Name: Key Per: 1,3,5 Date: 1/5/15
Serafino ▪ Precalculus

4.4 Writing Equations from Graphs

Part 1: From Graphs **Part 2:** From Real World Models
Classwork /Notes Packet

Four Simple steps to write any equation from a graph:

1. Find "D". That should be easiest. Just find the middle of the graph. Boom.
   Or, if you need to do some math, you can take the average of the Max and min: $D = \frac{M+m}{2}$

2. Find "A" → Also easy. Just count how far up/down you go from the middle.
   Or, you can take the half the difference of the Max and min $A = \frac{|M-m|}{2}$

3. Find "B" → You may be able to see how many cycles are completed by 2 π, but it's probably easier to just find "P" first. To get P, just see how long it takes to complete one full cycle.
   Then, you already know the NP of the function. Then solve for B. $P = \frac{NP}{B}$ so $B = \frac{NP}{P}$

You're practically there! A, B, and D will be the same for EVERY function you write. Then, "C" will change depending on which trig function and which "starting point" you chose.

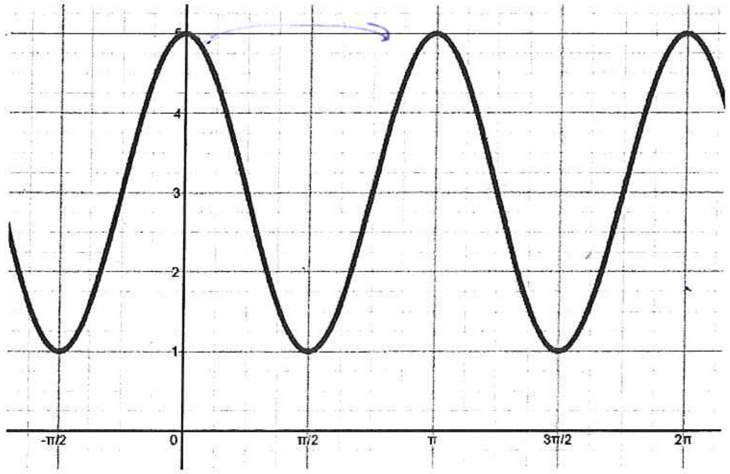
4. Find "C" → You already know B from above. "x" is the starting point you chose. $Bx + C = 0$
   You could set Bx + C = 0. OR you can put B(x+ h), and distribute. Same thing. $B(x + h)$

For each graph, you'll write four equations; one of each cofunction, one of each sign (±).
Check by typing each function into Desmos.com. If the info & graph match up, you win! ☺

