Name: Answer Key No. 0 Per: 1 Date: _____

Serafino · Algebra 2E

M T W R (F)

4A

Polynomial Functions

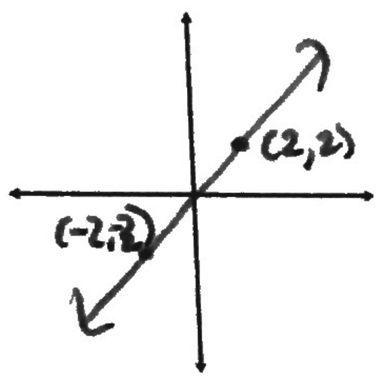
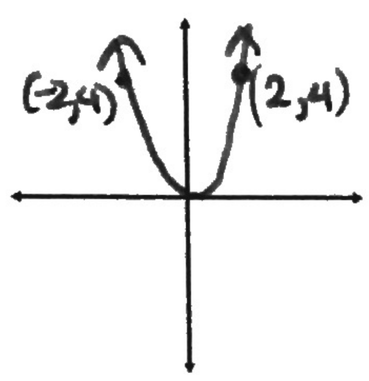
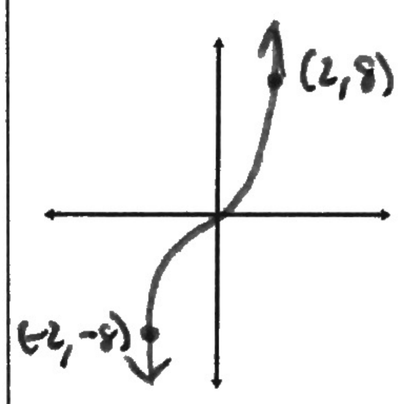
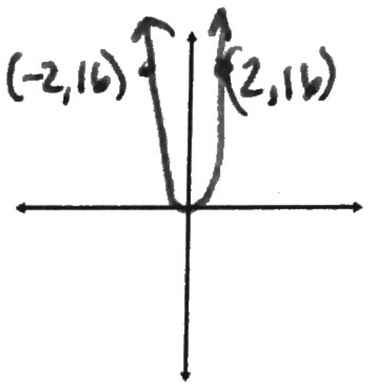
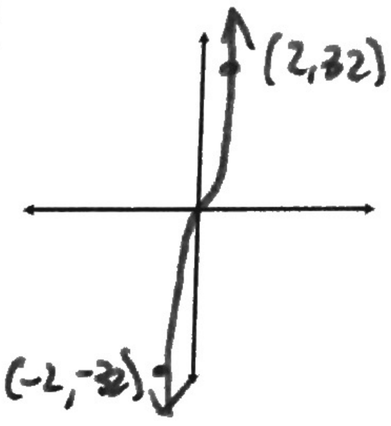
Intercept Form: Notes & Practice Packet

Intercept: $f(x) = a(x-r_1)(x-r_2)(x-r_3)(x-r_4)...$

Standard: $f(x) = ax^n + bx^{n-1} + cx^{n-2} - ... + k$

r is a root, n is a natural number, k is a constant

They've all been polynomials!

Family	**Linear**	**Quadratic**	**Cubic**	**Quartic**	**Quintic**
Parent	$f(x) = x$	$f(x) = x^2$	$f(x) = x^3$	$f(x) = x^4$	$f(x) = x^5$
Graph					
Shape at x-ints	cross	bounce	squiggle	bounce	squiggle
End Behavior	"down / up"	"up / up"	"down / up"	"up / up"	"down / up"
As....	$x \to -\infty$, $f(x) \to -\infty$ $x \to \infty$, $f(x) \to \infty$	$x \to -\infty$, $f(x) \to \infty$ $x \to \infty$, $f(x) \to \infty$	$x \to -\infty$, $f(x) \to -\infty$ $x \to \infty$, $f(x) \to \infty$	$x \to -\infty$, $f(x) \to \infty$ $x \to \infty$, $f(x) \to \infty$	$x \to -\infty$, $f(x) \to -\infty$ $x \to \infty$, $f(x) \to \infty$

So in summary...

