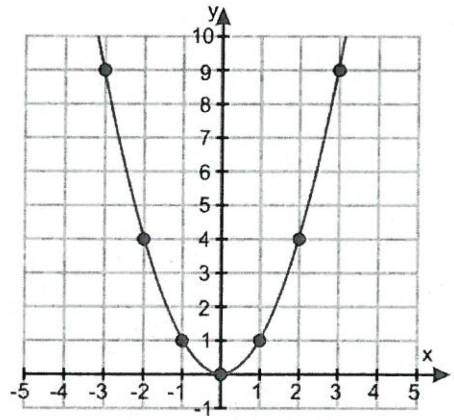
Name: Key No. 0 Per: 1 Date: 11/10/15
Serafino · Algebra 2E M T W R F

3A

Quadratic Functions (Vertex Form)

Notes & Practice Packet

Parent: $f(x) = x^2$



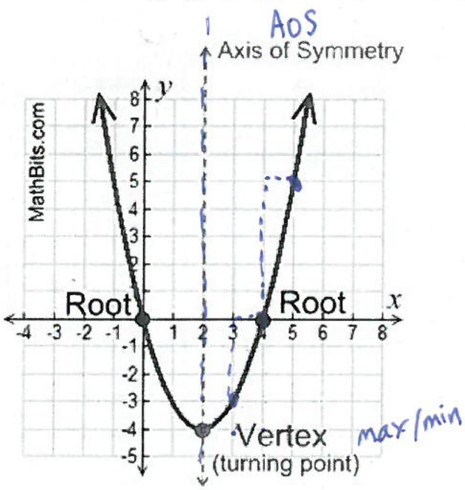
Later, Lines! We have explored all the kinds of functions that have a constant rate of change; a consistent growth rate from point to point. For the rest of this course, the functions we explore are going to have rates of change that *vary* from point to point – they will be *curvy*.

Quadratic are polynomials with a degree of 2; their graphs are called parabolas.

Vertex Form:	$y = a(x-h)^2 + k$
Intercept Form:	$y = a(x-x_1)(x-x_2)$
Standard Form:	$y = ax^2 + bx + c$

Quadratics in Vertex Form: $y = a(x-h)^2 + k$

Same growth pattern rules apply. You start at the vertex and then put "in" the input by moving that many units to the left or right, square that input… then multiply by "a"



$f(x) = 1(x-2)^2 - 4$ * *always double check "a"* set = 0

a. Domain: $x \in \mathbb{R}$ Range: $y \geq -4$ AOS: $x = 2$

b. Vertex: $(2,-4)$ Max/Min y-int: $(0,0)$ No. of # x-int: 2

$y = (0-2)^2 - 4$
$(-2)^2 - 4$
$4 - 4 = 0$

d. Use the graph to find the average ROC between…

$2 < x < 3$ $= 1$ $3 < x < 4$ $= 3$ $4 < x < 5$ $= 5$

e. Evaluate: $f(6) = (6-2)^2 - 4$ $4^2 - 4 = 16 - 4$ $= 12$ $f(10) = (10-2)^2 - 4$ $(8)^2 - 4$ $64 - 4$ $= 60$ Average ROC between those points? $\frac{\Delta y}{\Delta x}$ $\frac{48}{4}$ 12

f. Solve:

x–intercepts: $(x-2)^2 - 4 = 0$; $\sqrt{(x-2)^2} = \sqrt{4}$; $(x-2) = \pm 2$; $x = 2 \pm 2$ ↗ 4, 0; $(0,0)$ $(4,0)$

$f(x) = 7$: $(x-2)^2 - 4 = 7$; $\sqrt{(x-2)^2} = \sqrt{11}$; $x = 2 \pm \sqrt{11}$; $(2+\sqrt{11}, 7)$ $(2-\sqrt{11}, 7)$

$f(x) = -4$: $(x-2)^2 - \cancel{4} = -\cancel{4}$; $\sqrt{(x-2)^2} = 0$; $x = 2$; $(2,-4)$

$f(x) = -6$: $(x-2)^2 - 4 = -6$; $\sqrt{(x-2)^2} = \sqrt{-2}$; $\pm$; $x = 2 \pm i\sqrt{2}$

What do we notice about the AOS and the solution(s)? It's in the center!
How can we use the equation in vertex form to pre the number and/or type of solutions?