$Bx+C=0$

1. D = 3 A= 2 SP₁: $x=0$ $2(0)+C=0$

P = π B = 2 $B=\frac{NP}{B}=\frac{2\pi}{\pi}$ $y_1 = 2\cos(2x)+3$



$2(\frac{\pi}{2})+C=0$

SP₂: $x=\frac{\pi}{2}$

$y_2 = -2\cos(2x-\pi)+3$

$2(-\frac{\pi}{4})+C=0$

SP₃: $x=-\frac{\pi}{4}$

$y_3 = 2\sin(2x-\frac{\pi}{2})+3$

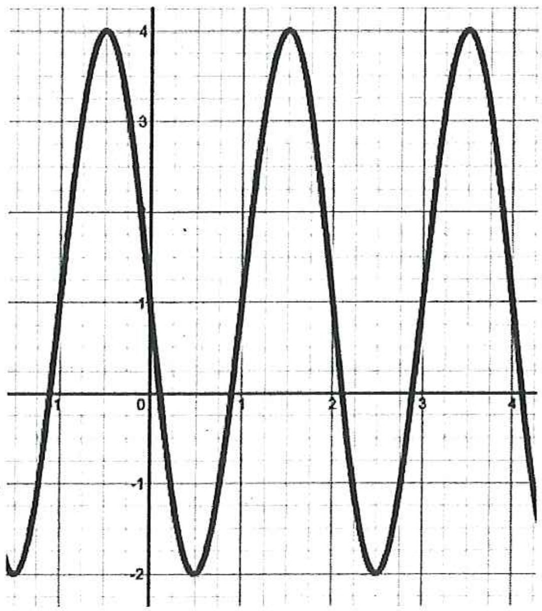
$2(\frac{\pi}{4})+C=0$

SP₄: $x=\frac{\pi}{4}$

$y_4 = -2\sin(2x-\frac{\pi}{2})+3$

2. D = 1 A= 3

P = 2 B = π



SP$_1$: $x=0$

$y_1 = -3\sin(\pi x)+1$

$Bx+C$

$\pi(1)+C=0$

SP$_2$: $x=1$

$y_2 = 3\sin(\pi x-\pi)+1$

$\pi(\frac{1}{2})+C=0$

SP$_3$: $x=\frac{1}{2}$

$y_3 = -3\cos(\pi x-\frac{\pi}{2})+1$

SP$_4$: $x=\frac{3}{2}$

$y_4 = 3\cos(\pi x-\frac{3\pi}{2})+1$

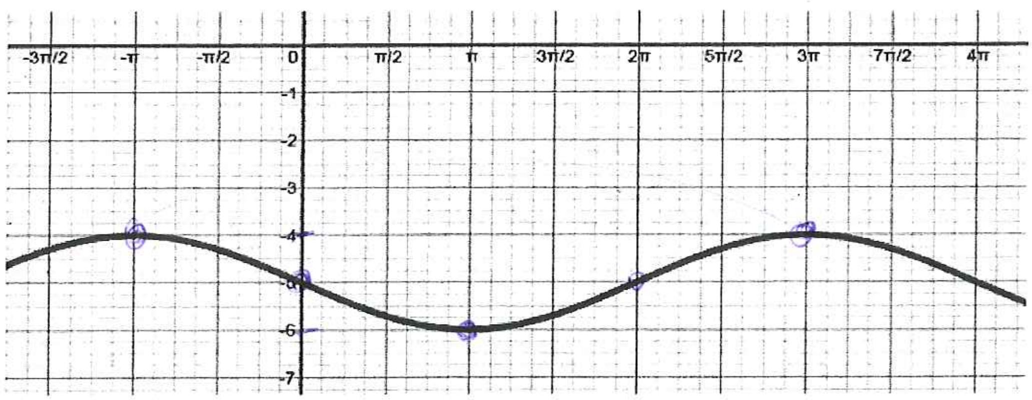
3.

D = -5

A = 1

P = 4π

B = $\frac{1}{2}$ $\frac{2\pi}{4\pi}=\frac{1}{2}$



$\frac{1}{2}(-\pi)+C=0$

SP$_1$: $x=-\pi$

$y_1 = \cos(\frac{1}{2}x+\frac{\pi}{2})-5$

SP$_2$: $x=0$

$y_2 = -\sin(\frac{1}{2}x)-5$

SP$_3$: $x=\pi$

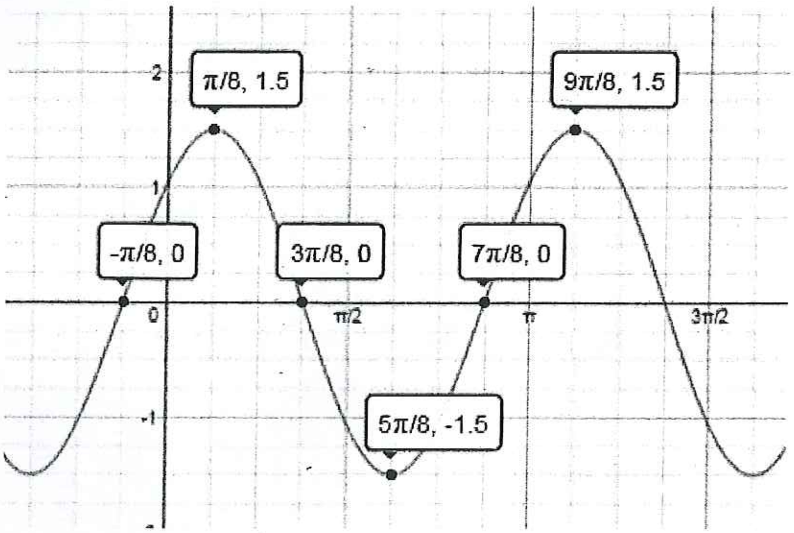
$y_3 = -\cos(\frac{1}{2}x-\frac{\pi}{2})-5$

