

2 new pages!

3C:1 Quadratic Functions in Standard Form

$$f(x) = ax^2 + bx + c$$

Unit 3C1 Skills:

- | | |
|---|--|
| <input type="checkbox"/> Classifying polynomials | <input type="checkbox"/> Perfect Square Trinomial, Complete the Square |
| <input type="checkbox"/> Converting from Vertex & Intercept Form | <input type="checkbox"/> Graphing using the $-b/2a$ shortcut & growth rate |
| <input type="checkbox"/> Getting the y-intercept from Standard Form | <input type="checkbox"/> Rewriting in vertex form |
| | <input type="checkbox"/> Transformations |

CLASSIFYING POLYNOMIALS: Classifications need two names: 1) the function family 2) the number of terms

$y = 2x + 6$
 Linear binomial
 y-int: (0, 6)

$y = 2x^2 + 4x - 2$
 Quadratic trinomial
 y-int: (0, -2)

$y = 3x$
 Linear monomial
 y-int: (0, 0)

$y = 2x^2 + 9$
 Quadratic binomial
 y-int: (0, 9)

1. For each of the following, classify by function & terms, and state the y-intercept:

a. $y = 2x^2 + 3x - 6$ (0, -6)
 quad. trinomial

d. $y = .07x^2 - 5.2$ (0, -5.2)
 quad. binomial

g. $y = .002x^2$ (0, 0)
 quad. monomial

b. $y = x^2 + 109x - 23$ (0, -23)
 quad. trinomial

e. $y = 49x - 1$ (0, -1)
 linear binomial

h. $y = -\frac{2}{5}x + \frac{6}{5}$ (0, $\frac{6}{5}$)
 linear binomial

c. $y = x^2$ (0, 0)
 quad. monomial

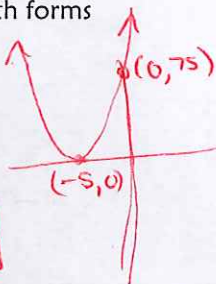
f. $y = 4x^2 + 9x - 2$ (0, -2)
 quad. trinomial

i. $y = -604x^2 + 2x$ (0, 0)
 quad. binomial

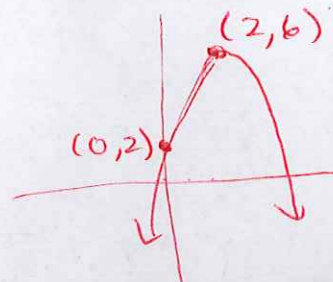
CONVERTING FROM OTHER FORMS: Just Multiply (remember PEMDAS!)

2. Convert the following from Vertex Form from Standard Form. Then state the y-intercept and sketch based on the information you have from both forms

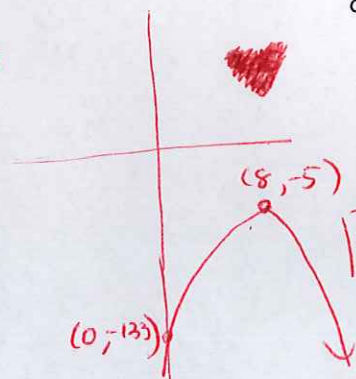
a. $y = 3(x+5)^2$
 $3(x+5)(x+5)$
 $3(x^2+10x+25)$
 $y = 3x^2 + 30x + 75$



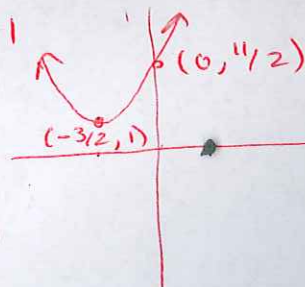
c. $y = -(x-2)^2 + 6$
 $-(x-2)(x-2) + 6$
 $-(x^2 - 4x + 4) + 6$
 $-x^2 + 4x - 4 + 6$
 $y = -x^2 + 4x + 2$



b. $y = -2(x-8)^2 - 5$
 $-2(x-8)(x-8) - 5$
 $-2(x^2 - 16x + 64) - 5$
 $-2x^2 + 32x - 128 - 5$
 $y = -2x^2 + 32x - 133$



d. $y = \frac{1}{2}(2x+3)^2 + 1$
 $\frac{1}{2}(2x+3)(2x+3) + 1$
 $\frac{1}{2}(4x^2 + 12x + 9) + 1$
 $y = 2x^2 + 6x + \frac{11}{2}$