2 rational 2 irrational 1 rational none! (2 imaginary)

Part I: Analyzing & Graphing in Vertex Form
Fill out the information BEFORE graphing

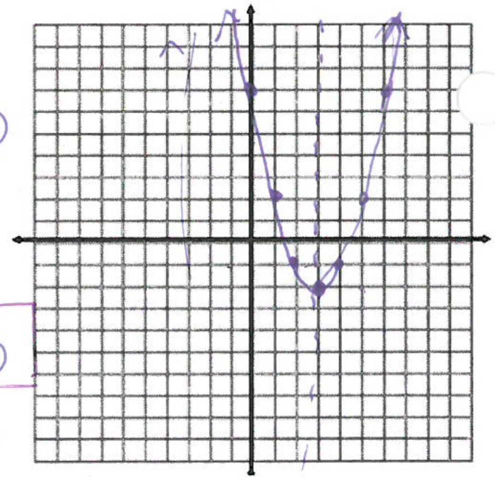
1. $f(x) = (x-3)^2 - 2$ AOS: $x = 3$ Vertex: $(3, -2)$ Max/Min

Range: $y \geq -2$ y–int: $(0-3)^2 - 2$
$9 - 2 = (0, 7)$

x–int: 2

Solve for x–int: $(x-3)^2 - 2 = 0$
$\sqrt{(x-3)^2} = \sqrt{2}$
$x = 3 \pm \sqrt{2}$

$\boxed{(3+\sqrt{2}, 0)\ (3-\sqrt{2}, 0)}$
4.~ 1..~



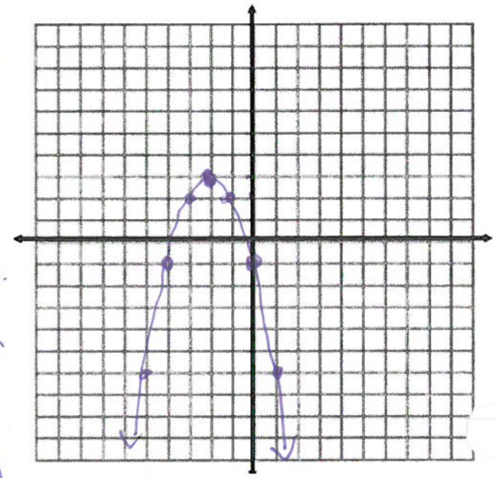
2. $f(x) = -(x+2)^2 + 3$ AOS: $x = -2$ Vertex: $(-2, 3)$ Max/Min

Range: $y \leq 3$ y–int: $-(0+2)^2 + 3$
$-4 + 3 = -1$

x–int: 2

Solve for x–int: $-(x+2)^2 + 3 = 0$
$+(x+2)^2 = +3$
$x = -2 \pm \sqrt{3}$

$\boxed{(-2+\sqrt{3}, 0)\ \ (-2-\sqrt{3}, 0)}$ $\approx 0.\sim$ $\approx -4.\sim$



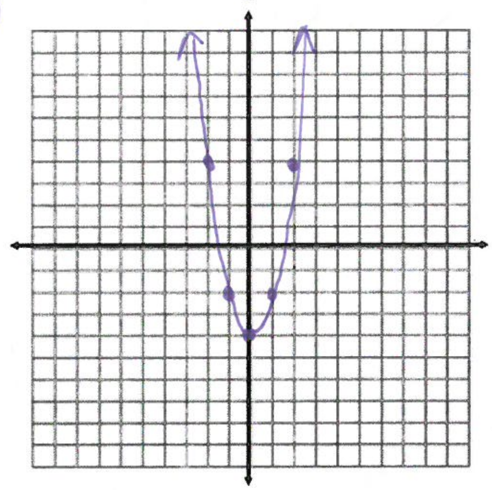
3. $f(x) = 2x^2 - 4$ AOS: $x = 0$ Vertex: $(0, -4)$ Max/Min

Range: $y \geq -4$ y–int: $(0, -4)$

x–int: 2

Solve for x–int: $2x^2 - 4 = 0$
$2x^2 = 4$
$\sqrt{x^2} = \sqrt{2}$
$x = \pm\sqrt{2}$