SP$_4$: $x=2\pi$ $\frac{1}{2}(2\pi)$

$y_4 = \sin(\frac{1}{2}x-\pi)-5$

4. D = 0 A = 1.5

P = π B = 2



SP₁: $x = \frac{5\pi}{8}$

$y_1 = -1.5\cos\left(2x - \frac{5\pi}{4}\right)$

SP₂: $x = \frac{3\pi}{8}$

$y_2 = -1.5\sin\left(2x - \frac{3\pi}{4}\right)$

SP₃: $x = \frac{\pi}{8}$

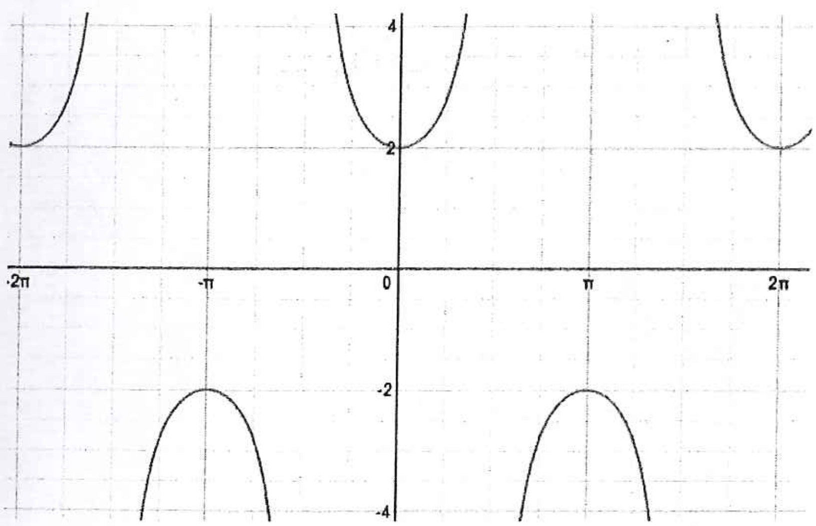
$y_3 = 1.5\cos\left(2x - \frac{\pi}{4}\right)$

SP₄: $-\frac{\pi}{8}$

$y_4 = 1.5\sin\left(2x + \frac{\pi}{4}\right)$

5. D = 0 A = 2

P = 2π B = 1



SP₁: $x = 0$

$y_1 = 2\sec(x)$

SP₂: $x = \pi$

$y_2 = -2\sec(x - \pi)$

SP₃: $x = \frac{\pi}{2}$

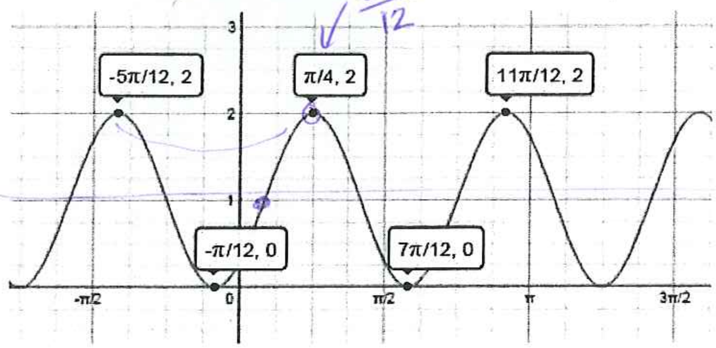
$y_3 = -2\csc\left(x - \frac{\pi}{2}\right)$

SP₄: $x = -\frac{\pi}{2}$

$y_4 = 2\csc\left(x + \frac{\pi}{2}\right)$

$\frac{8\pi}{12} = \frac{2\pi}{3}$

6.



D = 1

A = 1

P = $\frac{2\pi}{3}$

B = 3

$\frac{2\pi}{B} = \frac{2\pi}{3}$

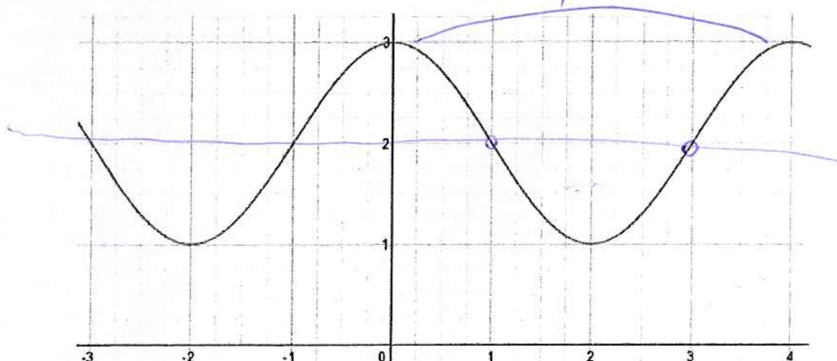
SP₁: $\frac{\pi}{4}$ $y_1 = \cos\left(3x - \frac{3\pi}{4}\right) + 1$

SP₂: $\frac{-\pi}{12}$ $y_2 = -\cos\left(3x + \frac{\pi}{4}\right) + 1$

SP₃: $\frac{-\pi}{4}$ $y_3 = -\sin\left(3x + \frac{3\pi}{4}\right) + 1$

SP₄: $\frac{\pi}{12}$ $y_4 = \sin\left(3x - \frac{\pi}{4}\right) + 1$

7.