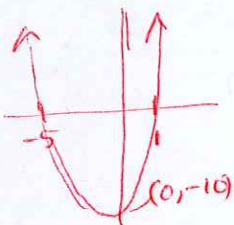


3. Convert the following quadratics from Intercept form Standard Form. Then state the y-intercept and sketch a graph, based on the information you found from both forms.

a. $y = 2(x + 5)(x - 1)$

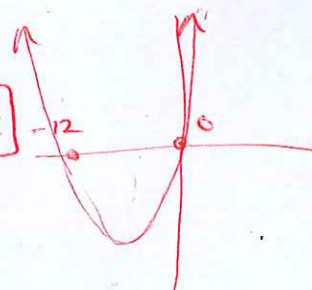
$2(x^2 - x + 5x - 5)$
 $+4x$

$y = 2x^2 + 8x - 10$



d. $y = -3x(x + 12)$

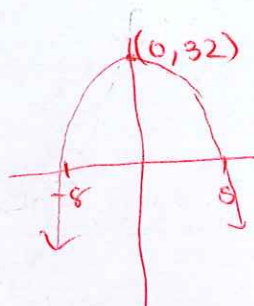
$y = -3x^2 - 36x$



b. $y = -\frac{1}{2}(x - 8)(x + 8)$

$-\frac{1}{2}(x^2 - 64)$

$y = -\frac{1}{2}x^2 + 32$

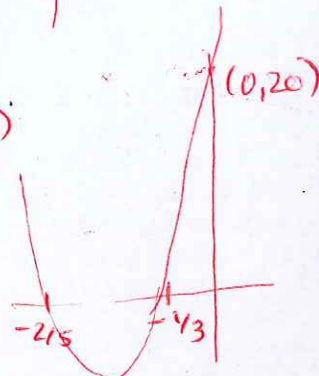


e. $y = 4(2x + 5)(3x + 1)$

$4(6x^2 + 2x + 15x + 5)$

$4(6x^2 + 17x + 5)$

$y = 24x^2 + 68x + 20$

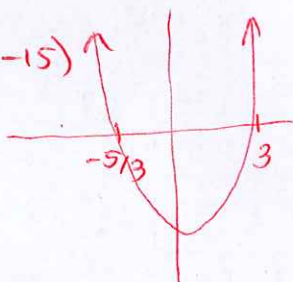


c. $y = \frac{1}{6}(3x + 5)(x - 3)$

$\frac{1}{6}(3x^2 - 9x + 5x - 15)$

$\frac{1}{6}(3x^2 - 4x - 15)$

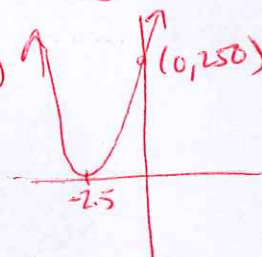
$y = \frac{1}{2}x^2 - \frac{2}{3}x - \frac{5}{2}$



f. $y = 10(2x + 5)(2x + 5)$

$10(4x^2 + 10x + 10x + 25)$

$y = 40x^2 + 200x + 250$



GRAPHING IN STANDARD FORM:

Using the Shortcut to get the AOS

$x = \frac{-b}{2a}$

To get the AOS, you calculate $-b/2a$. To get the y of the vertex, you plug it in.

