqrt{4} = \sqrt{(x-1)^2}$
$\pm 2 = x - 1$

AOS!
$x = 1 \pm 2$
$x = -1$ or 3
rational

4. $f(x) = 3(x+2)^2$ one

$\frac{0}{3} = \frac{3(x+2)^2}{3}$
$\sqrt{0} = \sqrt{(x+2)^2}$
$0 = x + 2$
$x = -2$

$x = -2$
rational

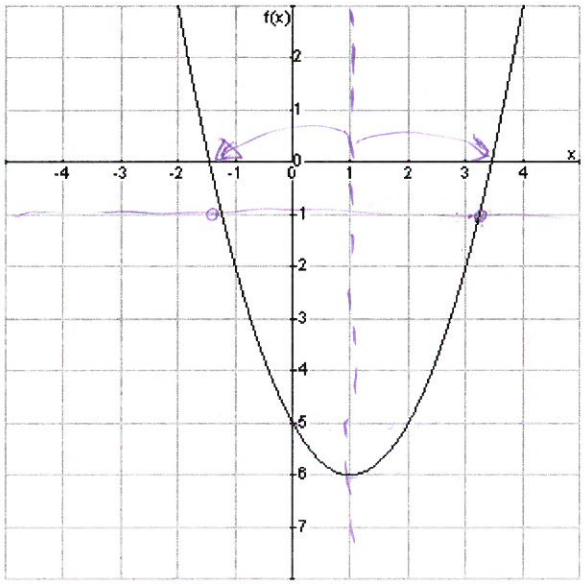
5. $f(x) = \frac{1}{2}(3x+5)^2 + 24$ none

$0 = \frac{1}{2}(3x+5)^2 + 24$
$-24 = \frac{1}{2}(3x+5)^2$
$-48 = (3x+5)^2$
$\pm 4i\sqrt{3} = 3x + 5$
$\frac{-5 \pm 4i\sqrt{3}}{3} = \frac{3x}{3}$

imaginary

$x = \frac{-5+4i\sqrt{3}}{3}$ or $\frac{-5-4i\sqrt{3}}{3}$

PART 2: ANALYZING FUNCTIONS



1. Write the equation of this function:

$y = (x-1)^2 - 6$

2. What is the AOS? $x = 1$

3. What is this function's max or min?

min @ $y = -6$

4. Where is f(x) decreasing?

$x < 1$ or $x \in (-\infty, 1)$

5. Where is f(x) increasing?

$x > 1$ or $x \in (1, \infty)$

6. f(x) = 0

$(x-1)^2 - 6 = 0$

$\sqrt{(x-1)^2} = \sqrt{6}$

$x - 1 = \pm\sqrt{6}$

$x = 1 + \sqrt{6} \approx 3.449$ or $x = 1 - \sqrt{6} \approx -1.449$

7. f(x) = – 6

$-6 = (x-1)^2 - 6$

$x = 1$

8. f(x) = –9

$-9 = (x-1)^2 - 6$

$\pm\sqrt{-3} = \sqrt{(x-1)^2}$

no solution

$x = 1 + i\sqrt{3}$, $1 - i\sqrt{3}$

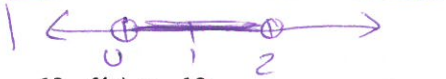
9. f(x) < –5

$(x-1)^2 - 6 < -5$

$(x-1)^2 < 1$

$x - 1 < 1$ and $x > 0$

$0 < x < 2$



10. f(x) < –1

$(x-1)^2 - 6 < -1$

$+6 \quad +6$

$\sqrt{(x-1)^2} < \sqrt{5}$

$x - 1 < \pm\sqrt{5}$

$x < 1 + \sqrt{5}$ and $x > 1 - \sqrt{5}$

$1 - \sqrt{5} < x < 1 + \sqrt{5}$

11. f(x) ≥ 115

$(x-1)^2 - 6 = 115$

$\sqrt{(x-1)^2} = \sqrt{121}$

$x - 1 = \pm 11$

$x = 1 \pm 11$

$x \leq -10$ or $x \geq 12$

12. f(x) > –10

$(x-1)^2 - 6 \geq -10$

$(x-1)^2 \geq -4$

$x \in \mathbb{R}$

13. Determine the max/min if f(x) was compressed by a factor of 1/3.

$\frac{1}{3}\left[(x-1)^2 - 6\right)$

$= \frac{1}{3}(x-1)^2 - 2$

min @ $y = -2$

14. Determine the y–intercept if f(x) was stretched by a factor of 2 and shifted 3 to the left?

$y = (x-1)^2 - 6$

$= 2(x-1)^2 - 12$

$+3$

$y = 2(x+2)^2 - 12$

$f(0) = -4$

(0, −4)

15. Determine the x–intercepts if f(x) was reflected over the x–axis.

$-\left[(x-1)^2 - 6\right] = -(x-1)^2 + 6$

$-(x-1)^2 = -6$

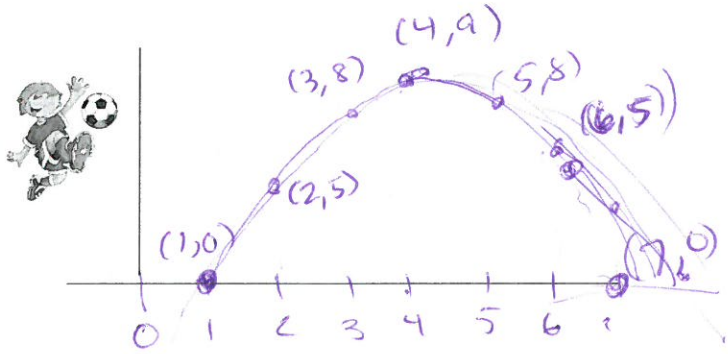
$\sqrt{(x-1)^2} = \sqrt{6}$

$x = 1 \pm \sqrt{6}$

they're the same!