D = 2

A = 1

P = 4

B = $\pi/2$ $\frac{2\pi}{4}$

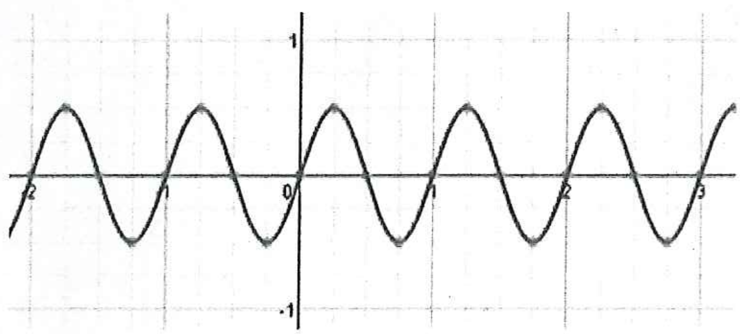
SP₁: $x=0$ $y_1 = \cos\left(\frac{\pi x}{2}\right) + 2$

SP₂: $x=1$ $y_2 = -\sin\left(\frac{\pi x}{2} - \frac{\pi}{2}\right) + 2$

SP₃: $x=2$ $y_3 = -\cos\left(\frac{\pi x}{2} - \pi\right) + 2$

SP₄: $x=3$ $y_4 = \sin\left(\frac{\pi x}{2} - \frac{3\pi}{2}\right) + 2$

8.



D = 0

A = $1/2$

P = 1

B = 2π $\frac{2\pi}{1}$

$\frac{3}{4} \cdot 2\pi$

SP₁: 0 $y_1 = \frac{1}{2}\sin(2\pi x)$

SP₂: $1/2$ $y_2 = -\frac{1}{2}\sin(2\pi x - \pi)$

SP₃: $1/4$ $y_3 = \frac{1}{2}\cos\left(2\pi x - \frac{\pi}{2}\right)$

SP₄: $3/4$ $y_4 = -\frac{1}{2}\cos\left(2\pi x - \frac{3\pi}{2}\right)$

$2\pi \cdot \frac{1}{2}$

$\frac{2\pi}{6\pi} = \frac{1}{3}$

9.

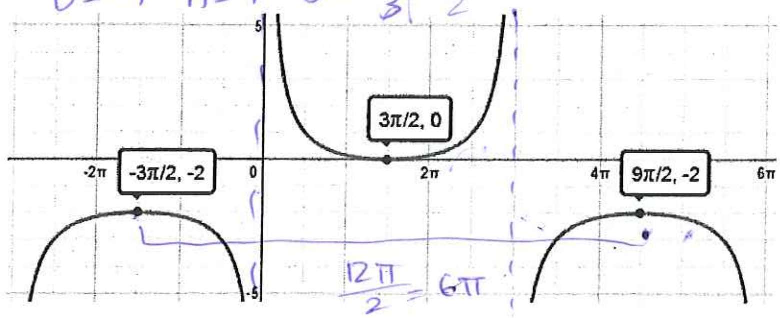
$D = -1 \quad A = 1 \quad B = \frac{1}{3} \quad -\frac{3\pi}{2}$

