Name: Answer key Per: ________ Date: 7/8
Serafino ▪ Algebra II

3D The Quadratic Formula & Complex/Imaginary Numbers

Finding imaginary intercepts & working with complex numbers

At this point, if I ask you for x–intercepts or x–values in a word problem, you have several options on how to get there. You can graph and kind of just look at them, you can complete the square and solve with square roots, or you factor. So many options!

This is all well and good… but what happens if the quadratic doesn't touch the x–axis? This happens a lot… in fact, it's been happening all chapter and we've just been ignoring the fact that we can't get certain x values. When you've tried to do those in vertex form, you ended taking a negative square root and then well… just stopped because you know we can't do that…

… or can we?

For ANY quadratic function, all you have to do it plug in a, b, and c into the quadratic formula, and voila! Enjoy your zeros!

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The –b / 2a part should look VERY familiar (axis of symmetry). But what about that other mess, under the radical? Well, that is called the **discriminant**. Ugly name; awesome information. It's the number that is produced when you do $b^2 - 4ac$. It gives us SO MUCH information we can use for our quadratic.

For example, $y = x^2 + 2x - 3$. Do the quadratic formula and see what it gets you.

$$\frac{-2 \pm \sqrt{4-4(1)(-3)}}{2(1)} = \frac{-2 \pm \sqrt{16}}{2} = \frac{-2 \pm 4}{2} \quad \frac{-2+4}{2} = 1 \quad \frac{-2-4}{2} = -3$$

Sweet. So those are x intercepts, right? The fact that you have two solutions tells the number of times it crosses the x–axis is two.

Now let's try this one: $y = x^2 + 6x + 9$.

$$\frac{-6 \pm \sqrt{36-4(1)(9)}}{2} = \frac{-6 \pm \sqrt{0}}{2} = \frac{-6}{2} = -3$$

So what happened? The thing under the radical is zero ... this means you're doing the AOS ± nothing.... which means you're left with the AOS as your only x intercept. If the only time your parabola touches the x–axis once is when it is also the vertex. Nifty, eh?

(so nifty!)

Let's practice reducing the quadratic formula to get the x–intercepts.

$\frac{1 \pm \sqrt{32}}{4}$

$\boxed{\frac{1 \pm 4\sqrt{2}}{4}}$ or $\boxed{\frac{1}{4} \pm \sqrt{2}}$

$\frac{12 \pm \sqrt{200}}{10}$

$\boxed{\frac{6 \pm 5\sqrt{2}}{5}}$ or $\boxed{\frac{6}{5} \pm \sqrt{2}}$

$\frac{3 \pm \sqrt{121}}{4}$

$\frac{3 \pm 11}{4}$

$\boxed{\frac{7}{2}, -2}$

$\frac{7 \pm \sqrt{8}}{14}$

$\boxed{\frac{7 \pm 2\sqrt{2}}{14}}$ or $\boxed{\frac{1}{2} \pm \frac{\sqrt{2}}{7}}$

$\frac{8 \pm \sqrt{100}}{16}$

$\frac{8 \pm 10}{16}$

$\boxed{\frac{9}{8}, \frac{-1}{8}}$

$\frac{5 \pm \sqrt{75}}{10}$

$\boxed{\frac{1 \pm \sqrt{3}}{2}}$ or $\boxed{\frac{1}{2} \pm \frac{\sqrt{3}}{2}}$

$\frac{9 \pm \sqrt{54}}{6}$

$\boxed{\frac{3 \pm \sqrt{6}}{2}}$ or $\boxed{\frac{3}{2} \pm \frac{\sqrt{6}}{2}}$

$\frac{15 \pm \sqrt{125}}{5}$

$\boxed{3 \pm \sqrt{5}}$

$\frac{14 \pm \sqrt{360}}{8}$

$\frac{7 \pm 3\sqrt{10}}{4}$ or $\boxed{\frac{7}{4} \pm \frac{3\sqrt{10}}{4}}$

$\frac{4 \pm \sqrt{28}}{6}$

oops

$\boxed{\frac{2 \pm \sqrt{7}}{3}}$ or $\boxed{\frac{2}{3} \pm \frac{\sqrt{7}}{3}}$

$\frac{7 \pm \sqrt{98}}{7}$

$\boxed{1 \pm \sqrt{2}}$

$\frac{10 \pm \sqrt{36}}{8}$

$\boxed{2, \frac{1}{2}}$

So what happens when we have a "floater"? Some quadratics don't have x–intercepts because they don't touch the x–axis.

