Name: Key Per: ________ Date: ____________

Serafino ▪ Precalculus

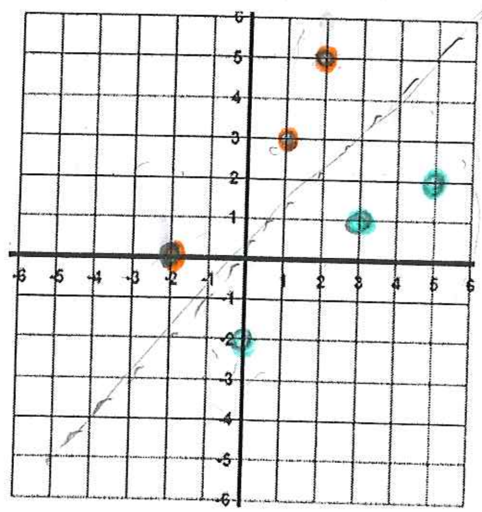
4.6 Inverse Functions – Graphing + Domain/Range

Classwork / Homework (Ch 4, video 7)

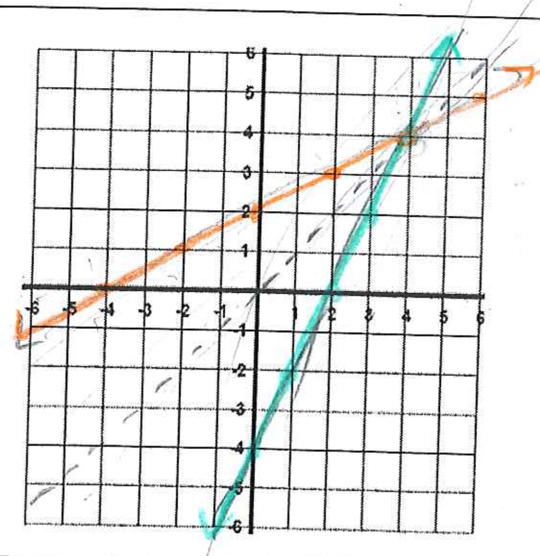
The Inverse of any relation or function is the switching of input and output.
To get points on the inverse, you switch x and y values. Plot the output values on the x–axis and the input values on the y–axis. To get the inverse equation, you switch x and y, and then isolate y.

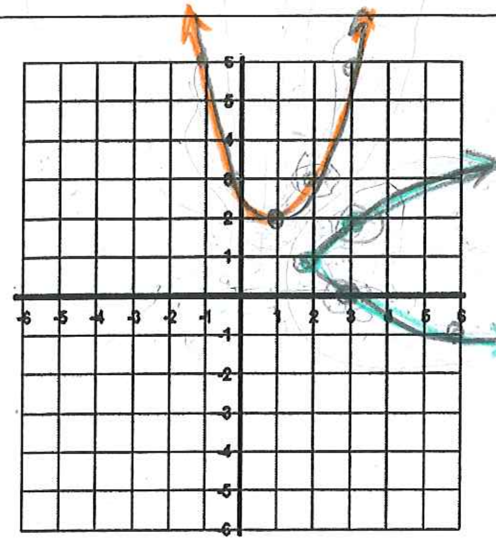
- Domain and Range switch: The domain of the original relation becomes the range of the inverse, and vice versa. Also, nicely, every point's coordinates switch places. All (a, b) becomes (b, a).
- The graphs are a reflection (have symmetry) over the line y = x.