SP₁: $-3\pi/2$ $y_1 = -\sec\left(\frac{1}{3}x + \frac{\pi}{2}\right) - 1$

SP₂: 0 $y_2 = \csc\left(\frac{1}{3}x\right) - 1$

SP₃: $3\pi/2$ $y_3 = \sec\left(\frac{1}{3}x - \frac{\pi}{2}\right) - 1$

SP₄: 3π $y_4 = -\csc\left(\frac{1}{3}x - \pi\right) - 1$



$\frac{12\pi}{2} = 6\pi$

10. $D = 1 \quad A = 1 \quad P = \pi \quad B = 2$

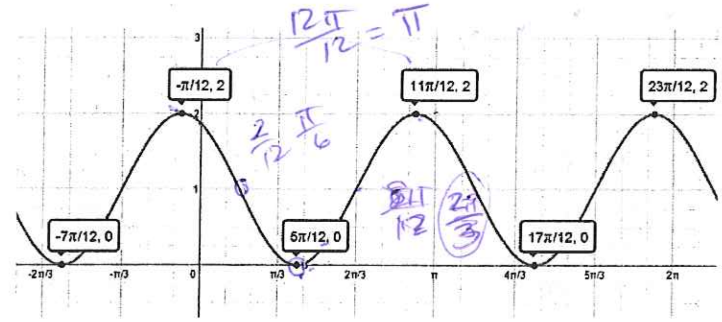
SP₁: $5\pi/12$ $y_1 = -\cos\left(2x - \frac{5\pi}{6}\right) + 1$

SP₂: $-\pi/12$ $y_2 = \cos\left(2x + \frac{\pi}{6}\right) + 1$

SP₃: $\pi/6$ $y_3 = -\sin\left(2x - \frac{\pi}{3}\right) + 1$

SP₄: $2\pi/3$ $y_4 = \sin\left(2x - \frac{4\pi}{3}\right) + 1$

$\frac{12\pi}{12} = \pi$



$\frac{2}{12} \quad \frac{\pi}{6} \quad \frac{8\pi}{12} \quad \frac{2\pi}{3}$

11. $D = -2 \quad A = 1 \quad B = 4/3$

$\frac{2\pi}{3\pi/2} \quad 2\pi \cdot \frac{2}{3\pi} = \frac{4}{3} \quad \frac{12\pi}{8} \quad \frac{3\pi}{2}$

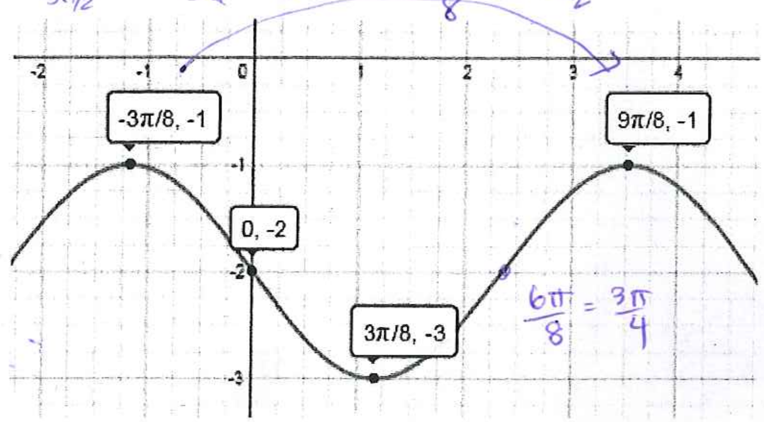
SP₁: $-3\pi/8$ $y_1 = \cos\left(\frac{4}{3}x + \frac{\pi}{2}\right) - 2$

SP₂: 0 $y_2 = -\sin\left(\frac{4}{3}x\right) - 2$

SP₃: $3\pi/8$ $y_3 = -\cos\left(\frac{4}{3}x - \frac{\pi}{2}\right) - 2$

SP₄: $3\pi/4$ $y_4 = \sin\left(\frac{4}{3}x - \pi\right) - 2$

$\frac{4}{3} \cdot \frac{3}{8}$



$\frac{6\pi}{8} = \frac{3\pi}{4}$

12.

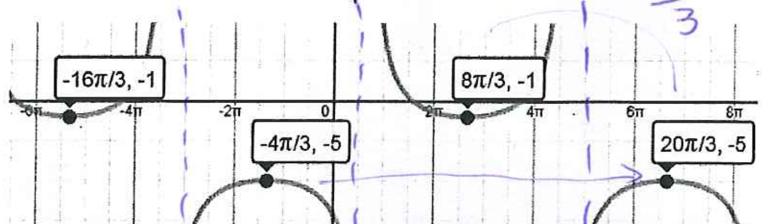
$\frac{2\pi}{3} \cdot \frac{1}{4} \quad \frac{14\pi}{3}$

