

Name: _____ Per: _____ Date: _____

Serafino ▪ Precalculus

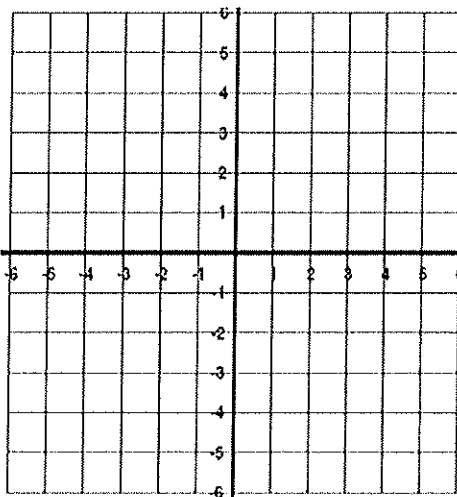
4.6 Inverse Functions – Graphing + Domain/Range

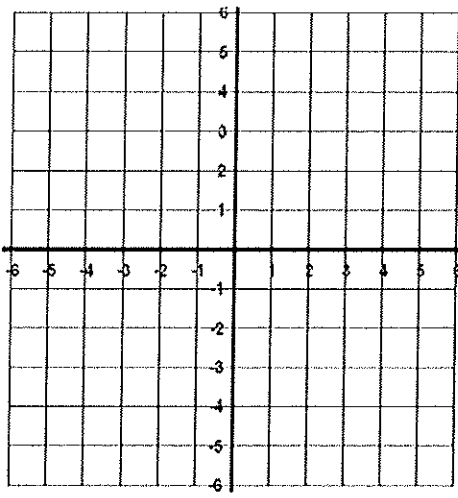
Classwork / Homework

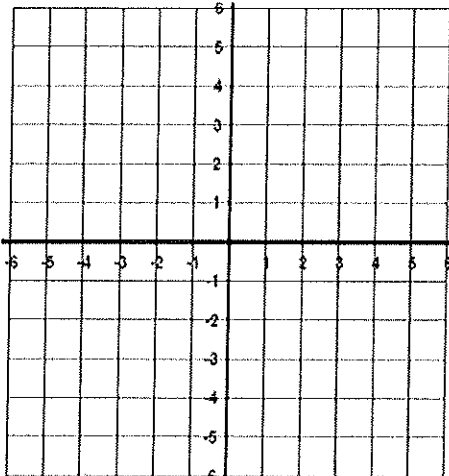
The Inverse of any relation or function is the switching of input and output.

To get points on the inverse, you switch x and y values. Plot the output values on the x-axis and the input values on the y-axis. To get the inverse equation, you switch x and y, and then isolate y.

- Domain and Range switch: The domain of the original relation becomes the range of the inverse, and vice versa. Also, nicely, every point's coordinates switch places. All (a, b) becomes (b, a).
- The graphs are a reflection (have symmetry) over the line $y = x$.

$(1,3), (2,5), (-2,0)$		THE INVERSE		
Function or Relation		Function or Relation		
D:		D:		
R:		R:		

$y = \frac{1}{2}x + 2$		THE INVERSE		
Function or Relation		Function or Relation		
D:		D:		
R:		R:		

$y = (x - 1)^2 + 2$		THE INVERSE		
Function or Relation		Function or Relation		
D:		D:		
R:		R:		

Where is my original relation at? 3. This means (2, 3) is a point on our original relation.

That should mean, if we look at 3 on the inverse, we should only get 2. Look and see what happens.

Gah! The inverse is not a function, because we get two values.

Remember what a function actually is: The relationship between input and output is such that one input value can only give you ONE possible output. If you can put in 3, for example, and get 0 or 2, it's not a function.

Let's say we wanted to FORCE the inverse to be a function... because square root functions are useful. That means we need to restrict the range such that we ONLY will get one possible output value per input value.

Restricting the range on the inverse means we have to restrict the domain on the original.