		THE INVERSE	
(1,3), (2, 5), (−2, 0)		(3,1) (5,2) (0,−2)	
Function or Relation		Function or Relation	
D:	$x \in \{-2, 1, 2\}$	D:	$x \in \{0, 3, 5\}$
R:	$y \in \{0, 3, 5\}$	R:	$y \in \{-2, 1, 2\}$



		THE INVERSE	
$y = \frac{1}{2}x + 2$ $x = \frac{1}{2}y + 2$ $-2 \qquad -2$ $2(x - 2 = \frac{y}{2})2$		$y^{-1} = 2x - 4$	
Function or Relation		Function or Relation	
D:	$x \in \mathbb{R}$	D:	$x \in \mathbb{R}$
R:	$y \in \mathbb{R}$	R:	$y \in \mathbb{R}$



$y=(x-1)^2+2$	THE INVERSE	
$x=(y-1)^2+2$ $-2 \quad -2$ $\pm(x-2)=\sqrt{(y-1)^2}$ $\boxed{\pm\sqrt{x-2}}=y-1$	$y^{-1}=\pm\sqrt{x-2}+1$ $+1 \quad +1$	
Function or Relation	Function or Relation	
D: $x\in\mathbb{R}$	D: $x\geq 2$	
R: $y\geq 2$	R: $y\in\mathbb{R}$	

Evaluate the original relation, above, at 2. You should get 3. This means (2, 3) is a point on our original relation.

That should mean, if we plug in 3 value into the inverse, we should get 2. Try it and see what happens.

Gah! The inverse is not a function, because we get two values.

Remember what a function actually is: The relationship between input and output is such that one input value can only give you ONE possible output. If you can put in 3, for example, and get 0 or 2, it's not a function.

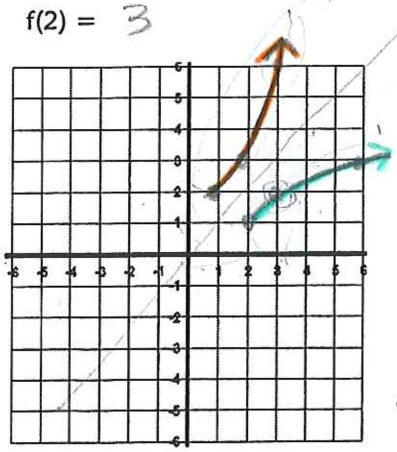
Let's say we wanted to FORCE the inverse to be a function… because square root functions are useful. That means we need to restrict the range such that we ONLY will get one possible output value per input value.

Restricting the range on the inverse means we have to restrict the domain on the original.

Graph $f(x)=(x-1)^2+2$

Restricted to $x\geq 1$

f(2) = 3



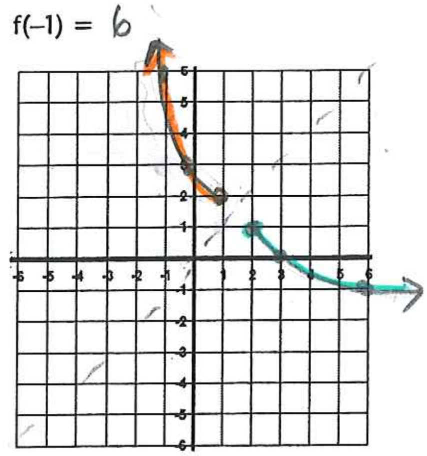
$f^{-1}(x) = \sqrt{x-2}+1$

$f^{-1}(3) = 2$

Graph $f(x)=(x-1)^2+2$

Restrict domain to $x\leq 1$

f(–1) = 6



$f^{-1}(x) = -\sqrt{x-2}+1$

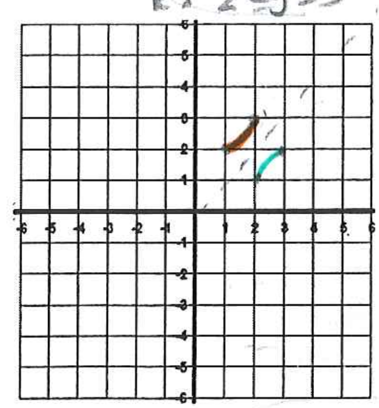
$f^{-1}(6) = -1$

Just for fun: $f(x)=(x-1)^2+2$

Restrict domain to $1\leq x\leq 2$

f(x)'s D: $1\leq x\leq 2$

R: $2\leq y\leq 3$



$f^{-1}(x)$'s D: $2\leq x\leq 3$

R: $1\leq x\leq 2$

They switched!

