

Name: Blank + Solutions (4B)
 Serafino • Precalculus

Period: 3/7

Date: 1/5/16

6.1 Solving Trigonometric Equations & Applications: Part 1: Equations Part 2: Applications

For each equation, you should be able to give the general solution in both degrees and radians (ALL possible solutions (+ something k)). You also should be able to list all solutions between 0° and 360° or 0 and 2π .

Part 1A: Equations with Linear Terms. *Give solutions between $0 \leq x < 2\pi$*

1) $\sin x - \frac{1}{2} = 0$

2) $2\cos x - \sqrt{3} = 0$

3) $4\sin x + 3 = 3\sin x + 2$

4) $2\sin x \cos x = \cos x$

5) $3\tan x = 3$

6) $2\cos x + 1 = 2$

7) $\sin x \cos x \tan x + \sin x \cos x = 0$

8) $2\cos x(\cos x + \frac{1}{2}) = 0$

9) $(\sec x + 1)(\sin x - 1)(\cot x - 1) = 0$

10) $\csc x \cot x + \cot x = 0$

11) $\tan x \cos x + \cos x = 0$

12) $(\cos x - 1)(\tan x - 1) = 0$

13) $\sec x \cos x + \sec x = 0$

14) $\frac{\sin x}{2} = \frac{\sin x}{3}$

Part 1B: Equations with Increased Frequency. *Give all solutions between $0 \leq \theta < 2\pi$*

1) $\sin 2\theta = -\frac{\sqrt{3}}{2}$

4) $\sin 2\theta = -1$

2) $\cos 2\theta = \frac{\sqrt{3}}{2}$

5) $\cos 3\theta = \frac{\sqrt{3}}{2}$

3) $\cos 2\theta = -\frac{1}{2}$

6) $\cos\left(\frac{1}{2}\theta\right) = \frac{\sqrt{2}}{2}$

Part 1C: Equations Requiring Factoring. Give all solutions between $0 \leq x < 2\pi$.

1) $\cos^2 x = \frac{3}{4}$

7) $6\cos^2 x - 3\cos x - 3 = 0$

2) $\sin^2 x - \frac{1}{4} = 0$

8) $2\sin^2 x - 3\sin x + 1 = 0$

3) $3\tan^2 x = 3$

9) $4\cos^2 x + 2\cos x - 2 = 0$

4) $4\sin^2 x - 3 = 0$

10) $2\cos^3 x + \cos^2 x - \cos x = 0$

5) $2\sin^2 x - \sin x - 1 = 0$

11) $\tan^4 x - \tan^2 x = 0$

6) $2\sin^2 x = 1 - \sin x$

12) $\cos^8 x - \cos^4 x = 0$

Part 1D: All Trig Equations: Find the general solution in both degrees AND radians:

1) $\sin 3x = \frac{\sqrt{3}}{2}$

2) $\cos^2 x = 1$

3) $\sin 2x = 0$

4) $\sin 4x = -\frac{1}{2}$

5) $\tan 2x = \sqrt{3}$

6) $\sin^2 x - 0.25 = 0$

7) $\tan x + \sqrt{3} = 0$

8) $\sin \frac{1}{2}x = -\frac{\sqrt{3}}{2}$

Part 1E: Trig Equations requiring calculator. Give all solutions between $0^\circ \leq x < 360^\circ$
You will either use factoring or the quadratic formula to solve:

1) $\sin^2 x + 3\sin x = 4$ *No calc

6) $10\sin^2 x + 9\sin x - 9 = 0$

2) $\tan^2 x - 5\tan x + 6 = 0$

7) $\tan^2 x + 5\tan x - 4 = 0$

3) $3\cos^2 x + 10\cos x + 2 = 10$

8) $2\sin^2 x + 5\sin x - 1$ ~~no calc~~ Calc needed!

4) $15\sin^2 x + 11\sin x + 2 = 0$

9) $3\cos^2 x - 10\cos x + 5 = 0$

5) $2\cos^2 x + 2\cos x - 12 = 0$ *No calc

4B Trig Equations Solutions & work

Note!