Try this: $x^2 - 2x + 5$. Stop when you get to the point when you calculate the discriminant.

$$\frac{2 \pm \sqrt{4-4(1)(5)}}{2} = \frac{2 \pm \sqrt{-16}}{2} = \frac{2 \pm 4i}{2} = \boxed{1 \pm 2i}$$

Ahhh!! The discriminant becomes negative... which means we have to take the square root of a negative number.

There is no *real* number that if you multiply it by itself that will give you a negative number... so we make up an imaginary unit, *i*, that will do just that.

The good news is that you will do everything exactly the same... except when you simplify or take the square root of a negative radical... you will now just pull out the negative sign as an I, and proceed as usual.

Let's practice: **SIMPLIFYING NEGATIVE RADICALS:**

$\sqrt{-2}$ $\boxed{i\sqrt{2}}$

$\sqrt{-4}$ $\boxed{2i}$

$\sqrt{-64}$ $\boxed{8i}$

$\sqrt{-200}$ $\boxed{10i\sqrt{2}}$

$\sqrt{-7}$ $\boxed{\sqrt{7}i}$

$\sqrt{-80}$ $\boxed{4i\sqrt{5}}$

$\frac{-4 \pm \sqrt{-12}}{8}$ $\boxed{\frac{1}{2} \pm \frac{\sqrt{3}i}{4}}$

$\frac{2 \pm \sqrt{-12}}{6}$ $\boxed{\frac{1}{3} \pm \frac{\sqrt{3}i}{3}}$

$\frac{1 \pm \sqrt{-40}}{2}$ $\boxed{\frac{1}{2} \pm \sqrt{10}i}$

OPERATIONS WITH COMPLEX NUMBERS:

Okay! So now that we can do that.... let's work strictly with imaginary numbers. When you have a <u>complex</u> number, it just means that some part of it is imaginary: $a \pm bi$ (real component first)

Working with them is kind of cool.

Adding & Subtracting: Business as usual. Combine like terms:

1. $3i + 2i$ $\boxed{5i}$

2. $4i - 6i$ $\boxed{-2i}$

3. $(6 + 15i) - (2 + 3i)$ $\boxed{4+2i}$

4. $(-6 + i) + (1 - 3i)$ $\boxed{-5-2i}$

5. $(4 - i) - (-2 + 6i)$ $\boxed{6-7i}$

6. $(3i + 6) + (2 + 3i)$ $\boxed{8+6i}$

Multiplying: Like working with variables except one little extra step.

1. $3i \cdot 2i$ $= 6i^2$ $\boxed{= -6}$

2. $-4i \cdot 6i$ $\boxed{= 24}$

3. $4i(2 + 3i)$ $= \boxed{-12+8i}$

4. $(-6 + i)(1 - 3i)$ $\boxed{-3+19i}$

5. $(4 - i)(-2 + 6i)$ $\boxed{-2+26i}$

6. $(2 + 3i)(2 + 3i)$ $\boxed{-5+12i}$

7. $-3i(-6 + 2i)$ $\boxed{6+8i}$

8. $i(4 - i)$ $\boxed{1+4i}$

9. $(5 + 3i)(5 - 3i)$ $\boxed{34}$

That last one is key. Notice what happened... the i disappeared! We have the power to make that happen any time we want if we have a i in the denominator of a division problem.

Division: Multiply by the conjugate to make the i disappear from the denominator!