Graph $f(x) = (x - 1)^2 + 2$

Graph $f(x) = (x - 1)^2 + 2$

Just for fun: $f(x) = (x - 1)^2 + 2$

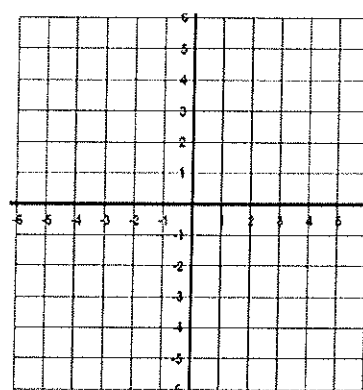
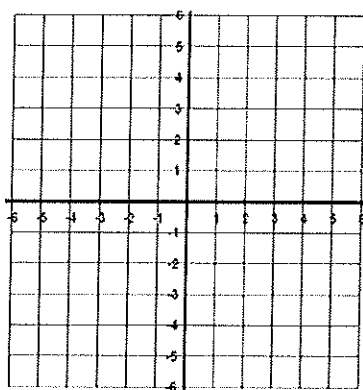
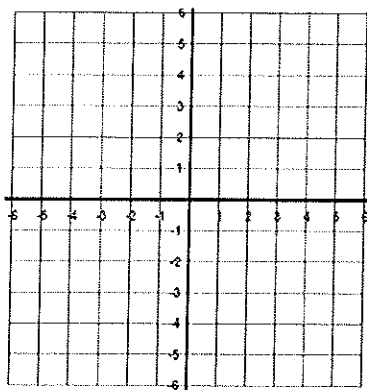
Restricted to $x \geq 1$

Restrict domain to $x \leq 1$

Restrict domain to $1 \leq x \leq 2$

$f(2) =$

$f(-1) =$



$f^{-1}(x) =$

$f^{-1}(x) =$

$f^{-1}(3) =$

$f^{-1}(6) =$

Why this matters for trigonometric functions:

In trigonometric functions (sin x, cos x, tan x, sec x, csc x, cot x): angles are the input → a ratio is the output (y)

In INVERSE trig functions, (sin⁻¹ x, cos⁻¹ x, tan⁻¹ x, sec⁻¹ x, csc⁻¹ x, cot⁻¹ x): ratios are the input → an angle is the output (y)

Evaluate: sin (30) = sin (150) = But if sin x = 1/2, We know x =

So arcsine is not a function, if we keep the entire domain of sine.



Hey! Hey you with the face!!
Don't you confuse sin⁻¹ x with (sin x)⁻¹!!
One of them is arcsin x, one is csc x!

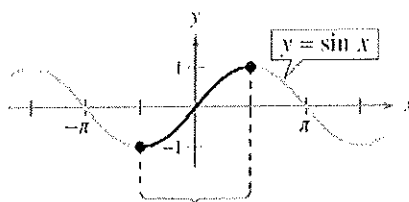
The Sine Inverse: sin⁻¹ x or arcsin x

Recall y = sin x

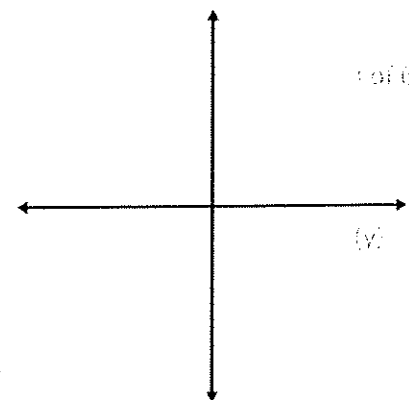
What points are critical values?

Domain:

Range:

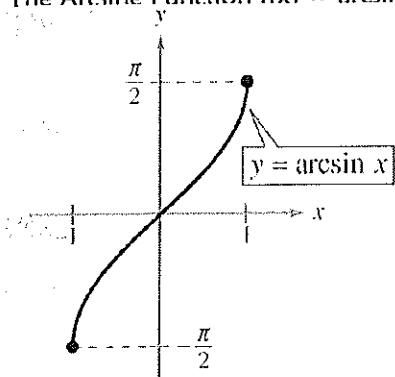


Graph sine and arcsine:



In order to make arcsine a function, we restrict sine's domain at the shown intervals.
We want the restriction to include a negative interval, one zero, and a positive interval.
That way, we will be restricting arcsine's answers to only give ONE.

The Arcsine Function f(x) = arcsin x or f(x) = sin⁻¹ x



Domain:

Note it's the range of the inverse

Range:

Note it's the restricted domain of the inverse

IMPORTANT:

Since the range of arcsin is between -90 and 90, that means any angle returned by that function **MUST** be in Quadrants I or IV

We've BEEN taking the arcsine of the absolute value of ratios so all we get is a REFERENCE angle, but we SHOULD be able to evaluate an inverse trig ratio on any interval between -1 and 1, because that is in the domain. Now we can, but we have to limit our outputs to values between -90 and 90, so only results in Quadrants I or IV.