$x = \text{degrees}$

$\theta = \text{radians}$

1A

① $\sin x - \frac{1}{2} = 0$

$$\sin x = \frac{1}{2}$$

$$x = 30^\circ, 150^\circ$$

$x = 30^\circ, 150^\circ$
$x = \frac{\pi}{6}, \frac{5\pi}{6}$

② $2 \cos x - \sqrt{3} = 0$

$$2 \cos x = \sqrt{3}$$

$$\cos x = \frac{\sqrt{3}}{2}$$

$$x = 30, 330$$

$x = 30^\circ, 330^\circ$
$\theta = \frac{\pi}{6}, \frac{11\pi}{6}$

③ $4 \sin x + 3 = 3 \sin x + 2$

$$\sin x = -1$$

$x = 270^\circ$
$\theta = 3\pi/2$

④ $2 \sin x \cos x = \cos x$

$$2 \sin x \cos x - \cos x = 0$$

$$\cos x (2 \sin x - 1) = 0$$

$$\cos x = 0 \quad \sin x = \frac{1}{2}$$

$$x = 90, 270, \quad x = 30, 150^\circ$$

$x = 90^\circ, 270^\circ, 30^\circ, 150^\circ$
$\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

⑤ $3 \tan x = 3$

$$\tan x = 1$$

$x = 45^\circ, 225^\circ$
$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$

$$\begin{aligned} \textcircled{6} \quad 2 \cos x + 1 &= 2 \\ 2 \cos x &= 1 \\ \cos x &= 1/2 \end{aligned}$$

$x = 60^\circ, 300^\circ$
$\theta = \pi/3, 5\pi/3$

$$\begin{aligned} \textcircled{7} \quad \sin x \cos x \tan x + \sin x \cos x &= 0 \\ \sin x \cos x (\tan x + 1) &= 0 \end{aligned}$$

$$\begin{aligned} \sin x = 0 \quad \cos x = 0 \quad \tan x = -1 \\ 0^\circ, 180^\circ; \quad 90^\circ, 270^\circ, \quad 135^\circ, 315^\circ \end{aligned}$$

$x = 0, 180^\circ, 135^\circ, 315^\circ$
$\theta = 0, \pi, 3\pi/4, 7\pi/4$

ext: $90^\circ, 270^\circ$
ext: $\pi/2, 3\pi/2$

$$\textcircled{8} \quad 2 \cos x (\cos x + 1/2) = 0$$

$$2 \cos x = 0 \quad \cos x + 1/2 = 0, \cos x = -1/2$$

$x = 90^\circ, 270^\circ, 120^\circ, 240^\circ$
$\theta = \pi/2, 3\pi/2, 2\pi/3, 4\pi/3$

$$\textcircled{9} \quad (\sec x + 1)(\sin x - 1)(\cot x - 1) = 0$$

$$\sec x = -1 \quad \sin x = 1 \quad \cot x = 1$$

$$\cancel{180^\circ} \quad \cancel{90^\circ} \quad 45^\circ, 215^\circ$$

$x = 45^\circ, 225^\circ$
$\theta = \pi/4, 5\pi/4$

ext: $90^\circ, 180^\circ$
ext: $\pi/2, \pi$

(1B) ① $\sin 2x = -\frac{\sqrt{3}}{2} \Rightarrow 2x = \frac{240}{2}, \frac{300}{2} + \frac{360k}{2}$
 $x = 120, 150 + 180^\circ k$

$x = 120^\circ, 150^\circ, 300^\circ, 330^\circ$
$\theta = \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{5\pi}{3}, \frac{11\pi}{6}$

② $\cos 2x = \frac{\sqrt{3}}{2} \quad 2x = \frac{30}{2}, \frac{330}{2} + \frac{360k}{2}$
 $15^\circ, 165^\circ + 180^\circ k$

$x = 15^\circ, 165^\circ, 195^\circ, 345^\circ$
$\theta = \frac{\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{23\pi}{12}$

③ $\cos 2x = -1/2$
 $2x = \frac{120}{2}, \frac{240}{2} + \frac{360k}{2}$
 $x = 60, 120 + 180^\circ k$

$x = 60, 120, 240, 300^\circ$
$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

④ $\sin 2x = -1$
 $2x = \frac{270}{2} + \frac{360k}{2}$
 $x = 135^\circ + 180k$

$x = 135^\circ, 315^\circ$
$\theta = 3\pi/4, 11\pi/4$

⑤ $\cos 3x = \sqrt{3}/2$
 $3x = \frac{30}{3}, \frac{330}{3} + \frac{360k}{3}$
 $x = 10^\circ, 110^\circ + 120^\circ k$