Why this matters for trigonometric functions:

In trigonometric functions (sin x, cos x, tan x, sec x, csc x, cot x): angles are the input → a ratio is the output (y)

In INVERSE trig functions, ($\sin^{-1} x$, $\cos^{-1} x$, $\tan^{-1} x$, $\sec^{-1} x$, $\csc^{-1} x$, $\cot^{-1} x$): ratios are the input → an angle is the output (y)

Evaluate: sin (30) = $\boxed{\frac{1}{2}}$ sin (150) = $\boxed{\frac{1}{2}}$ But if sin x = ½, We know x = $\{30^\circ, 150^\circ, 390^\circ, 510^\circ \ldots\}$ +360k

So arcsine is not a function, if we keep the domain of sine.

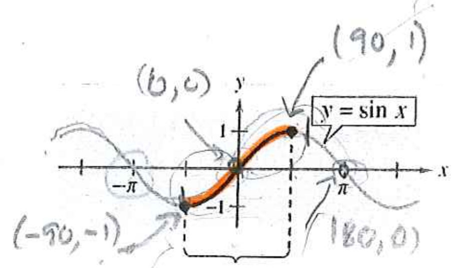
The Sine Inverse: $\sin^{-1} x$ or arcsin x

Recall y= sin x

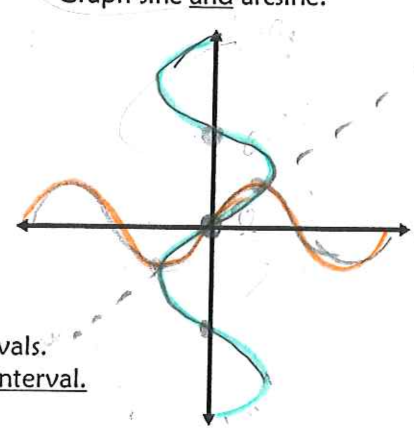
Domain: $x \in \mathbb{R}$

Range: $y \in [-1, 1]$

What points are critical values?

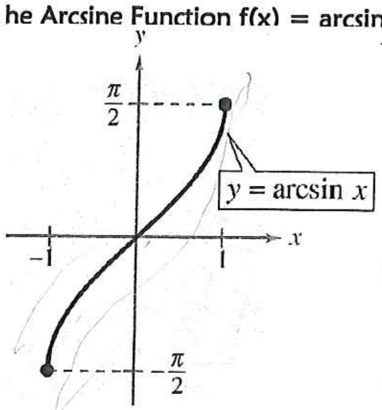


Graph sine and arcsine:



In order to make arcsine a function, we restrict sine's domain at the shown intervals.
We want the restriction to include a negative interval, one zero, and a positive interval.
That way, we will be restricting arcsine's answers to only give ONE.

The Arcsine Function f(x) = arcsin x or f(x) = $\sin^{-1}$ x



Domain: $x \in [-1, 1]$

Note it's the range of the inverse

Range: $y \in [-90^\circ, 90^\circ]$ or $[-\frac{\pi}{2}, \frac{\pi}{2}]$

Note it's the restricted domain of the inverse

IMPORTANT:
Since the range of arcsin is between –90 and 90, that means any angle returned by that function MUST be in Quadrants I or IV

We've BEEN taking the arcsine of the absolute value of ratios so all we get is a REFERENCE angle, but we SHOULD be able to evaluate an inverse trig ratio on any interval between –1 and 1, because that is in the domain. Now we can, but we have to limit our outputs to values between –90 and 90, so only results in Quadrants I or IV.

Now that f(x) = arcsin x is a TRUE function, we can evaluate it!

