Unit 4A

DO: 297 # 9, 10, 12, 18, 19, 20, 22, 23

5-1 Polynomial Functions

Objectives To classify polynomials
To graph polynomial functions and describe end behavior

Working backwards unlocks the patterns.

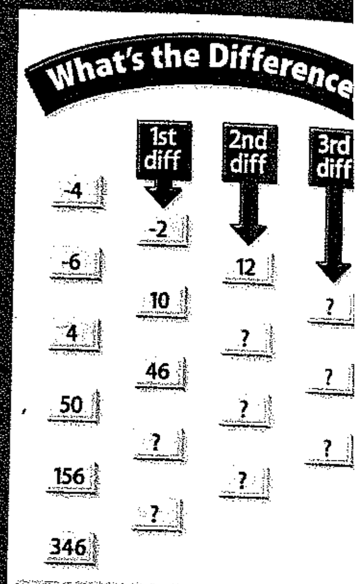
SOLVE IT! Getting Ready!

The first column shows a sequence of numbers. For 1st differences, subtract consecutive numbers in the sequence:

$-6 - (-4) = -2$, $4 - (-6) = 10$, and so on.

For 2nd differences, subtract consecutive 1st differences. For 3rd differences, subtract consecutive 2nd differences.

If the pattern suggested by the 3rd differences continues, what is the 8th number in the first column? Justify your reasoning.



The sequence of numbers in the first column in the Solve It are values of a particular *polynomial function.* For such a sequence, you can use patterns of 1st differences, 2nd differences, 3rd differences, and so on, to learn more about the polynomial function.

Lesson Vocabulary
- monomial
- degree of a monomial
- polynomial
- degree of a polynomial
- polynomial function
- standard form of a polynomial function
- turning point
- end behavior

Focus Question What is a polynomial function?

A **monomial** is a real number, a variable, or a product of a real number and one or more variables with whole-number exponents. The **degree of a monomial** in one variable is the exponent of the variable. A **polynomial** is a monomial or a sum of monomials. The **degree of a polynomial** in one variable is the greatest degree among its monomial terms.

Monomial: $3x^4$ — The degree is 4.

Polynomial: $2x^5 + 3x^4 - ($ — The degree is 5

A polynomial with the variable x defines a **polynomial function** of x. The degree of the polynomial function is the same as the degree of the polynomial.

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ıowing how to write quadratic function standard form can ːlp you remember ·w to write a ılynomial function standard form.

take note

Key Concept Standard Form of a Polynomial Function

The **standard form of a polynomial function** arranges the terms by degree in descending numerical order.

A polynomial function $P(x)$ in standard form is

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0,$$

where n is a nonnegative integer and $a_n, \ldots, a_0$ are real numbers.

$$P(x) = 4x^3 + 3x^2 + 5x - 2$$

Cubic term | Quadratic term | Linear term | Constant term

You can classify a polynomial by its degree or by its number of terms. Polynomials of degrees zero through five have specific names, shown in the table below.

Degree	Name Using Degree	Polynomial Example	Number of Terms	Name Using Number of Terms
0	constant	5	1	monomial
1	linear	$x + 4$	2	binomial
2	quadratic	$4x^2$	1	monomial
3	cubic	$4x^3 - 2x^2 + x$	3	trinomial
4	quartic	$2x^4 + 5x^2$	2	binomial
5	quintic	$-x^5 + 4x^2 + 2x + 1$	4	polynomial of 4 terms

Problem 1 Classifying Polynomials

Write each polynomial in standard form. What is the classification of each polynomial by degree and number of terms?

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nbine like terms if sible. Then write the ns with their degrees escending order.

A $3x + 9x^2 + 5$

$9x^2 + 3x + 5$

The polynomial has degree 2 and 3 terms. It is a quadratic trinomial.

B $4x - 6x^2 + x^4 + 10x^2 - 12$

$x^4 + 4x^2 + 4x - 12$

The polynomial has degree 4 and 4 terms. It is a quartic polynomial of 4 terms.

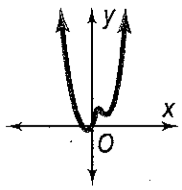
Got It? 1. Write each polynomial in standard form. What is the classification of each polynomial by degree and number of terms?

a. $3x^3 - x + 5x^4$ **b.** $3 - 4x^5 + 2x^2 + 10$

The degree of a polynomial function affects the shape of its graph. It determines the maximum number of **turning points**, or places where the graph changes direction. It also affects the **end behavior**, or the directions of the graph to the far left and to the far right.