If the **LEADING COEFFICIENT** of the polynomial is POSITIVE, the function will end up on right ↗

NEGATIVE, it will end down on right ↘

If the **DEGREE** of the polynomial is EVEN, the function will do same thing on left / right

Even: "up/up" or "down/down" ODD, the function will do diff. things on left right

If the **MULTIPLICITY OF A SOLUTION is:**

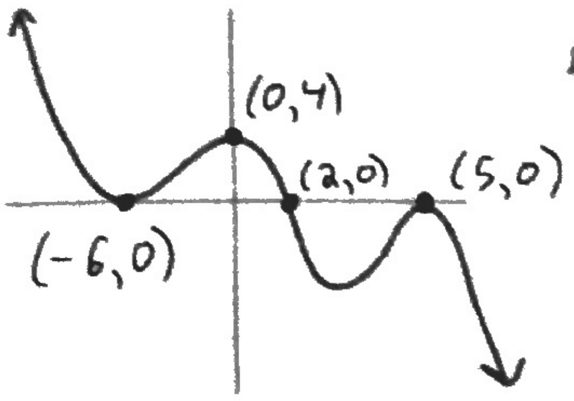
ONE: the function will cross

EVEN, the function will bounce

ODD ≥ 3, the function will squiggle

1. For each of the polynomial, identify the degree and the sign of the leading coefficient. Then write the equations in intercept form and find the average rate of change on the given interval.

a. Sign of LC: [−] Degree: [5]



$4 = a(0+6)^2(0-2)(0-5)^2$

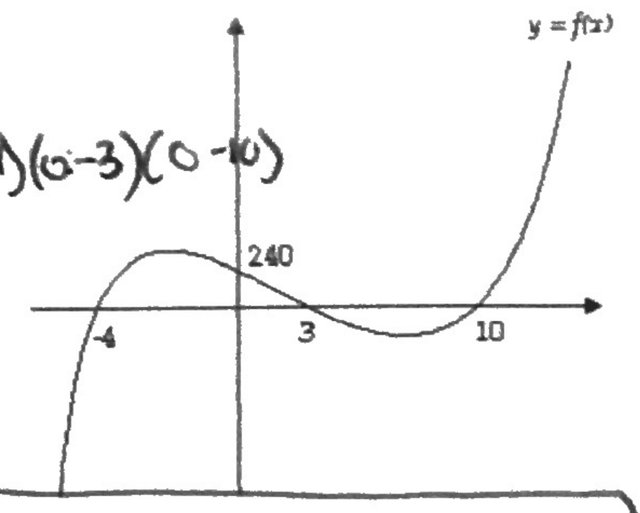
$4 = a(36)(-2)(25)$

Equation: $y = -\frac{1}{450}(x+6)^2(x-2)(x-5)^2$

Average ROC $3 < x < 5$ [.36]

$(3,-.72), (5,0)$ $\frac{.72}{2}$

b. Sign of LC: [+] Degree: [3]



$240 = a(0+4)(0-3)(0-10)$

$240 = 120a$

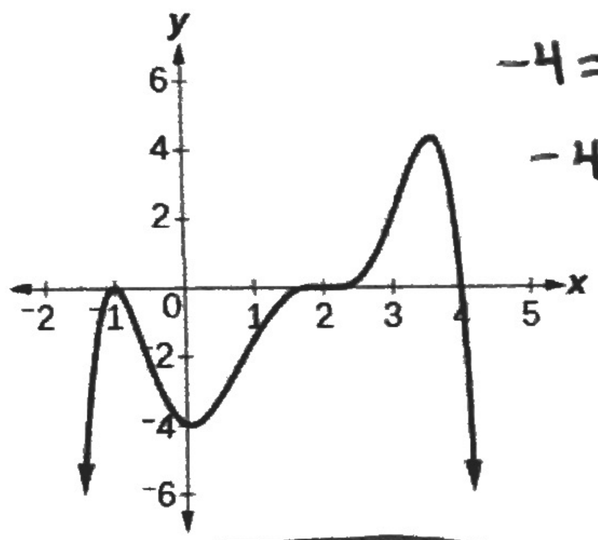
$a = 2$

Equation: $y = 2(x+4)(x-3)(x-10)$

Average ROC $6 < x < 10$ [60]

$(6,-240)(10,0)$

c Sign of LC: [−] Degree: [6]



$-4 = a(0+1)^2(0-2)^3(0-4)$

$-4 = a(32)$

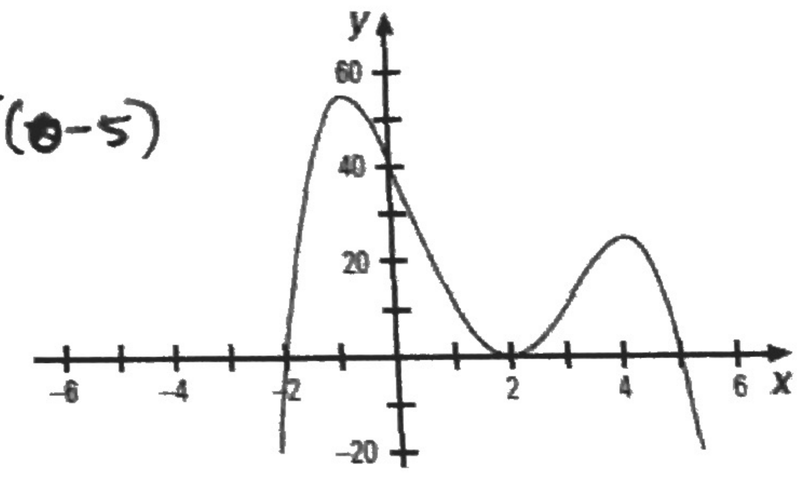
$a = -\frac{1}{8}$

Equation: $y = -\frac{1}{8}(x+1)^2(x-2)^3(x-4)$

Average ROC $3 < x < 5$ [−61.75]