$x = 10^\circ, 110^\circ, 130^\circ, 230^\circ, 250^\circ, 350^\circ$
$\theta = \frac{\pi}{18}, \frac{11\pi}{18}, \frac{13\pi}{18}, \frac{23\pi}{18}, \frac{25\pi}{18}, \frac{35\pi}{18}$

⑥ $\cos (1/2 x) = \sqrt{2}/2$
 $1/2 x = 45, 315 + 360^\circ k$
 $x = 90, 630 + 720^\circ k$

$x = 90^\circ, \theta = \frac{\pi}{2}$
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⑩ $\cot x (\csc x + 1) = 0$
 $\cot x = 0$ $\csc x = -1$

$x = 90^\circ, 270^\circ, 270$ $\theta = \pi/2, 3\pi/2$
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⑪ $\tan x \cos x + \cos x = 0$
 $\cos x (\tan x + 1) = 0$

$\cos x = 0$ $\tan x = -1$
 $x = \cancel{90^\circ}, \cancel{270^\circ} \quad 135^\circ, 315^\circ$

$x = 135^\circ, 315^\circ$ $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

ext: $90^\circ, 270^\circ$
ext: $\frac{\pi}{2}, \frac{3\pi}{2}$

⑫ $(\cos x - 1)(\tan x - 1) = 0$
 $\cos x = 1$ $\tan x = 1$
 $0, \quad 45, 135^\circ$

$x = 0^\circ, 45^\circ, 135^\circ$ $\theta = 0, \frac{\pi}{4}, \frac{5\pi}{4}$

⑬ $\sec x (\cos x + 1) = 0$
 $\sec x = 0$ $\cos x = -1$
 $\times \quad 180$

$x = 180^\circ$ $\theta = \pi$

⑭ $\frac{\sin x}{2} = \frac{\sin x}{3}$
 $\sin x = 0$

$x = 0^\circ, 180^\circ$ $\theta = 0, \pi$

1C

① $\sqrt{\cos^2 x} = \frac{3}{4}$
 $\cos x = \pm \frac{3}{4}$
 $x = 30^\circ, 330^\circ, 150^\circ, 210^\circ$

$x = 30^\circ, 330^\circ, 150^\circ, 210^\circ$
$\theta = \frac{\pi}{6}, \frac{11\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$

② $\sin^2 x - \frac{1}{4} = 0$
 $\sqrt{\sin^2 x} = \frac{1}{4}$
 $\sin x = \pm \frac{1}{2}$

$x = 30^\circ, 330^\circ, 150^\circ, 210^\circ$
$\theta = \frac{\pi}{6}, \frac{11\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$

③ $3 + \tan^2 x = 3$
 $\tan^2 x = 1$
 $\tan x = \pm 1$

$x = 45^\circ, 135^\circ, 225^\circ, 315^\circ$
$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

④ $4 \sin^2 x - 3 = 0$
 $\sqrt{\sin^2 x} = \frac{3}{4}$
 $\sin x = \pm \frac{3}{4}$

$x = 60^\circ, 120^\circ, 240^\circ, 300^\circ$
$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

⑤ $2 \sin^2 x - \sin x - 1 = 0$ $2x^2 - x - 1 = 0$ $\frac{-2}{-2} \pm 1$
 $(2x-2)(2x+1) = 0$
 $\sin x = 1$ $\sin x = -\frac{1}{2}$ $(x-1)(2x+1) \quad x = 1, -1/2$

$x = 90^\circ, 210^\circ, 330^\circ$	$\theta = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$
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⑥ $2 \sin^2 x = 1 - \sin x$
 $\sin x = -1, \sin x = -1/2$

$2x^2 + x - 1 = 0$
 $(2x+2)(2x-1) = 0$
 $(x+1)(2x-1) = 0$
 $x = -1, 1/2$

$x = 270^\circ, 30^\circ, 150^\circ$	$\theta = \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$
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⑦ $6 \cos^2 x - 3 \cos x - 3 = 0$
 $\cos x = 1, \cos x = -1/2$