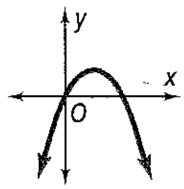
For polynomial functions of degree one or greater, there are four types of end behavior as you move to the left and move to the right, away from the origin:

Up and Up



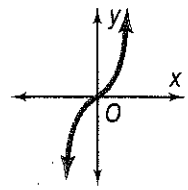
$y = x^4 - 3x^3 + 5x$

Down and Down



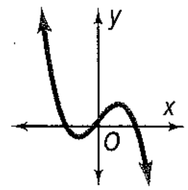
$y = -x^2 + 6x$

Down and Up



$y = x^3$

Up and Down



$y = -0.3x^3 + 4x + 2$

Hint

leading term
The leading term of a polynomial function in standard form is the first term.

You can determine the end behavior of a polynomial function of degree n from the leading term ax^n of the standard form.

End Behavior of a Polynomial Function of Degree n With Leading Term ax^n (Moving Away From the Origin)

	n Even	n Odd
a Positive	Up and Up	Down and Up
a Negative	Down and Down	Up and Down

Problem 2 Describing End Behavior of Polynomial Functions

Consider the leading term of each polynomial function. What is the end behavior of the graph? Check your answer with a graphing calculator.

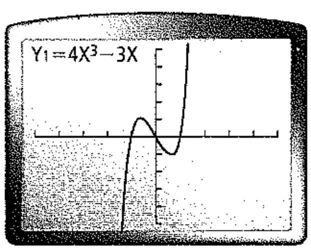
Think

What do a and n represent?
a is the coefficient of the leading term. n is the exponent of the leading term.

A $y = 4x^3 - 3x$

The leading term is $4x^3$. Since n is odd and a is positive, the end behavior is down and up.

Check

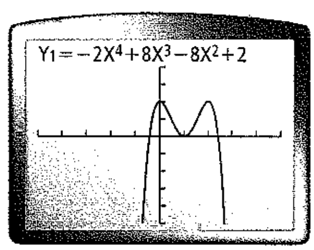


The solution checks.

B $y = -2x^4 + 8x^3 - 8x^2 + 2$

The leading term is $-2x^4$. Since n is even and a is negative, the end behavior is down and down.

Check



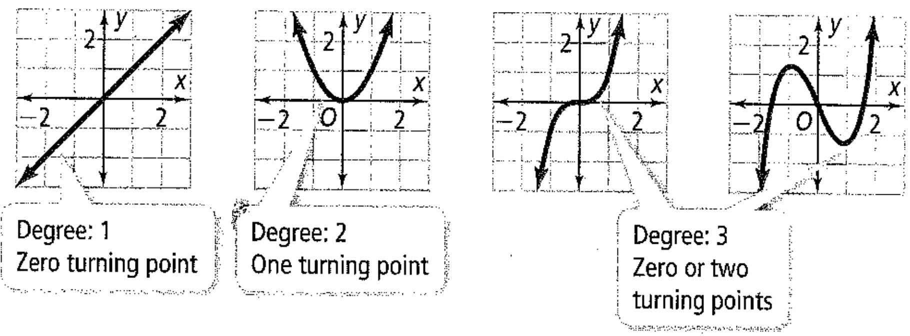
The solution checks.

Got It? 2. Consider the leading term of $y = -4x^3 + 2x^2 + 7$. What is the end behavior of the graph?

In general, the graph of a polynomial function of degree n (where $n \geq 1$) has at most $n - 1$ turning points. The graph of a polynomial function of odd degree has an even number of turning points. The graph of a polynomial function of even degree has an odd number of turning points. This information, combined with end behavior, determines possible shapes that the graph of a polynomial function can have.

Hint

Compare the turning point relationships with the end behavior relationships on the previous page.



Problem 3 Graphing Cubic Functions

What is the graph of each cubic function? Describe the graph.

Plan

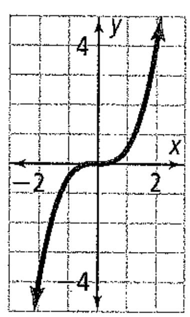
How can you graph a polynomial function? Make a table of values to help you sketch the graph near the origin. Use what you know about end behavior to sketch the graph moving out from the origin.