Now that f(x) = arcsin x is a TRUE function, we can evaluate it!

sin⁻¹ (√3/2)

arcsin (-1/2)

arcsin (-√2/2)

sin⁻¹(0)

sin⁻¹ (-2/5)

We can also take composition of functions

cos (sin⁻¹(0))

cos (sin⁻¹(-1/2))

tan (sin⁻¹(√2/2))

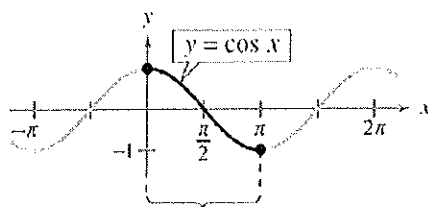
The Cosine Inverse: $\cos^{-1} x$ or $\arccos x$

Recall $y = \cos x$

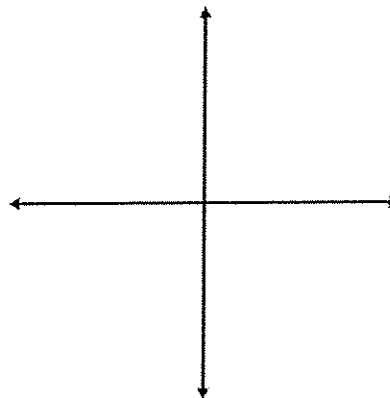
What points are critical values?

Domain:

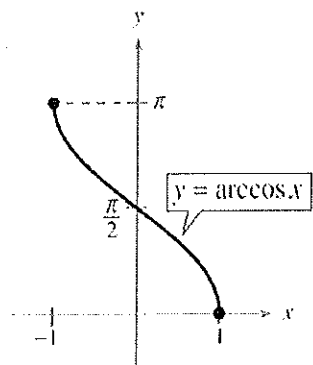
Range:



Graph cosine and arccosine:



The Arccosine Function $f(x) = \arccos x$ or $f(x) = \cos^{-1} x$



Domain:

Note it's the range of the inverse

Range:

Note it's the restricted domain of the inverse

IMPORTANT:

Since the range of arccos is between 0 and 180, that means any angle returned by that function **MUST** be in Quadrants I or II

Evaluate the following:

$$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\cos^{-1}\left(-\frac{1}{2}\right)$$

$$\cos^{-1}(0)$$

$$\cos^{-1}\left(-\frac{5}{7}\right)$$

We can also take composition of functions

$$\sin(\cos^{-1}(-1))$$

$$\tan(\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right))$$

$$\sin(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right))$$

$$\sec(\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right))$$

$$\tan(\cos^{-1}\left(-\frac{\sqrt{1}}{2}\right))$$

$$\csc(\cos^{-1}\left(\frac{\sqrt{3}}{2}\right))$$

When if the ratio is rational, you can draw out the triangle to evaluate the "missing" side.

$$\sin(\cos^{-1}\left(\frac{4}{5}\right))$$

$$\cot(\cos^{-1}\left(-\frac{5}{12}\right))$$

$$\csc(\cos^{-1}\left(\frac{3}{5}\right))$$

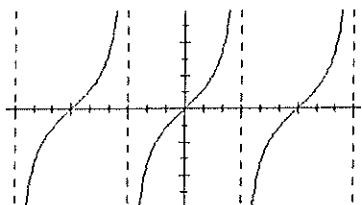
The Tangent Inverse: $\tan^{-1} x$ or $\arctan x$

Recall $y = \tan x$

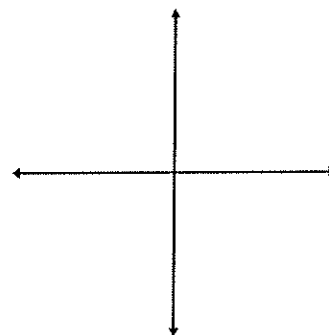
What points are critical values?