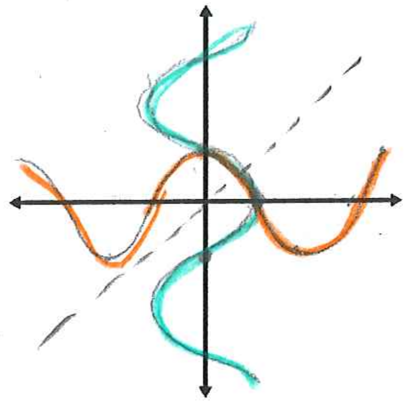
$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ $\boxed{60^\circ}$ $\arcsin\left(-\frac{1}{2}\right)$ $\boxed{-30^\circ}$ $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$ $\boxed{-45^\circ}$ $\sin^{-1}(0)$ $\boxed{0}$ $\sin^{-1}\left(-\frac{2}{5}\right)$ $\boxed{-23.58^\circ}$

We can also take composition of functions

$\cos(\sin^{-1}(0))$ $\boxed{1}$ $\cos(\sin^{-1}(-\frac{1}{2}))$ $\boxed{\frac{\sqrt{3}}{2}}$ $\tan(\sin^{-1}(\frac{\sqrt{2}}{2}))$ $\boxed{1}$

The Cosine Inverse: $\cos^{-1} x$ or $\arccos x$

Graph cosine and arccossine:

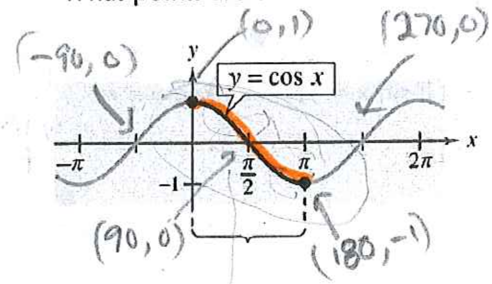


Recall y= cos x

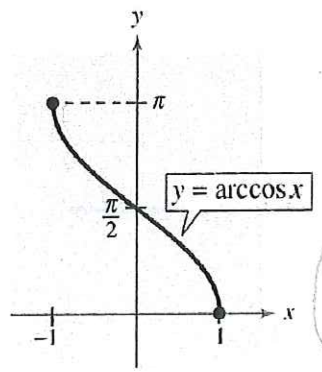
Domain: $x \in \mathbb{R}$

Range: $y \in [-1, 1]$

What points are critical values?



The Arccosine Function f(x) = arccos x or f(x) = cos^{-1} x



Domain: $x \in [-1, 1]$

Note it's the range of the inverse

Range: $y \in [0^\circ, 180^\circ]$ or $[0, \pi]$

Note it's the restricted domain of the inverse

IMPORTANT:
Since the range of arccos is between 0 and 180, that means any angle returned by that function MUST be in Quadrants I or II

Evaluate the following:

$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ $\boxed{30^\circ}$ $\quad$ $\cos^{-1}\left(-\frac{1}{2}\right)$ $\boxed{120^\circ}$ $\quad$ $\cos^{-1}(0)$ $\boxed{90^\circ}$ $\quad$ $\cos^{-1}\left(-\frac{5}{7}\right)$ $\boxed{135.58^\circ}$

We can also take composition of functions

$\sin(\cos^{-1}(-1))$ $\boxed{0}$ 180

$\tan\left(\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right)$ $\boxed{-1}$ 135

* shortcut #1
$\sin\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$ $\boxed{\frac{\sqrt{3}}{2}}$

* short cut #2
$\sec\left(\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right)$ $\boxed{-\sqrt{2}}$

$\tan\left(\cos^{-1}\left(-\frac{\sqrt{1}}{2}\right)\right)$ $\boxed{-\sqrt{3}}$

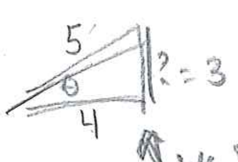
$\csc\left(\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$ $\boxed{2}$

When if the ratio is rational, you can draw out the triangle to evaluate the "missing" side.