A $y = \frac{1}{2}x^3$

Step 1 Make a table of values.

Step 2 Plot the points and sketch the graph.

x	y
−2	−4
−1	−0.5
0	0
1	0.5
2	4



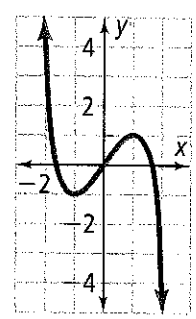
Step 3 The end behavior is down and up. There are no turning points.

B $y = 2x - x^3$

Step 1 Make a table of values.

Step 2 Plot the points and sketch the graph.

x	y
−2	4
−1	−1
0	0
1	1
2	−4



Step 3 The end behavior is up and down. There are two turning points.

Got It? 3. What is the graph of each cubic function? Describe the graph.

a. $y = -x^3 + 2x^2 - x - 2$ **b.** $y = x^3 - 1$

Suppose you are given a set of function outputs. You know that their corresponding inputs are an ordered set of x-values in which consecutive x-values differ by a constant. By analyzing the differences of consecutive y-values, it is possible to determine the least-degree polynomial function that could generate the data.

If the first differences are constant, the function is linear. If the second differences (but not the first) are constant, the function is quadratic. If the third differences (but not the second) are constant, the function is cubic, and so on.

Problem 4 Using Differences to Determine Degree

What is the degree of the polynomial function that generates the data shown at the right?

x	y
−3	−1
−2	−7
−1	−3
0	5
1	11
2	9
3	−7

Know
A set of polynomial function values

Need
Degree of the polynomial function

Plan
- Check first differences of y-values.
- Check second differences, third differences, and so on until they are constant.

y-value	1st difference	2nd difference	3rd difference
−1			
	−6		
−7		10	
	4		−6
−3		4	
	8		−6
5		−2	
	6		−6
11		−8	
	−2		−6
9		−14	
	−16		
−7			

The third differences are constant.

Think
How do you find the second differences?
Subtract the consecutive first differences.

The 3rd differences are constant, so the degree of the polynomial function is 3.

Got It? 4. a. What is the degree of the polynomial function that generates the data shown at the right?

b. Reasoning What is an example of a polynomial function whose fifth differences are constant but whose fourth differences are not constant?

x	y
−3	23
−2	−16
−1	−15
0	−10
1	−13
2	−12
3	29

Focus Question What is a polynomial function?

Answer A polynomial function $P(x)$ is a sum of monomial terms with the variable x classified by degree and number of terms. The algebraic form of a polynomial function and the behavior of its graph are related. Use one to find the other.

Lesson Check

Do you know HOW?

Classify each polynomial by degree and by number of terms.

1. $5x^3$
2. $6x^2 + 4x - 2$

Write each polynomial in standard form.

3. $7x + 3 + 5x^2$
4. $-3 + 9x$

Do you UNDERSTAND?

5. **Vocabulary** Describe the end behavior of the function $y = -2x^7 - 8x$.
6. **Reasoning** Can the graph of a polynomial function be a straight line? If so, give an example.
7. **Error Analysis** Your friend claims the graph of the function $y = 4x^3 + 4$ has only one turning point. Describe the error your friend made and give the correct number of turning points.

Practice and Problem-Solving Exercises

A Practice

Write each polynomial in standard form. Then classify it by degree and by number of terms. See Problem 1.

Guided Practice

To start, write the terms of the polynomial with their degrees in descending order.

8. $2m^2 - 3 + 7m$

$2m^2 + 7m - 3$

9. $7x + 3x + 5$
10. $5 - 3x$
11. $-x^3 + x^4 + x$
12. $-4p + 3p + 2p^2$
13. $5a^2 + 3a^3 + 1$
14. $-x^5$
15. $3 + 12x^4$
16. $7x^3 - 10x^3 + x^3$
17. $4x + 5x^2 + 8$

Determine the end behavior of the graph of each polynomial function. See Problem 2.

18. $y = -7x^3 + 8x^2 + x$
19. $y = -3x + 6x^2 - 1$
20. $y = 1 - 4x - 6x^3 - 15x^6$
21. $y = 8x^{11} - 2x^9 + 3x^6 + 4$
22. $y = -3 - 6x^5 - 9x^8$
23. $y = x^4 - 7x^2 + 3$