Domain:

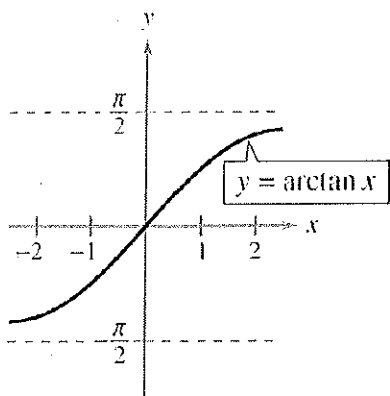
Range:



Graph tangent and arctangent:



The Arctangent Function $f(x) = \arctan x$ or $f(x) = \tan^{-1} x$



Domain:

Note it's the range of the inverse

Range:

Note it's the restricted domain of the inverse

IMPORTANT:

Since the range of arctan is between -90 and 90 , that means any angle returned by that function **MUST** be in Quadrants I or IV

Evaluate the following:

$$\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$$

$$\tan^{-1}(-1)$$

$$\tan^{-1}(-\sqrt{3})$$

$$\tan^{-1}(-516)$$

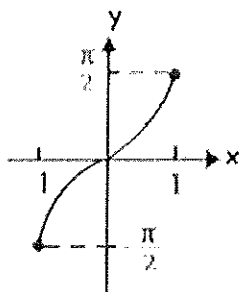
$$\sin(\tan^{-1}(1))$$

$$\csc(\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right))$$

$$\sec(\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right))$$

$$\sin(\tan^{-1}\left(\frac{12}{5}\right))$$

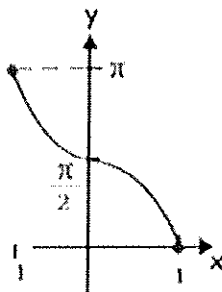
arcsine:
Q 1 (+) & 4(-)



Domain : $-1 \leq x \leq 1$

Range : $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

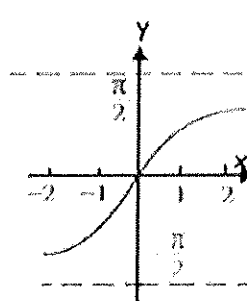
arccosine
Q 1 (+) & 2(-)



Domain : $-1 \leq x \leq 1$

Range : $0 \leq y \leq \pi$

arctangent
Q 1 (+) & 4(-)



Domain : $-\infty < x < \infty$

Range : $-\frac{\pi}{2} < y < \frac{\pi}{2}$

Practice Evaluating Inverse Trig Functions:

Shortcut #1: The trig function of its inverse function is the original ratio.

Shortcut #2: The trig function of its reciprocal function is the reciprocal ratio

1. $\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$

2. $\tan\left(\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right)$

3. $\csc\left(\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)\right)$

4. $\sec\left(\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$

5. $\cos\left(\sin^{-1}\left(\frac{4}{5}\right)\right)$

6. $\sec\left(\tan^{-1}\left(\frac{12}{5}\right)\right)$

7. $\cos\left(\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)\right)$

8. $\csc\left(\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)\right)$

9. $\sin^{-1}\left(\sin\left(\frac{7\pi}{4}\right)\right)$

10. $\sin^{-1}(\cos \pi)$

11. $\cos\left(\sin^{-1}\left(-\frac{1}{2}\right)\right)$

12. $\cos\left(\sin^{-1}\left(\frac{3}{5}\right)\right)$

13. $\tan\left(\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$

12. $\cos\left(\tan^{-1}(\sqrt{3})\right)$

13. $\cot\left(\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$

14. $\left(\tan^{-1}\left(\cos\left(\sin^{-1}\left(\frac{1}{2}\right)\right)\right)\right)$

15. $\left(\cos^{-1}\left(\sin\left(\tan^{-1}(-1)\right)\right)\right)$

16. $\sec\left(\tan^{-1}\left(\cos\left(\sin^{-1}(0)\right)\right)\right)$

17. Draw out the triangles for these problems:

$\tan\left(\sin^{-1}\left(-\frac{4}{5}\right)\right)$	$\cos\left(\tan^{-1}\left(\frac{5}{12}\right)\right)$	$\cot\left(\cos^{-1}\left(-\frac{7}{25}\right)\right)$
$\sec\left(\cos^{-1}\left(\frac{15}{17}\right)\right)$	$\csc\left(\sin^{-1}\left(\frac{1}{5}\right)\right)$	$\cos\left(\tan^{-1}\left(-\frac{5}{7}\right)\right)$

4.6 Problem set:

Evaluation without a calculator: #5–23 & 63–71; Simplification of "X" problems without a calculator: #75–83

Evaluation with a calculator: #25–37

You can now also do the Ch 4 Test on page 232, #25–40 (you should have already done # 9 – 23