$\boxed{(\sqrt{2}, 0)\ \ (-\sqrt{2}, 0)}$



4. $f(x) = -3x^2 + 5$ AOS: $x = 0$ Vertex: $(0, 5)$ Max/Min

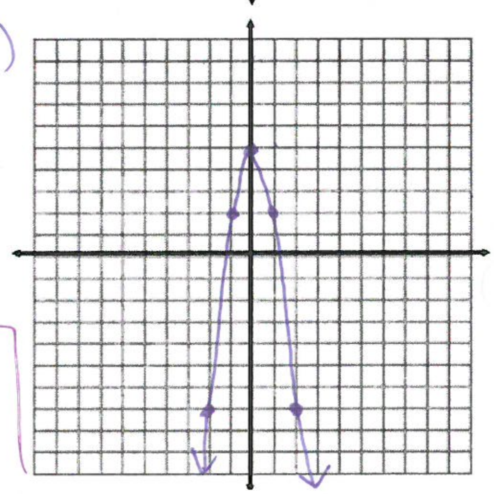
Range: $y \leq 5$ y–int: $(0, 5)$

x–int: 2

Solve for x–int: $-3x^2 + 5 = 0$
$-3x^2 = -5$
$3x^2 = 5$
$\sqrt{x^2} = \sqrt{\frac{5}{3}}$

$x = \frac{\sqrt{5}\,\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}} = \frac{\sqrt{15}}{3}$

$\boxed{\left(\frac{\sqrt{15}}{3}, 0\right)\ \ \left(-\frac{\sqrt{15}}{3}, 0\right)}$

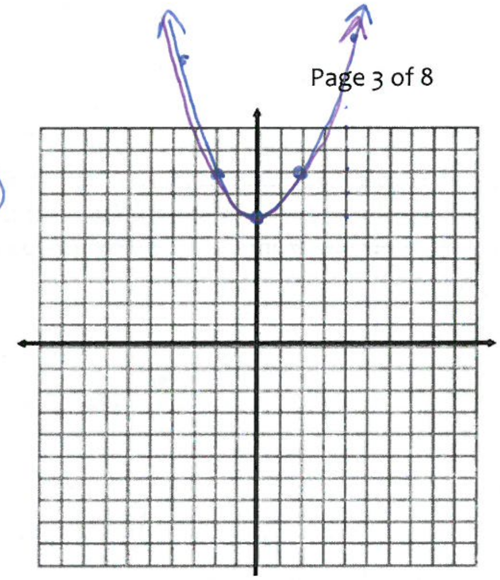


5. $f(x) = \frac{1}{2}(x-2)^2 + 4$ AOS: $x = 2$ Vertex: Max/Min $(2,4)$

Range: $y \geq 4$ y–int: $\frac{1}{2}(0-2)^2 + 4$
$\frac{1}{2}(4) + 4$
$2 + 4 = 6$
$(0,6)$

x–int: 0

Solve for x–int:
$\frac{1}{2}(x-2)^2 + 4 = 0$
$\frac{1}{2}(x-2)^2 = -4$
$\sqrt{(x-2)^2} = \sqrt{-8} = i\sqrt{8} = 2i\sqrt{2}$
$\boxed{x = 2 \pm 2i\sqrt{2}}$

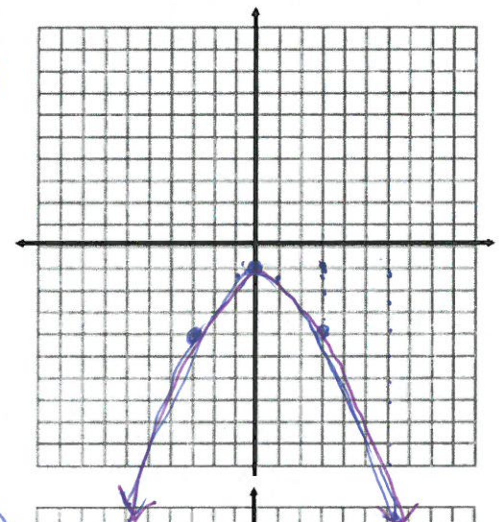


6. $f(x) = -\frac{1}{3}x^2 - 1$ AOS: $x = 0$ Vertex: $(0,-1)$ Max/Min

Range: $y \leq -1$ y–int: $(0,-1)$

x–int: none ! 0 !