4. Find the vertex of the following functions. Then state how many real roots it will have based on where the vertex is and which way the function opens.

a. $y = -x^2 + 2x + 1$

$\frac{-b}{2a} = \frac{-2}{2(-1)} = \frac{-2}{-2}$

$x = 1$
 $V: (1, 2)$ | two real roots

c. $y = 2x^2 - 16x + 33$

$\frac{16}{2(2)} = \frac{16}{4} = 4$

$V: (4, 1)$ | no real roots

b. $y = \frac{1}{2}x^2 - 8x + 14$

$\frac{8}{2(\frac{1}{2})} = 8$

$V: (8, -18)$
| two real roots

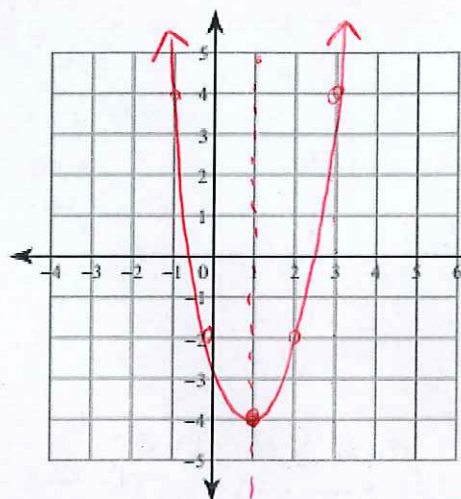
d. $y = x^2 + 10x + 25$

$\frac{-10}{2} = -5$

$V: (-5, 0)$
| one real root

5. For each parabola, use the AOS shortcut to find the vertex. Then graph, using the vertex and the parabola's growth pattern. Check to see if your y-intercept makes sense, given the equation. Then, put the function into vertex form, solve for the x-intercepts AND classify them.

a. $y = 2x^2 - 4x - 2$ $\frac{4}{2(2)} = \frac{4}{4} = 1$
 AOS: $x = 1$ Vertex: $(1, -4)$



$$y = 2(x-1)^2 - 4$$

$$0 = 2(x-1)^2 - 4$$

$$4 = 2(x-1)^2$$

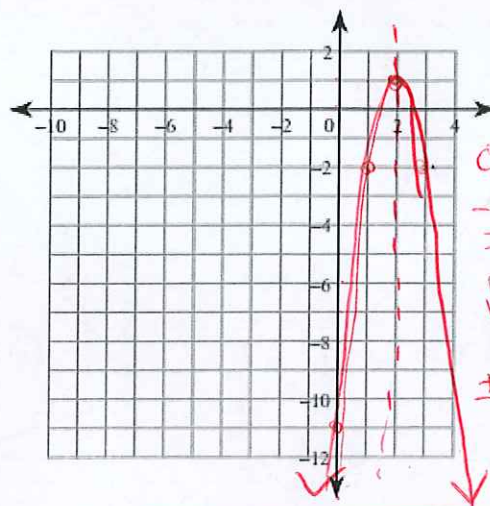
$$\sqrt{2} = \sqrt{(x-1)^2}$$

$$\pm\sqrt{2} = x-1$$

$$x = 1 \pm \sqrt{2}$$

Roots? $x = 1 + \sqrt{2}, x = 1 - \sqrt{2}$
 $\approx 2.414 \quad \approx -0.414$

b. $y = -3x^2 + 12x - 11$ $\frac{-12}{2(-3)} = \frac{-12}{-6} = 2$
 AOS: $x = 2$ Vertex: $(2, 1)$



$$0 = -3(x-2)^2 + 1$$

$$-1 = -3(x-2)^2$$

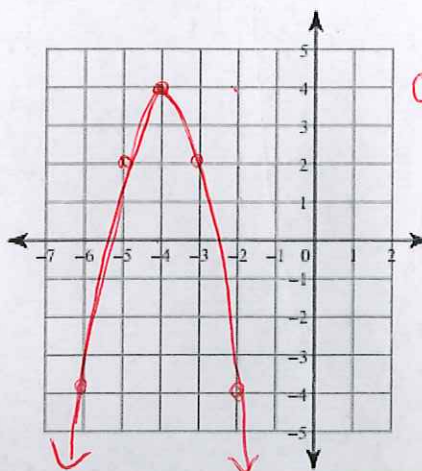
$$\frac{1}{3} = \sqrt{(x-2)^2}$$

$$\pm\frac{1}{\sqrt{3}} = x-2$$

$$x = 2 \pm \frac{\sqrt{3}}{3}$$

Roots? $2 + \frac{\sqrt{3}}{3}, 2 - \frac{\sqrt{3}}{3}$
 $\approx 2.577, \approx 1.422$