SP₁: $-4\pi/3$ $y_1 = -2\sec\left(\frac{x}{4} + \frac{\pi}{3}\right) - 3$

SP₂: $2\pi/3$ $y_2 = 2\csc\left(\frac{x}{4} - \frac{\pi}{6}\right) - 3$

SP₃: $8\pi/3$ $y_3 = 2\sec\left(\frac{x}{4} - \frac{2\pi}{3}\right) - 3$

SP₄: $14\pi/3$ $y_4 = -\csc\left(\frac{x}{4} - \frac{7\pi}{6}\right) - 3$



$\frac{1}{4} \cdot \frac{4}{3}$

$D = -3$
$A = 2$

$\frac{24\pi}{3} = 8\pi$

$2\frac{8\pi}{3} \cdot \frac{1}{4}$

$B = \frac{2\pi}{8\pi} = \frac{1}{4}$

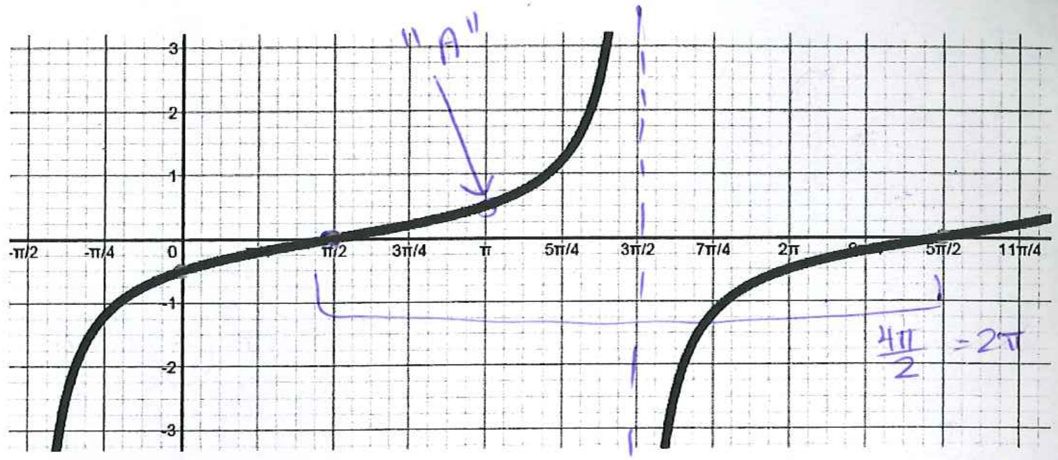
Gah!! Tangent? Relax. In fact, it cuts your work in half – because you can only have 2 possibilities. Well… I guess you could start from different places but let's just keep it to two.

13.

A = $\frac{1}{2}$

P = 2π

B = $\frac{1}{2}$



SP₁: $\frac{\pi}{2}$ y_1 = $\frac{1}{2}\tan\left(\frac{x}{2} - \frac{\pi}{4}\right)$ SP₂: $\frac{3\pi}{2}$ y_2 = $-\frac{1}{2}\cot\left(\frac{x}{2} - \frac{3\pi}{4}\right)$

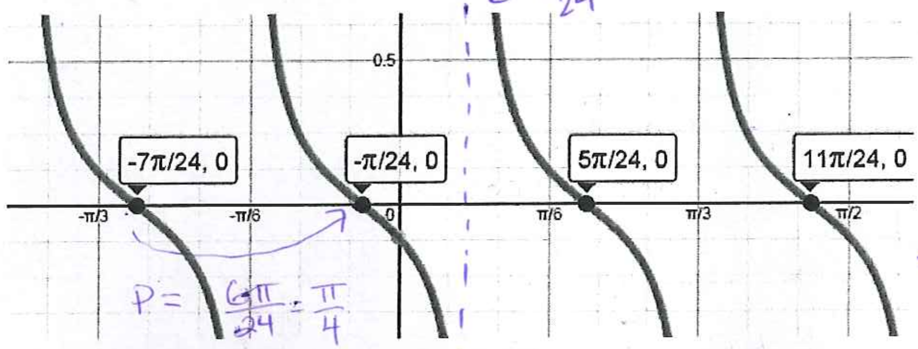
14. "Amplitude" is hard to see here, so, I'll just give it to you.