Solve for x–int:
$-\frac{1}{3}x^2 - 1 = 0$
$-\frac{1}{3}x^2 = 1$
$\sqrt{x^2} = \sqrt{-3}$
$\boxed{x = \pm i\sqrt{3}}$

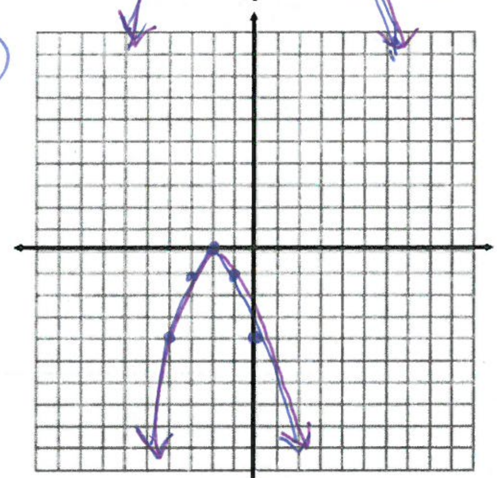


7. $f(x) = -(x+2)^2$ AOS: $x = -2$ Vertex: $(-2,0)$ Max/Min

Range: $y \leq 0$ y–int: $-(0+2)^2$
$-(4) = -4$

x–int: 1 (ON the ground)

Solve for x–int:
$-(x+2)^2 = 0$
$\sqrt{(x+2)^2} = \sqrt{0}$
$\boxed{x = -2}$

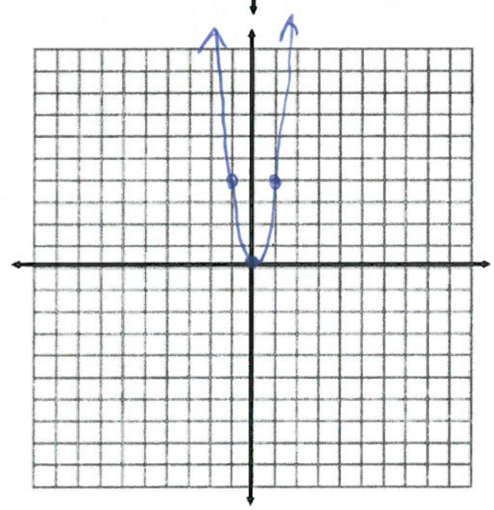


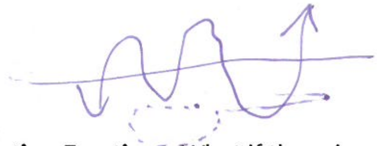
8. $f(x) = 4x^2$ AOS: $x = 0$ Vertex: $(0,0)$ Max/Min

Range: $y \geq 0$ y–int: $(0,0)$

x–int: 1 (on the ground)

Solve for x–int:
$4x^2 = 0$
$x^2 = 0$
$\boxed{x = 0}$



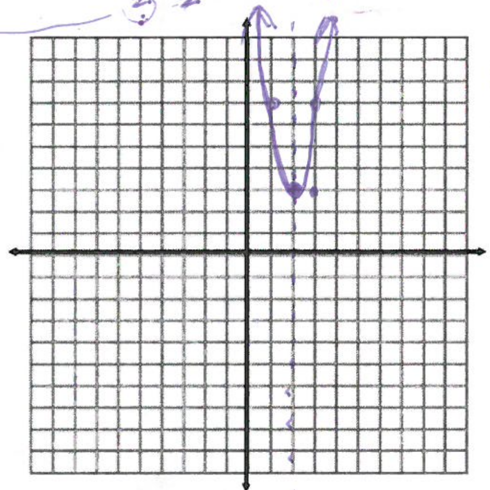


Axis of Symmetry that's a Fraction: What if there is more complicated stuff in the parentheses?
To get the AOS, you can still do the "opposite of the constant over the coefficient".
But for the growth pattern, be careful!! Your true "a" is hiding in the parentheses! When you rescue the leading coefficient from the parentheses, you HAVE to follow the function family rule for what that does to it.