$3(2x^2 - x - 1) = 0$
 $x = 1, x = -1/2$

$x = 0^\circ, 120^\circ, 240^\circ$	$\theta = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$
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$$\textcircled{8} \quad 2\sin^2 x - 3\sin x + 1$$

$$\boxed{x = 90^\circ, 30^\circ, 150^\circ}$$

$$\boxed{\theta = \pi/2, \pi/6, 5\pi/6}$$

$$2x^2 - 3x + 1$$

$$(2x-2)(2x-1)$$

$$(x-1)(2x-1) \quad x=1, 1/2$$

$$\textcircled{9} \quad 4\cos^2 x + 2\cos x - 2 = 0$$

$$\boxed{x = 180^\circ, 60^\circ, 300^\circ}$$

$$\boxed{\theta = \pi, \pi/3, 5\pi/3}$$

$$2(2x^2 + x - 1)$$

$$(2x-1)(x+1) \quad x = -1, 1/2$$

$$\textcircled{10} \quad 2\cos^3 x + \cos^2 x - \cos x = 0$$

$$\boxed{x = 90^\circ, 270^\circ, 180^\circ, 60^\circ, 300^\circ}$$

$$\boxed{\theta = \pi/2, 3\pi/2, \pi, \pi/3, 5\pi/3}$$

$$2x^3 + x^2 - x = 0$$

$$x(2x^2 + x - 1)$$

$$x = 0, -1, 1/2$$

$$\textcircled{11} \quad \tan^4 x - \tan^2 x = 0$$

$$\tan x = 0 \quad \tan x = \pm 1$$

$$\boxed{x = 0, 180, 45, 135, 225, 315^\circ}$$

$$\boxed{\theta = 0, \pi, \pi/4, 3\pi/4, 5\pi/4, 7\pi/4}$$

$$x^4 - x^2 = 0$$

$$x^2(x^2 - 1) = 0$$

$$x^2 = 1 \quad x = \pm 1$$

$$x = 0, 1, -1$$

$$\textcircled{12} \quad \cos^8 x - \cos^4 x = 0$$

$$\boxed{x = 0, 180^\circ, 90^\circ, 270^\circ}$$

$$\boxed{\theta = 0, \pi, \pi/2, 3\pi/2}$$

$$x^8 - x^4 = 0$$

$$x^4(x^4 - 1) = 0 \quad x = 0, \pm 1$$

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① $\sin 3x = \frac{\sqrt{3}}{2}$
 $3x = 60, 120 + 360k$
 $x = 20, 40 + 120k$

$$x \in \{20^\circ, 40^\circ\} + 120^\circ k$$

$$\theta \in \left\{ \frac{\pi}{9}, \frac{2\pi}{9} \right\} + \frac{2\pi}{3} k$$

② $\cos^2 x = 1$
 $\cos x = \pm 1$

$$x \in \{180^\circ k\}$$

$$\theta \in \{\pi k\}$$

③ $\sin 2x = 0$
 $2x = 0, 180 + 360$

$$x \in \{0^\circ, 90^\circ\} + 180^\circ k \quad | \quad \theta = \{0, \frac{\pi}{2}\} + \pi k$$

or, simply, $x \in 90^\circ k \quad | \quad \theta = \frac{\pi}{2} k$

④ $\sin 4x = -\frac{1}{2}$
 $4x = 210, 330 + 360k$

$$x \in \{52.5, 82.5\} + 90^\circ k$$

$$\theta = \left\{ \frac{7\pi}{24}, \frac{11\pi}{24} \right\} + \frac{\pi}{2} k$$

⑤ $\tan 2x = \sqrt{3}$
 $\frac{2x}{2} = \frac{60^\circ}{2} + \frac{180^\circ k}{2}$

$$x \in \{30^\circ + 90^\circ k\}$$

$$\theta \in \left\{ \frac{\pi}{6} + \frac{\pi}{2} k \right\}$$

⑥ $\sin^2 x - 0.25 = 0$
 $\sqrt{\sin^2} = \sqrt{\frac{1}{4}} = \pm \frac{1}{2}$

$$x \in \{30^\circ, 150^\circ\} + 180^\circ k$$

$$\theta \in \left\{ \frac{\pi}{6}, \frac{5\pi}{6} \right\} + \pi k$$

⑦ $\tan x + \sqrt{3} = 0$
 $\tan x = -\sqrt{3}$

$$x \in 120^\circ + 180^\circ k \quad | \quad \theta = \frac{2\pi}{3} + \pi k$$

⑧ $\sin \frac{1}{2} x = -\frac{\sqrt{3}}{2}$
 $\frac{1}{2} x = 240, 300 + 360k$
 $x = 480, 60 + 720^\circ k$