A = 1/5

P = $\frac{\pi}{4}$

B = 4

$\frac{\pi}{\frac{\pi}{4}} = \frac{\pi}{1} \cdot \frac{4}{\pi}$



SP₁: $-\frac{\pi}{24}$ y_1 = $-\frac{1}{5}\tan\left(4x + \frac{\pi}{6}\right)$ SP₂: $\pi/12$ y_2 = $\frac{1}{5}\cot\left(4x - \frac{\pi}{3}\right)$

$4\left(-\frac{\pi}{24}\right)$ $\frac{\pi}{12}$

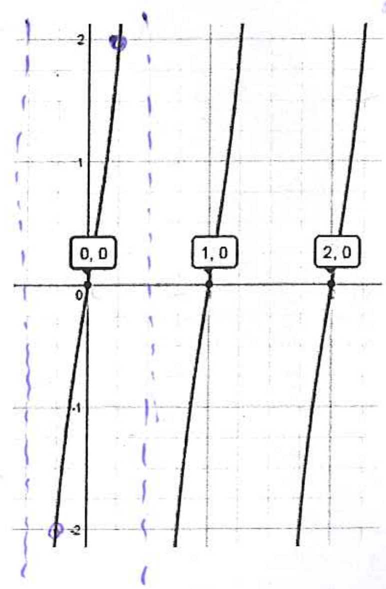
15.

A = 2 P = 1 B = π $\frac{\pi}{1} = \pi$

SP₁: 0 y_1 = $2\tan(\pi x)$

SP₂: $\frac{1}{2}$ y_2 = $-2\cot\left(\pi x - \frac{\pi}{2}\right)$

$\pi\left(\frac{1}{2}\right)$

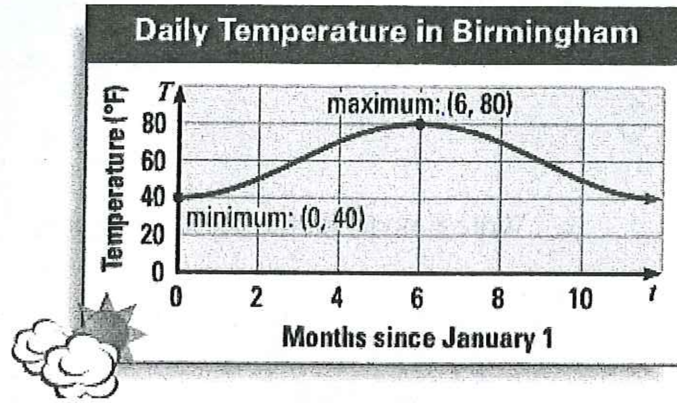


PART 2: Modeling Trig Functions in Real-World Scenarios: Stuff goes up and down in cycles. The sun, electricity currents, temperature, populations, the ocean tides, wheels, etc. We use sinusoids (the name of the sine or cosine graph, like parabola is the graph of a quadratic) to model these scenarios. This helps us solve problems and make predictions.

16. Yearly Temperature: Here is a chart of the average temperature in Birmingham, AL, throughout the year. Let's use the graph to write a sinusoidal model.

Average Temp: D = 60

Variation: A = 20



Here, period (P) is not a function of an angle, but time. How long to complete one cycle? Well, how long does it take for weather cycle to repeat itself? A year. How many months is that? That's the period.

Also, max → min = 6
so P = 12

P = 12

Use $2\pi/B = P$ to solve for B.

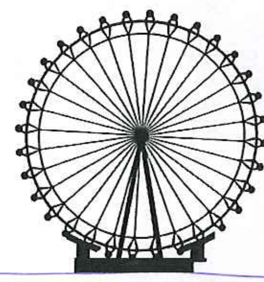
B = $\pi/6$

$\boxed{t(m) = -20\cos\left(\frac{\pi}{6}m\right) + 60}$

17. The London Eye: My friend studied abroad in London and always took dates on the famous Ferris wheel. In addition to the beautiful view, one full rotation took 30 minutes, which was enough time to see if the date should end or continue. The boarding platform is 6 feet above the ground and the wheel's diameter is 444 feet. Express a rider's height as a function of time.

First off, think about where the rider starts in relation to the average height of the ride. Because it's BELOW, then that tells us which trig function we will use.

Average height of ride = D = 228



Amplitude = 222

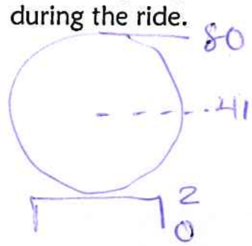
P = 30

Period: The input is minutes... so how many minutes is one full cycle of the wheel?

Now solve to get B = $\frac{\pi}{15}$

$\boxed{h(m) = -222\cos\left(\frac{\pi}{15}m\right) + 228}$

18. The very first ferris wheel was the Chicago Wheel. It was 80 meters tall, and the boarding platform was 2 meters off the ground when you got on at the very bottom. Your ticket got you got a 20 minute trip that involved going around twice. Find an equation that would tell you your height off the ground at any time during the ride.