$2x-4=0$
$\frac{2x}{2}=\frac{4}{2}$
$x=2$

V: (2, 3)

9. $f(x) = (2x-4)^2 + 3$ → $4(x-2)^2+3$



AOS $x=2$

V: (2, 3)

x-int: none!

$(2x-4)^2+3=0$

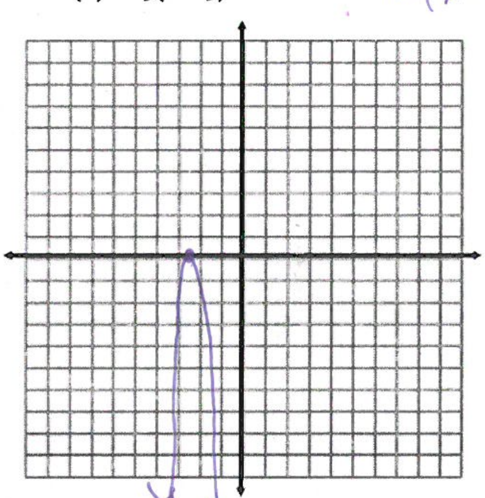
$\sqrt{(2x-4)^2}=\sqrt{-3}$

$2x-4=\pm i\sqrt{3}$

$\frac{2x}{2}=\frac{4\pm i\sqrt{3}}{2}$

$\boxed{x=2\pm\frac{i\sqrt{3}}{2}}$

10. $f(x) = -3(2x+5)^2$ → $-12\left(x+\frac{5}{2}\right)^2$

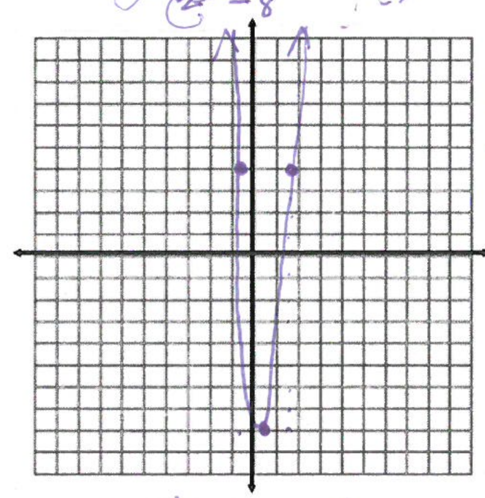


x-int: one!

$-3(2x+5)^2=0$

$\boxed{x=-\frac{5}{2}}$

11. $f(x) = 3(2x-1)^2 + 3$ -8 → $12\left(x-\frac{1}{2}\right)^2-8$



AOS $x=\frac{1}{2}$

V: $\left(\frac{1}{2},-8\right)$

x-int: two!

$3(2x-1)^2-8=0$

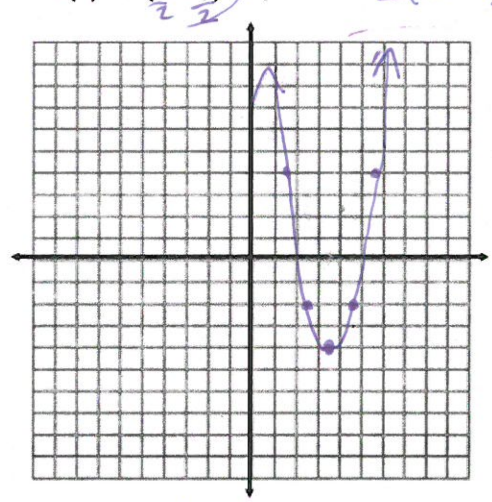
$\frac{3(2x-1)^2}{3}=\sqrt{\frac{8}{3}}$

$\frac{2\sqrt{2}\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}}=\frac{2\sqrt{6}}{3}$

$2x=1\pm\frac{2\sqrt{6}}{3}$

$\boxed{x=\frac{1}{2}\pm\frac{\sqrt{6}}{3}}$

12. $f(x) = ½(2x-7)^2 - 4$ → $2\left(x-\frac{7}{2}\right)^2-4$



x-int: two!