$\sin\left(\cos^{-1}\left(\frac{4}{5}\right)\right)$ $\boxed{\frac{3}{5}}$

$\cot\left(\cos^{-1}\left(-\frac{5}{12}\right)\right)$ $\boxed{\frac{-5\sqrt{119}}{119}}$ $= \frac{-5}{\sqrt{119}}$

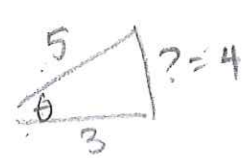
$\csc\left(\cos^{-1}\left(\frac{3}{5}\right)\right)$ $\boxed{\frac{5}{4}}$



↑ b/c positive ratio

$\sqrt{12^2-5^2} = \sqrt{119}$ →

↖ b/c neg. ratio



Tangent $\tan^{-1}x$ arctan x

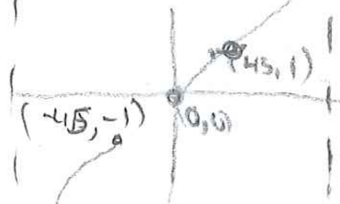
The ~~Cosine~~ Inverse: ~~$\cos^{-1}$~~ x or ~~arccos~~ x

Recall y= tan x

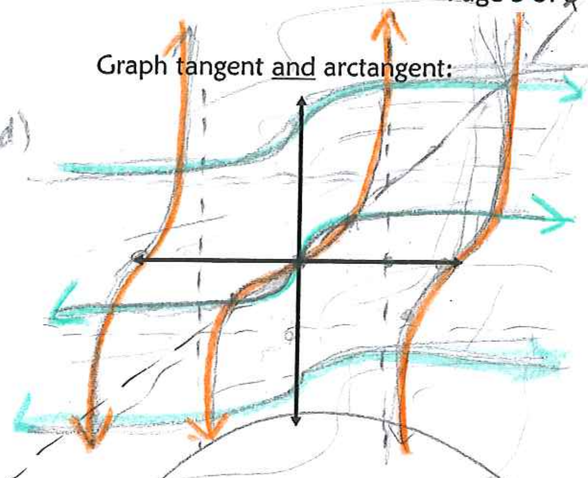
Domain: $x \neq 90 + 180k$

Range: $y \in \mathbb{R}$

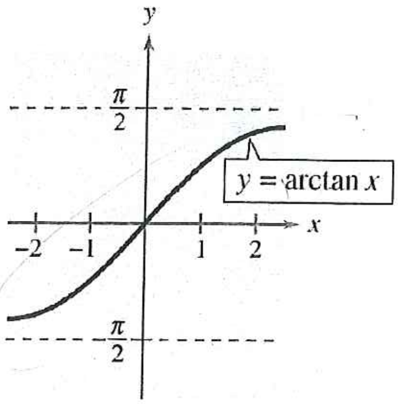
What points are critical values? (90, und)



Graph tangent and arctangent:



The Arctangent Function f(x) = arctan x or f(x) = ~~$\cos^{-1}$~~ x $\tan^{-1}$



Domain: $x \in \mathbb{R}$

Note it's the range of the inverse

Range: $y \in (-90, 90)$ Not including!

Note it's the restricted domain of the inverse

IMPORTANT:
Since the range of arctan is between −90 and 90, that means any angle returned by that function MUST be in Quadrants I or I~~I~~V

valuate the following:

$\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$ $\boxed{30°}$

$\tan^{-1}(-1)$ $\boxed{-45°}$

$\tan^{-1}(-\sqrt{3})$ $\boxed{-60°}$

$\tan^{-1}(-516)$ $\boxed{-89.89°}$

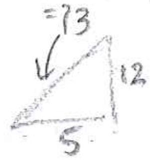
$\sin(\tan^{-1}(1))$ $\boxed{\frac{\sqrt{2}}{2}}$

$\csc\left(\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)\right)$ $\boxed{-2}$

$\sec\left(\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)\right)$ $\boxed{\frac{2\sqrt{3}}{3}}$