c. $y = -2x^2 - 16x - 28$ $\frac{16}{2(-2)} = \frac{16}{-4} = -4$
 AOS: $x = -4$ Vertex: $(-4, 4)$



$$0 = -2(x+4)^2 + 4$$

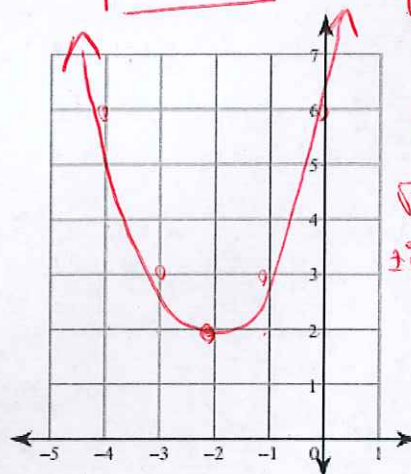
$$-4 = -2(x+4)^2$$

$$\sqrt{2} = \sqrt{(x+4)^2}$$

$$x = -4 \pm \sqrt{2}$$

Roots? $x = -4 - \sqrt{2}, x = -4 + \sqrt{2}$
 $\approx -5.414 \quad \approx -2.586$

d. $y = x^2 + 4x + 6$ $\frac{-4}{2(1)} = -2$
 AOS: $x = -2$ Vertex: $(-2, 2)$



$$0 = (x+2)^2 + 2$$

$$\sqrt{-2} = \sqrt{(x+2)^2}$$

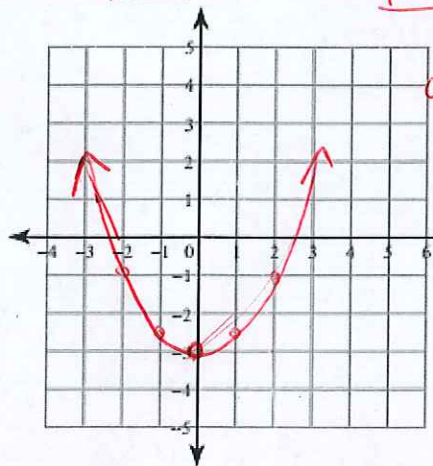
$$\pm i\sqrt{2} = x+2$$

$$x = -2 \pm i\sqrt{2}$$

Roots? $-2 + i\sqrt{2}, -2 - i\sqrt{2}$

e. $y = \frac{1}{2}x^2 - 3$

AOS: $x=0$ Vertex: $(0, -3)$

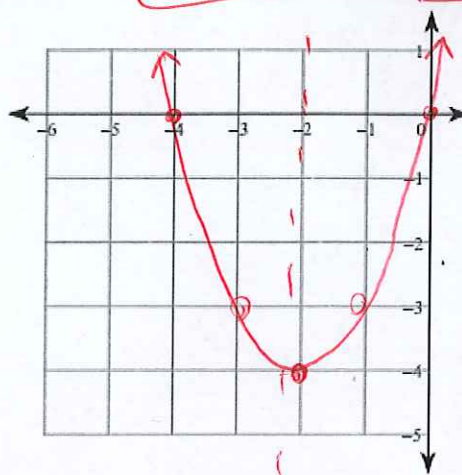


$0 = \frac{1}{2}x^2 - 3$
 $3 = \frac{1}{2}x^2$
 $\sqrt{6} = \sqrt{x^2}$
 $x = \pm\sqrt{6}$