$\frac{1}{2}(2x-7)^2-4=0$

$\frac{1}{2}(2x-7)^2=4$

$\sqrt{(2x-7)^2}=\sqrt{8}$

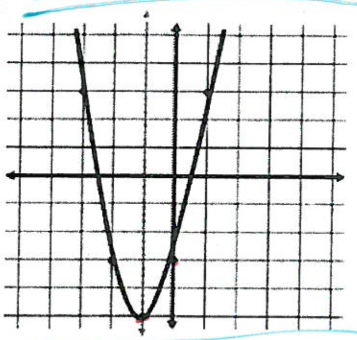
$2x-7=\pm2\sqrt{2}$

$\frac{2x}{2}=\frac{7\pm2\sqrt{2}}{2}$

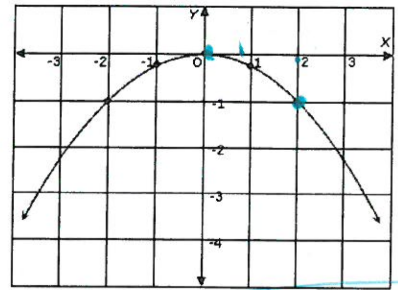
$\boxed{x=\frac{7}{2}\pm\sqrt{2}}$

Part II: Writing Equations of Graphs in Vertex Form:

1. $y = 2(x+1)^2 - 5$

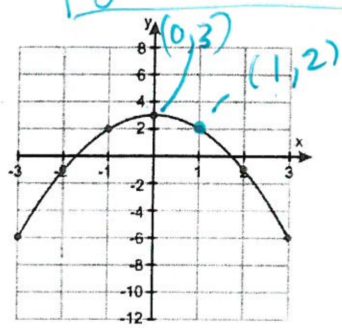


2. $y = -\frac{1}{4}x^2$

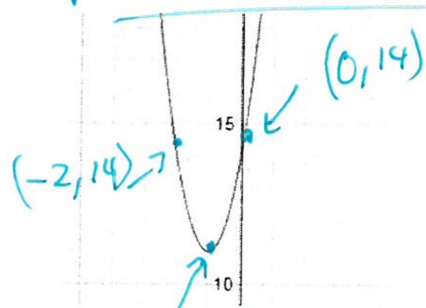


$\frac{4a}{4} = \frac{-1}{4}$

3. $y = -x^2 + 3$



4. $y = 3(x+1)^2 + 11$



$y = a(x-h)^2 + k$

5. Vertex: $\overset{h,k}{(-3, -2)}$ Point: $\overset{x,y}{(2, 5)}$

$y = a(x+3)^2 - 2$

$5 = a(2+3)^2 - 2$
$\quad +2 \quad +2$

$7 = a(25)$

$a = \frac{7}{25}$

$\boxed{y = \frac{7}{25}(x+3)^2 - 2}$

6. Vertex: (2, 4) Point: (6, –4)

$y = -\frac{1}{2}(x-2)^2 + 4$

7. Vertex: (–4, 1) Point: (–2, 13)

$y = 3(x+4)^2 + 1$

8. Vertex: (0, 4) Point: (3, 1)

$y = -\frac{1}{3}x^2 + 4$

9. Vertex: (5, 0) Point: (–2, –3)

$y = \frac{-3}{49}(x-5)^2$

10. Vertex: (0, 0) Point: (–5, 3)

$y = \frac{3}{25}x^2$

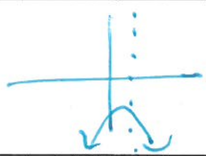
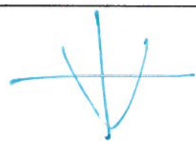
11. Vertex: (–1, 8) Point: (–2, 11)

$y = 3(x+1)^2 + 8$

12. Vertex: (–70, 58) Point: (–59, 48)

$y = \frac{-10}{121}(x+70)^2 + 58$

13. Give a sketch and a function that satisfies the given requirements.

Sketch	Function	Range	AOS
	anything positive $y = (x+3)^2 + 2$	$y \in [2, \infty)$	$x = -3$
	anything neg. $y = -2(x-1)^2 - 3$	$y \in (-\infty. -3]$	$x = 1$
	$y = -x^2 + 6$	$y \in (-\infty, 6]$	$x = 0$
	$y = x^2 - 5$	$y \in [-5, \infty)$	$x = 0$