A = 39

D = 41

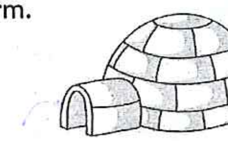
$\boxed{h(t) = -39\cos\left(\frac{\pi}{5}t\right) + 41}$

P = 20 min goes twice, so

P = 10

$\frac{2\pi}{10} = \frac{\pi}{5}$

19. Cool Igloos. Have you ever wondered how much warmer Igloos actually are compared to outside temperatures? The graph shows the temperatures inside and outside an igloo throughout a typical winter day: the outside temp, the temp on the igloo floor, and the temp at the sleeping platform.



a. Write a model for the <u>outside</u> temperature T (in degrees Fahrenheit) as a function of the time of day (t hours from midnight).

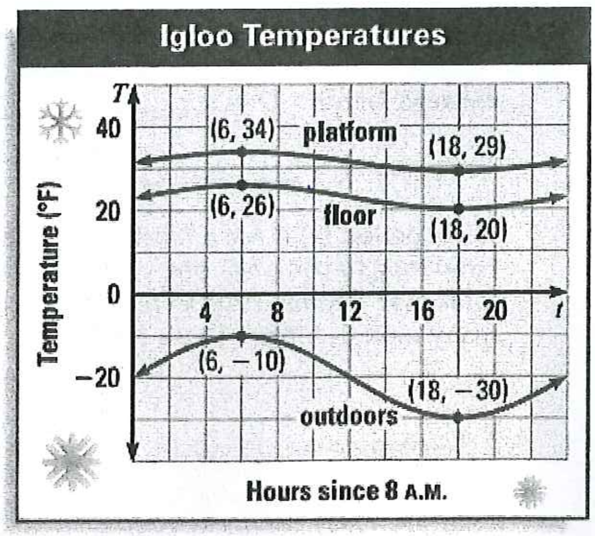
$f(x) = 10 \sin\left(\frac{\pi}{12}x\right) - 20$

b. Write a model for the floor-level temperature.

$f(x) = 3 \sin\left(\frac{\pi}{12}x\right) + 23$

c. Write a model for the sleeping platform.

$f(x) = 2.5 \sin\left(\frac{\pi}{12}x\right) + 31.5$



20. Elk Population: A population of elk in a forest oscillates 150 above/below a yearly average of 720 elk.

a. If the lowest elk population is in January, Find an equation for the population, *P*, in terms of the months since January, *t*.

$f(x) = -150 \cos\left(\frac{\pi}{6}x\right) + 720$

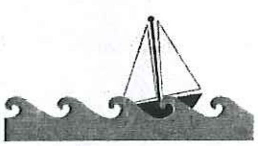
b. Write a function if the lowest value of the elk population occurred in March instead.

$f(x) = -150 \cos\left(\frac{\pi}{6}x - \frac{\pi}{3}\right) + 720$

21. Ocean Tides: The height of the water in a bay in Maine varies sinusoidally over time. On a certain day off a high tide of 10 feet occurred at 5:00 A.M. and a low tide of 2 feet occurred at 1:00 P.M. Write a model for the height h (in feet) of the water as a function of time t (in hours since midnight).

(5,10) Max → min = 8 hours, so per = 16

(13,2) $B = \frac{2\pi}{16} = \frac{\pi}{8}$



D = 6 A = 4

$\frac{\pi}{8}(5) + c = 0$

$4 \cos\left(\frac{\pi}{8}t - \frac{5\pi}{8}\right) + 6$

or $-4 \cos\left(\frac{\pi}{8}t - \frac{13\pi}{8}\right) + 6$

22. Summer Day: Outside temperature over a day can be modeled as a sinusoidal function. Suppose you know the high temperature for the day is 92 degrees and the low temperature of 78 degrees occurs at 4 AM. Assuming *t* is the number of hours since midnight, find an equation for the temperature, in terms of *t*.

at 3:00 pm

(15, 92)

Max → Min = 11 hours

(4, 78) Per = 22

$\frac{2\pi}{22} = \frac{\pi}{11}$

$-7 \cos\left(\frac{\pi t}{11} - \frac{4\pi}{11}\right) + 85$

or

$7 \cos\left(\frac{\pi}{11}t - \frac{15\pi}{11}\right) + 85$