$$x \in \{240^\circ, 300^\circ\} + 720^\circ$$

$$\theta \in \left\{ \frac{8\pi}{3}, \frac{10\pi}{3} \right\} + 4\pi k$$

$$\boxed{1E} \textcircled{1} \sin^2 x + 3 \sin x = 4$$

$$\sin x = -4 \quad \sin x = 1$$

$$\times \quad \boxed{x = 90^\circ \mid \theta = \frac{\pi}{2}}$$

$$x^2 + 3x - 4 = 0$$

$$(x+4)(x-1) = 0$$

$$x = -4, 1$$

$$\textcircled{2} \tan^2 x - 5 \tan x + 6 = 0$$

$$\tan x = 2 \quad \tan x = 3$$

$$x' = 63.44^\circ + 180^\circ \quad x' = 71.57^\circ + 180^\circ$$

$$x^2 - 5x + 6 = 0$$

$$(x-2)(x-3) = 0$$

$$x = 2, x = 3$$

$$\boxed{x = 63.44^\circ, 243.44^\circ, 71.57^\circ, 251.57^\circ}$$

$$\textcircled{3} 3 \cos^2 x + 10 \cos x + 2 = 16$$

$$\cos x = -4 \quad \cos x = 2/3$$

$$\times \quad x' = 48.189^\circ$$

$$\boxed{x = 48.189^\circ, 311.8103^\circ}$$

$$3x^2 + 10x - 8 = 0$$

$$\left(\frac{3x+12}{3}\right)\left(\frac{3x-2}{3}\right)$$

$$(x+4)(3x-2)$$

$$x = -4, x = 2/3$$

$$\textcircled{4} 15x^2 + 11x + 2 = 0$$

$$\left(\frac{15x+5}{5}\right)\left(\frac{15x+6}{3}\right)$$

$$(3x+1)(5x+2) \quad x = -\frac{1}{3}, -\frac{2}{5}$$

$$\sin x = -\frac{1}{3} \quad x' = 19.4712^\circ$$

$$x = 199.47^\circ, 340.53^\circ$$

$$x = 203.58^\circ, 336.42^\circ$$

$$\sin x = -\frac{2}{5} \quad x' = 23.58^\circ$$

$$\textcircled{5} 2(\cos^2 x + \cos x - 6)$$

$$2(x+3)(x-2) \quad \cos x = -3$$

$$\cos x = 2$$

No solution

$$\textcircled{6} 10x^2 + 9x - 9 = 0$$

$$\left(\frac{10x+15}{5}\right)\left(\frac{10x-6}{2}\right)$$

$$(2x+3)(5x-3)$$

$$x = -3/2, 3/5$$

$$\sin x = -3/2 \quad \sin x = 3/5$$

$$\times \quad \boxed{x = 36.87^\circ, 143.13^\circ}$$

$$\textcircled{7} \quad \tan^2 x + 5 \tan x - 4 = 0 \quad x^2 + 5x - 4 = 0 \quad \frac{-4}{1/4}$$

Not factorable

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{25 - 4(1)(-4)}}{2} = \frac{-5 \pm \sqrt{41}}{2}$$

$$\frac{-5 + \sqrt{41}}{2} \approx .7016 \rightarrow \tan x = .7016$$

$$x' = 35.053, 180 + x'$$

$$x = 35.053^\circ$$

$$= 215.053^\circ$$

$$\frac{-5 - \sqrt{41}}{2} \approx -5.7016$$

$$\tan x = -5.7016$$

$$= 99.948^\circ$$

$$x' = 80.052, 180 - x', 360 - x'$$

$$= 279.948^\circ$$

* Calc Req.

$$\textcircled{8} \quad 2 \sin^2 x + 5 \sin x - 1 = 0 \quad -5 \pm \sqrt{25 - 4(2)(-1)}$$

Not factorable 2(2)

$$\sin x = -2.686 \quad \times$$

$$= \frac{-5 \pm \sqrt{33}}{4} \rightarrow -2.686$$

$$\sin x = 0.186$$

$$\rightarrow 0.186$$

$$\boxed{x = 10.719^\circ, 169.201^\circ}$$

$$9. \quad 3x^2 - 10x + 5 = 0$$

$$\begin{array}{r} 5 \\ 1 \overline{) 15} \\ 3 \overline{) 5} \end{array}$$

$$\frac{10 \pm \sqrt{100 - 4(3)(5)}}{2(3)} = \frac{10 \pm \sqrt{40}}{6} \rightarrow 0.61257$$

$$\rightarrow 2.721 \times$$

$$\cos x = 0.61257$$

$$x' = 52.22$$

$$\boxed{x = 52.22, 367.776^\circ}$$