$(3,2)$ $(5,-121.5)$

d. Sign of LC: [−] Degree: [4]



$40 = a(0+2)^1(0-2)^2(0-5)$

$40 = a(2)(4)(5)$

$a = -1$

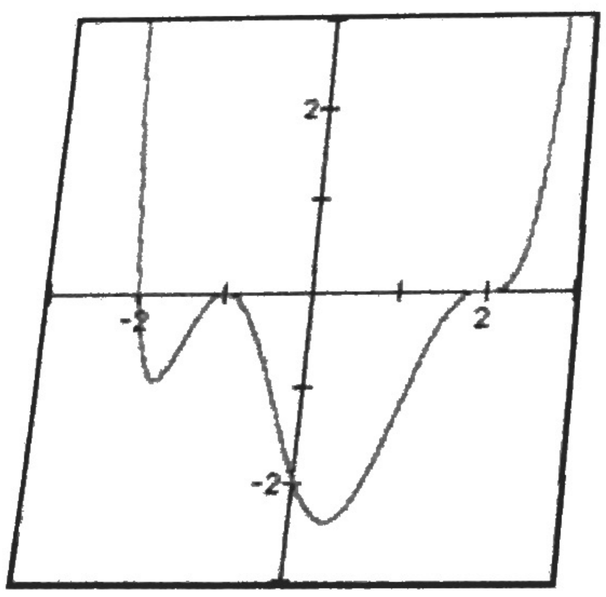
Equation: $y = -(x+2)(x-2)^2(x-5)$

Average ROC $6 < x < 10$ [−928]

$(6,-128$

$(10,-384$

e Sign of LC: [+] Degree: [6]



$-2 = a(2)(1)(-8)$

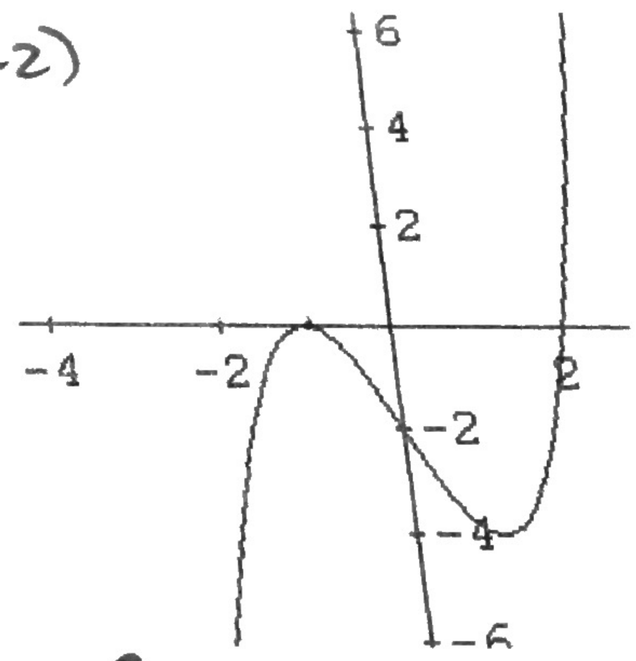
$a = 8$

Equation: $y = 8(x+2)(x+1)^2(x-2)^3$

Average ROC $-1 < x < 3$ [160]

$(-1,0)(3,640)$ $\frac{640}{4}$

f. Sign of LC: [+] Degree: [3]