$\sin\left(\tan^{-1}\left(\frac{12}{5}\right)\right)$ $\boxed{\frac{12}{13}}$

=13



Practice Working with Inverse Trig Functions:
Evaluate / Simplify each expression:

1. $\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$ $\boxed{\frac{1}{2}}$ 60

2. $\tan\left(\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right)$ $\boxed{-1}$

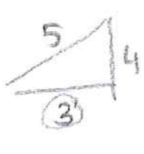
3. $\csc\left(\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)\right)$ $\boxed{-2}$

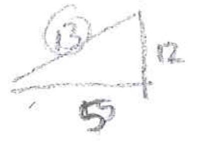
* shortcut # 2

4. $\sec\left(\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$ $\boxed{\frac{2\sqrt{3}}{3}}$ 30

5. $\cos\left(\sin^{-1}\left(\frac{4}{5}\right)\right)$ $\boxed{\frac{3}{5}}$

6. $\sec\left(\tan^{-1}\left(\frac{12}{5}\right)\right)$ $\boxed{\frac{13}{5}}$





7. $\cos\left(\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)\right)$ $\boxed{\frac{\sqrt{2}}{2}}$

45

8. $\csc\left(\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)\right)$ $\boxed{2}$

30

9. $\sin^{-1}\left(\sin\left(\frac{7\pi}{4}\right)\right)$ $\boxed{-45^\circ}$

$-\frac{\sqrt{2}}{2}$

10. $\sin^{-1}(\cos\pi)$ $\boxed{-90}$

11. $\cos\left(\sin^{-1}\left(-\frac{1}{2}\right)\right)$ $\boxed{\frac{-\sqrt{3}}{2}}$

-30

12. $\cos\left(\sin^{-1}\left(\frac{3}{5}\right)\right)$ $\boxed{\frac{4}{5}}$

5, 3, 4

13. $\tan\left(\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$ $\boxed{-\sqrt{3}}$

-60

12. $\cos\left(\tan^{-1}\left(\sqrt{3}\right)\right)$ $\boxed{\frac{1}{2}}$

60

13. $\cot\left(\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$ $\boxed{-\sqrt{3}}$

150

14. $\left(\tan^{-1}\left(\cos\left(\sin^{-1}\left(\frac{1}{2}\right)\right)\right)\right)$ $\boxed{40.893}$

30 $\quad \frac{\sqrt{3}}{2}$

needed a calc

15. $\left(\cos^{-1}\left(\sin\left(\tan^{-1}(-1)\right)\right)\right)$ $\boxed{135^\circ}$

-45 $\quad -\frac{\sqrt{2}}{2}$

16. $\sec\left(\tan^{-1}\left(\cos\left(\sin^{-1}(0)\right)\right)\right)$ $\boxed{\frac{\sqrt{2}}{2}}$

45 1 0

17. Draw out the triangles for these problems:

$\tan\left(\sin^{-1}\left(-\frac{4}{5}\right)\right)$ 3, -4, 5 $\boxed{\frac{-4}{3}}$	$\cos\left(\tan^{-1}\left(\frac{5}{12}\right)\right)$ 13, 5, 12 $\boxed{\frac{12}{13}}$	$\cot\left(\cos^{-1}\left(-\frac{7}{25}\right)\right)$ 25, -7, 24, $\sqrt{25^2-7^2}$ $\boxed{\frac{-7}{24}}$
* shortcut #2 $\sec\left(\cos^{-1}\left(\frac{15}{17}\right)\right)$ $\boxed{\frac{17}{15}}$	$\csc\left(\sin^{-1}\left(\frac{1}{5}\right)\right)$ $\sqrt{5^2-1^2}$ $\sqrt{24}$ $2\sqrt{6}$ 5, 1, $2\sqrt{6}$ $\boxed{5}$	$\cos\left(\tan^{-1}\left(-\frac{5}{7}\right)\right)$ 7, -5, $\sqrt{74}$, $\sqrt{5^2+7^2}$ $\boxed{\frac{-5\sqrt{74}}{74}}$