1. $\frac{6}{5i} \cdot \frac{i}{i}$

$\frac{6i}{5i^2} = \boxed{-\frac{6i}{5}}$

2. $\frac{2}{3+i} \cdot \frac{(3-i)}{(3-i)}$

$\frac{6-2i}{9-i^2}$ $+1$ $\frac{6-2i}{10}$

$\boxed{\frac{3}{5} - \frac{i}{5}}$

3. $\frac{1-3i}{2+5i} \cdot \frac{(2-5i)}{(2-5i)}$ $4-25i^2$

-16

$2-5i-6i+15i^2$

$\frac{-13-11i}{29}$

$\boxed{\frac{-13}{29} - \frac{11i}{29}}$

4. $\frac{6i}{1-2i}$

$\boxed{\frac{-12}{5} + \frac{6i}{5}}$

5. $\frac{6}{5-2i}$

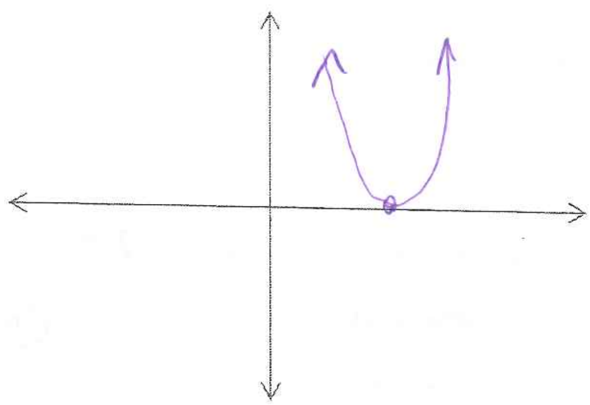
$\boxed{\frac{30}{29} + \frac{2i}{29}}$

6. $\frac{2+3i}{4-i}$

$\boxed{\frac{5}{17} + \frac{14i}{17}}$

PUT IT ALL TOGETHER:

1. $y = 4x^2 - 12x + 9$



a) What is the discriminant? 0

b) What kinds of solutions will I have? real or imaginary

c) How many solutions will I have? 1

d) How many times will my parabola cross the x-axis? 1

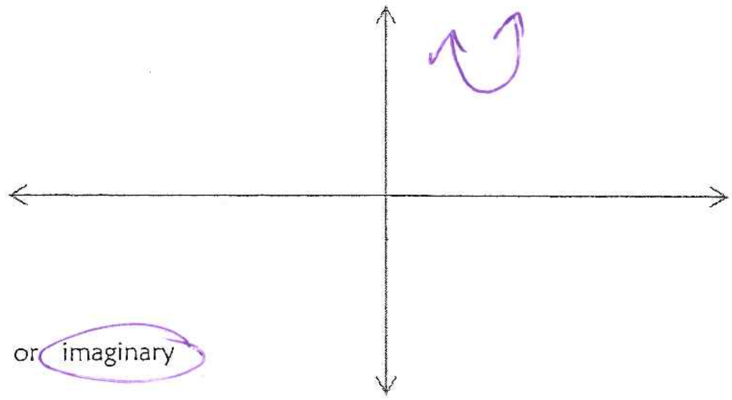
e) What is/are the solution(s)? 3/2

f) Please give decimal approximations, if possible: 1.5

g) Axis of Symmetry? x = 3/2

h) Vertex? (3/2, 0)

2. $y = 5x^2 - 2x + 4$



a) What is the discriminant? -76

b) What **kinds** of solutions will I have? real or imaginary

c) How many solutions will I have? 2

d) How many times will my parabola cross the x-axis? none

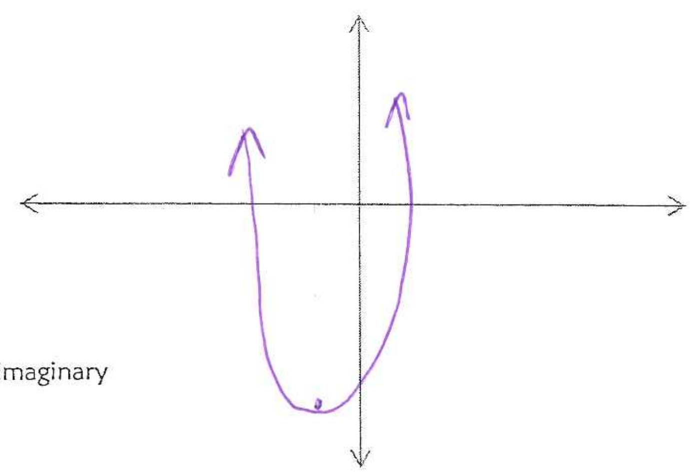
e) What is/are the solution(s)? $\frac{1}{5} \pm \frac{\sqrt{19}\, i}{5}$

f) Please give decimal approximations, if possible: n/a

g) Axis of Symmetry? $x = 1/5$

h) Vertex? $(0.2, 3.8)$

3. $y = 3x^2 + x - 2$



a) What is the discriminant? 25

b) What **kinds** of solutions will I have? real or imaginary

c) How many solutions will I have? 2

d) How many times will my parabola cross the x-axis? 2

e) What is/are the solution(s)? $1, -4/3$

f) Please give decimal approximations, if possible: $1, -1.33$

g) Axis of Symmetry? $x = -1/6$

h) Vertex? $(-0.167, -4.083)$