Roots? $\sqrt{6}, -\sqrt{6}$
 $\approx 2.449, -2.449$

f. $y = x^2 + 4x$

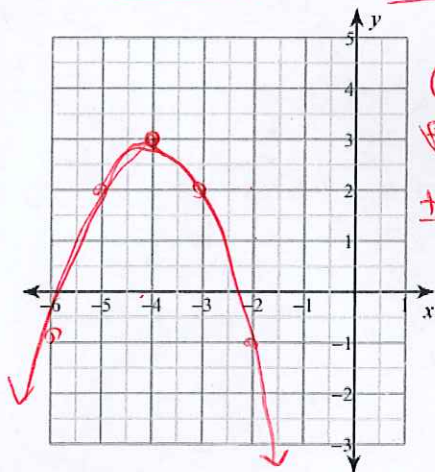
AOS: $x=-2$ Vertex: $(-2, -4)$



Roots? $x=0, x=-4$

g. $y = -x^2 - 8x - 13$ $\frac{8}{2(-1)} = -2 = -4$

AOS: $x=-4$ Vertex: $(-4, 3)$

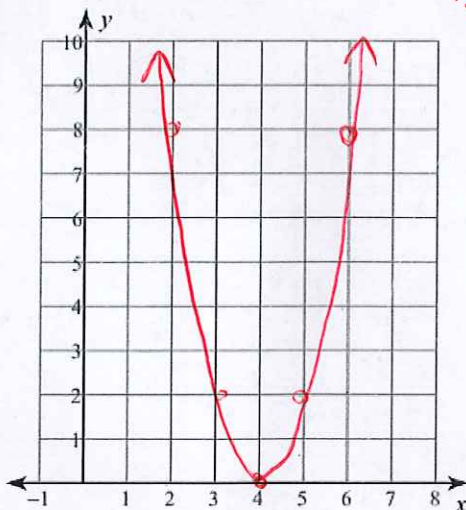


$0 = -(x+4)^2 + 3$
 $\sqrt{3} = \sqrt{(x+4)^2}$
 $\pm\sqrt{3} = x+4$
 $x = -4 \pm \sqrt{3}$

Roots? $x = -4 - \sqrt{3}, x = -4 + \sqrt{3}$
 $\approx -5.73 \quad \approx -2.268$

h. $y = 2x^2 + 16x + 32$ $\frac{+16}{2(2)} = \frac{+16}{4} = +4$
oops \perp make neg.

AOS: $x=4$ Vertex: $(4, 0)$



Roots? $x=4$ (multiplicity 2)

TRANSFORMING FUNCTIONS IN STANDARD FORM:

6. Transform the following functions, $f(x)$ to become $3f(x) + 2$

a. $f(x) = 2x^2 + 3x + 5$

$$3f(x) = 6x^2 + 9x + 15$$

$$3f(x) + 2 = \boxed{6x^2 + 9x + 17}$$

b. $f(x) = 4x^2 - 5x$

$$= \boxed{12x^2 - 15x + 2}$$

c. $f(x) = -\frac{1}{2}x^2 + 3x + 6$

$$-\frac{3}{2}x^2 + 9x + 18$$

$$+ 2$$

$$= \boxed{-\frac{3}{2}x^2 + 9x + 20}$$

7. Transform the following functions, $f(x)$ to become $-f(x+1)$

a. $f(x) = 2x^2 + 3x + 5$

$$-2x^2 - 3x - 5$$

$$-2(x+1)^2 - 3(x+1) - 5$$

$$-2(x+1)(x+1) - 3(x+1) - 5$$

$$-2[x^2 + 2x + 1] - 3(x+1) - 5$$

$$-2x^2 - 4x - 2 - 3x - 3 - 5$$

$$= \boxed{-2x^2 - 7x - 10}$$

b. $f(x) = 4x^2 - 5x$

$$-4x^2 + 5x$$

$$-4(x+1)(x+1) + 5(x+1)$$

$$-4[x^2 + 2x + 1] + 5(x+1)$$

$$-4x^2 - 8x - 4 + 5x + 5$$

$$= \boxed{-4x^2 - 3x + 1}$$

c. $f(x) = -\frac{1}{2}x^2 + 3x + 6$

$$-\frac{1}{2}x^2 - 3x - 6$$

$$\frac{1}{2}(x+1)(x+1) - 3(x+1) - 6$$

$$\frac{1}{2}[x^2 + 2x + 1] - 3(x+1) - 6$$

$$\frac{1}{2}x^2 + x + 0.5 - 3x - 3 - 6$$

$$= \boxed{\frac{1}{2}x^2 - 2x - 8.5}$$

8. Transform the following functions, $f(x)$ to become $\frac{2}{3}f(x) - 2$

a. $f(x) = 2x^2 + 3x + 5$

$$\frac{2}{3}(2x^2 + 3x + 5)$$

$$\frac{4}{3}x^2 + 2x + \frac{10}{3} - 2\left(\frac{2}{3}\right)$$

$$= \boxed{\frac{4}{3}x^2 + 2x + \frac{4}{3}}$$

b. $f(x) = 4x^2 - 5x$

$$\frac{2}{3}(4x^2 - 5x)$$

$$= \boxed{\frac{8}{3}x^2 - \frac{10}{3}x - 2}$$

c. $f(x) = -\frac{1}{2}x^2 + 3x + 6$

$$\frac{2}{3}\left(-\frac{1}{2}x^2 + 3x + 6\right)$$

$$\left(-\frac{1}{3}x^2 + 2x + 4\right) - 2$$

$$= \boxed{-\frac{1}{3}x^2 + 2x + 2}$$

PERFECT SQUARE TRINOMIALS: A trinomial you can smooch up into a squared binomial!

9. Rewrite in intercept form and multiply out to standard form. Then see if you can find a pattern!!!

a. $y = (x + 5)^2$

$(x+5)(x+5)$

$y = x^2 + 10x + 25$

b. $y = (x - 3)^2$

$(x-3)(x-3)$

$y = x^2 - 6x + 9$

c. $y = (x - 9)^2$

$(x-9)(x-9)$

$y = x^2 - 18x + 81$

d. $y = (x + \frac{1}{2})^2$

$(x+\frac{1}{2})(x+\frac{1}{2})$

$y = x^2 + x + \frac{1}{4}$

e. $y = (x - 1)^2$

$(x-1)(x-1)$

$y = x^2 - 2x + 1$

f. $y = (x - \frac{5}{2})^2$

$(x-\frac{5}{2})(x-\frac{5}{2})$

$y = x^2 - 5x + \frac{25}{4}$

10. **IMPORTANT:** How do you recognize a Perfect Square Trinomial?

cut "b" in half
and square it
to get "c"!

$c = (\frac{b}{2})^2$

11. Find the value of c that would make it a perfect square trinomial. Then rewrite as a squared binomial.

a. $y = x^2 - 14x + c$

$c = 49$

$y = (x-7)^2$

b. $y = x^2 + 22x + c$

$c = 121$

$y = (x+11)^2$

c. $y = x^2 - 20x + c$

$c = 100$

$y = (x-10)^2$

d. $y = x^2 + \frac{7}{2}x + c$

$c = \frac{49}{4}$

$y = (x + \frac{7}{2})^2$

e. $y = x^2 - \frac{1}{3}x + c$

$c = \frac{1}{36}$

$y = (x - \frac{1}{6})^2$

f. $y = x^2 - 5x + c$

$c = \frac{25}{4}$

$y = (x - \frac{5}{2})^2$

12. Are the following perfect square trinomials? If yes, rewrite as a binomial.

a. $y = x^2 - 2x + 4$

no!

b. $y = x^2 + 3x + \frac{9}{4}$

yes!
 $y = (x + \frac{3}{2})^2$

c. $y = x^2 - x + 1$

no!

e. $y = x^2 + 9x + \frac{81}{2}$

no!

e. $y = x^2 - \frac{5}{3}x + \frac{25}{36}$

yes!
 $y = (x - \frac{5}{6})^2$

f. $y = x^2 - 6x - 9$

no!