Part III: Transformations: $a(x) = 4x^2 - 8$ $b(x) = -2(x-1)^2 + 4$

1. Use function notation to illustrate the given transformations:

a) from $a(x) \to b(x)$ $-2x^2 + 4$

$$\boxed{b(x) = -\frac{1}{2}a(x-1)}$$

b) from $b(x) \to a(x)$

$$\boxed{a(x) = -2\,b(x+1)}$$

2. If $c(x) = -3f(x+5)^2 + 1$, list the transformations: reflect over x-axis, stretch vert, left 5, up 1

a) $c(x) = -3\,a(x+5)^2 + 1$

$-3[4x^2 - 8] = -12x^2 + 24$ $+5$ $+1$

$$\boxed{c(x) = -12(x+5)^2 + 25}$$

b) $c(x) = -3\,b(x+5)^2 + 1$

$-3[-2(x-1)^2 + 4]$

$6(x-1)^2 - 12$ $+5$ $+1$

$$\boxed{c(x) = 6(x+4)^2 - 11}$$

2. If $d(x) = ½\,f(x-2)^2 - 10$, list the transformations: vertical compression, right 2, down 10

a) $d(x) = ½\,a(x-2)^2 - 10$

$\frac{1}{2}(4x^2 - 8)$

$2x^2 - 4$ -10

$$\boxed{d(x) = 2(x-2)^2 - 14}$$

b) $d(x) = ½\,b(x-2)^2 - 10$

$\frac{1}{2}[-2(x-1)^2 + 4)$

$-1(x-1)^2 + 2$

$$\boxed{d(x) = -(x-3)^2 - 8}$$

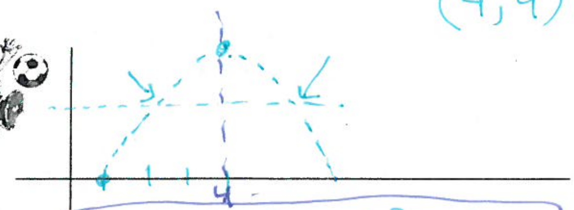
PART IV: APPLICATIONS

1. Dean is playing soccer. The whistle blows... and he takes a running start to kick the ball. He kicks the ball 1 second after the whistle blows and it reaches a maximum height of 9 feet, after 4 seconds.

$(1,0)$
$(4,9)$

a. Write the equation of the path of the ball:

$y=a(x-h)^2+k$
$0=a(1-4)^2+9$
$-9 \qquad -9$
$-9=a9$
$a=1$



b. How long is the ball in the air?

$\boxed{6 \text{ seconds}}$ from 1 sec to 7 sec

$\boxed{d(x) = -(x-4)^2+9}$ Dean

c. Dean is 5 feet tall. When is the ball eye level? ~~air?~~

$5=-(x-4)^2+9$
$-9 \qquad -9$
$\sqrt{-4}=\sqrt{-(x-4)^2}$
$\pm 2 = x-4$
$x=4\pm 2$

at 2 sec & 6 sec after the kick

d. When is the ball higher than 8 feet in the air?

$8=-(x-4)^2+9$
$\sqrt{1}=\sqrt{(x-4)^2}$
$x=4\pm 1$ 5, 3

Between 3 and 5 sec

e. When is the ball 6 inches off the ground? ~~the~~

$-\frac{18}{2}\ \frac{1}{2}=-(x-4)^2+9$
-9
$\sqrt{\frac{+17}{2}}=\sqrt{(x-4)}$
$x=4\pm\frac{\sqrt{34}}{2}$
$4+\frac{\sqrt{34}}{2}=6.92$ sec and
$4-\frac{\sqrt{34}}{2}=1.08$ sec

f. What equation would model the path of the ball if Dean kicked it 10 seconds later?

instead of (1,0) its (11,0)

$y=-(x-4)^2+9$
-10

$\boxed{y=-(x-14)^2+9}$

2. Jumping Jennifer tried to catch the ball mid–air.
She jumps off the ground at 6 seconds, reaches a maximum height of 3 feet one second after that.