PART 3: APPLICATIONS

1. Billy is playing soccer. The whistle blows and he takes a running start to kick the ball. He kicks the ball 1 second after the whistle blows and it reaches a maximum height of 9 feet, 4 seconds after started running to kick it.



a. Write the equation of the path of the ball:

$\frac{\Delta y}{\Delta x^2} = \frac{9}{3^2} = \frac{9}{9} = 1$

$y = -(x-4)^2 + 9$

b. How long is the ball in the air?

$-(x-4)^2 + 9 = 0$ $x = 4 \pm 3$

$-(x-4)^2 = -9$ $x = 1$ or 7

$(x-4)^2 = 9$

$x - 4 = \pm 3$

6 seconds.

c. Billy is 5 feet tall. When is the ball eye level?

$-(x-4)^2 + 9 = 5$ $x = 4 \pm 2$

$-(x-4)^2 = -4$ $x = 2$ sec or $x = 6$ sec

$\sqrt{(x-4)^2} = \sqrt{4}$

$x - 4 = \pm 2$

d. When is the ball higher than 8 feet in the air?

$-(x-4)^2 + 9 > 8$ $3 < x < 5$

$-(x-4)^2 > -1$ between 3 and 5 seconds

$\sqrt{(x-4)^2} < \sqrt{1}$

$x - 4 < \pm 1$

4 ± 1

e. When is the ball 6 inches off the ground?

$-(x-4)^2 + 9 = .5$ $x \approx 6.9155$ or 1.0845 secons

$\sqrt{(x-4)^2} = \sqrt{8.5}$

$x - 4 = \pm 2.9155$

$x = 4 \pm 2.9155$

f. What equation would model the path of the ball if Billy kicked it 10 seconds later?

$y = -(x-14)^2 + 9$

f. Jumping Jennifer tried to catch the ball mid–air. She jumps off the ground at 6 seconds, reaches a maximum height of 3 feet one second after that.

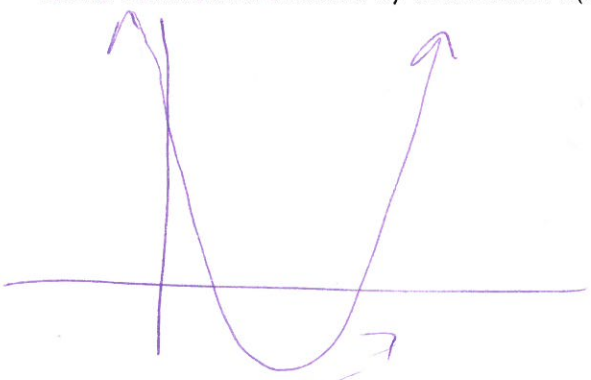
$(6,0)$ $(7,3)$ $a = \frac{3}{1^2} = 3$ $j(x) = -3(x-7)^2 + 3$

What equation is modeled by the Jennifer's path?

Use your calculator's "calculate intersection" function to determine if the jumper catches the ball. If so, at how many seconds and how high up in the air?

Yes, they intsect at 6.564 seconds, 2.428 feet in the air

7. A pelican flying above the ocean, sees Nutritious Nemo in the waters below and dives to get him. The pelican's location is modeled by $p(t) = 1.8\,(t-5)^2 - 4.7$, where t represents seconds after he sees Nemo. Nemo's location is modeled by the function $n(t) = 0.58t - 7.8$.



a. How high up in the air was the pelican when he first saw Nemo?

$p(0) = 1.8(0-5)^2 - 4.7$

$= 40.3$ feet in the air

b. How far below the surface was Nemo when the pelican saw him?

$n(0) = -7.8$ | 7.8 feet below the surface

c. How many seconds does it take for the pelican to get to the water's surface?

$1.8(x-5)^2 - 4.7 = 0$

$1.8(x-5)^2 = 4.7$

$\sqrt{(x-5)^2} = \sqrt{2.6111}$

$x - 5 = \pm 1.61589$

$x = 5 \pm 1.61589$

≈ 3.384 or ≈ 6.615

3.384 seconds

d. Does the pelican reach Nemo? If yes, say when. If no, by how much does the pelican miss catching Nemo?

$p(5) = -4.7$

$n(5) = -4.9$

He misses Nemo by 0.2 feet, or 2.4 inches

(functions do not intersect in calculator, so no!)

e. How long is the pelican holding his breath?

Enters water at (3.384, 0)

Leaves at (6.615, 0)

holds breath for 3.231 seconds

f. How high in the air will the pelican be when Nemo comes up to look around at the surface to see what just happened?

$n(t) = 0.58t - 7.8$

$0.58t - 7.8 = 0$

$0.58t = 7.8$

$n(t) = 0$

$t = 13.448$ sec.

$p(13.448) =$

123.764 ft in the air