$(6,0) \quad v:(7,3)$

$y=a(x-h)^2+k$
$0=a(6-7)^2+3$
$-3=a(-1)^2 \quad a=-3$

a) What equation is modeled by the Jennifer's path?

$\boxed{j(x)=-3(x-7)^2+3}$ ← Jen

b) What is the soccer ball's average speed (rate of change) between... Evaluate $d(x)$

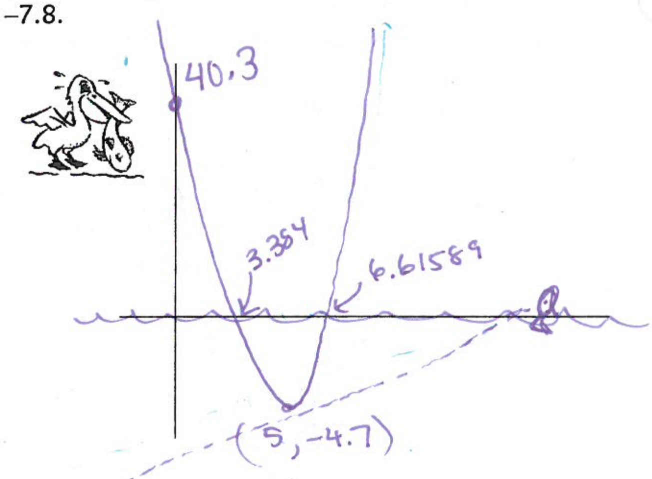
1 and 1.5 seconds?	1 and 3 seconds?	2 and 4 seconds?	3 and 5 seconds?
(1,0) (1.5, 2.75)	(1,0) (3,8)	(2,5) (4,9)	(3,8) (5,8)
≈ 1.83 ft/sec	= 4 ft/sec	= 2 ft/sec	0 ft/sec

c) Graph the two functions on your calculator. Use your calculator's "calculate intersection" function to determine if the jumper catches the ball. If so, after at many seconds after the whistle was blown and how high up in the air?

(6.564, 2.428)

Yes, she catches it after 6.564 seconds, when it is 2.428 ft high

3. A pelican flying above the ocean, sees Nutritious Nemo in the waters below and dives to get him. The pelican's location is modeled by $p(t) = 1.8\,(t-5)^2 - 4.7$, where t represents seconds after he sees Nemo. Nemo's location is modeled by the function $n(t) = 0.58t - 7.8$.



a. How high up in the air was the pelican when he first saw Nemo?

$p(0) = 1.8(0-5)^2 - 4.7$

$\boxed{= 40.3 \text{ ft above}}$

b. How far below the surface was Nemo when the pelican first saw him?

$n(0) = 0.58(t) - 7.8$

$\boxed{= -7.8 \text{ ft below}}$

c. How many seconds does it take for the pelican to get to the water's surface?

$0 = 1.8(t-5)^2 - 4.7$

$4.7 = 1.8(t-5)^2$

$2.611 = (t-5)^2$

$t = 5 \pm 1.6159$

$\boxed{\approx 3.384 \text{ seconds}}$

d. Graph both functions to see if the graphs intersect. Does the pelican eat Nemo at the lowest point of his dive? If yes, say after how many seconds and at what depth. If no, by how much does the pelican miss catching Nemo?

whew! so close!

$p(5) = -4.7$

$n(5) = -4.9$

misses by .2 of an inch, or 2.4 inches

e. How long is the pelican holding his breath before he flies back out of the water?

$6.61589 - 3.384$

$\boxed{\approx 3.232 \text{ seconds}}$

compositional functions!

f. How high in the air will the pelican be when Nemo surfaces to see what just happened?

we need to see when Nemo will be at surface.

$0 = .58t - 7.8$

$t = 13.448$

$p(13.448) = \boxed{123.77 \text{ ft in air}}$

g. What is the pelican's average speed between…

0 and 1 seconds?

(0, 40.3) (1, 24.1)

$\boxed{16.2 \text{ ft/sec}}$

2 and 3 seconds?

(2, 11.5) (3, 2.5)

$\boxed{9 \text{ ft/sec}}$

3 and 4 seconds?

(3, 2.5) (4, −2.9)

$\boxed{5.4 \text{ ft/sec}}$