$-2 = a(0+1)(0-2)$

$-2 = -2a$

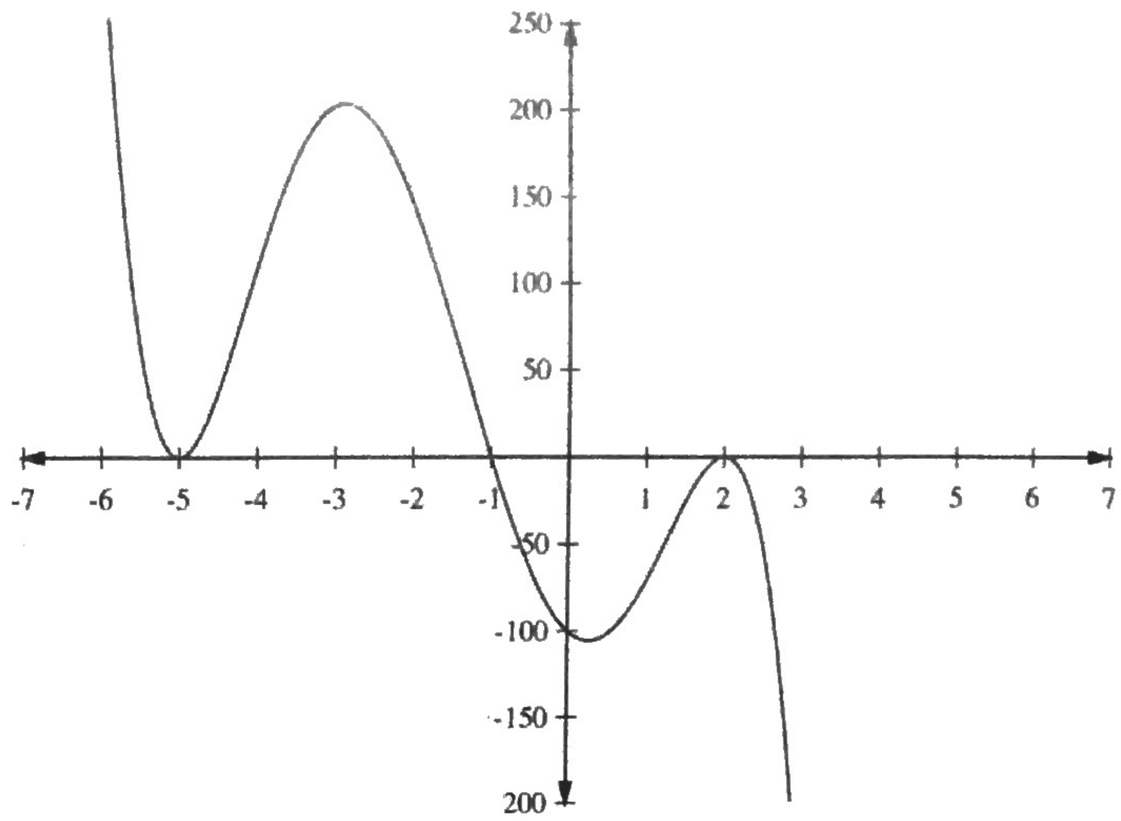
$a = 1$

Equation: $y = (x+1)^2(x-2)$

Average ROC $-2 < x < 2$ [0]

$(-2,0)$ $(2,0)$

2. **Analyze the following graph:**



a. State the official end behavior: (As x → ..)

As $x \to -\infty$, $f(x) \to \infty$
$x \to \infty$, $f(x) \to -\infty$

b. State the equation of this function, in intercept form:

$-100 = a(25)(1)(4)$

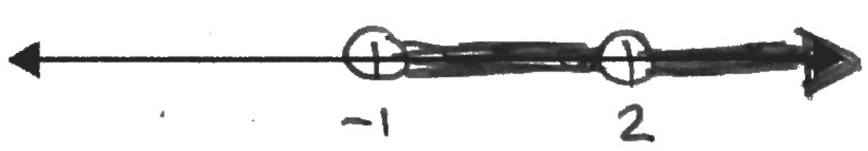
$f(x) = -(x+5)^2(x+1)(x-2)^2$

c. When looking at the graph, Ernie assumes $f(-3) = 200$
Is he right? Is not, how much is he off by?

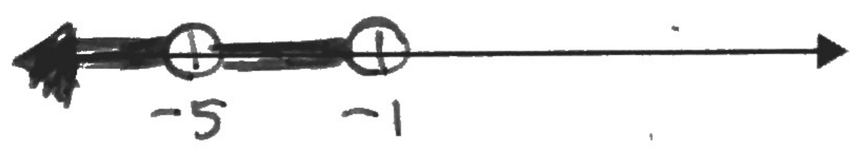
He is right! $f(-3) = 200$

d. For what intervals is/are the following true? State in inequality/interval notation and graph it:

f(x) < 0 $-1 < x < 2$ or $x > 2$
$x \in (-1, 2) \cup (2, \infty)$

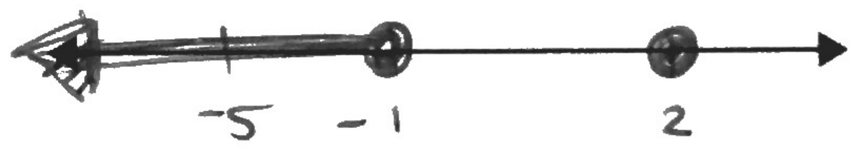


f(x) is > 0 $x < -5$ or $-5 < x < -1$
$x \in (-\infty, -5) \cup (-5, -1)$



f(x) is ≤ 0 $x = -5$ or $x \geq -1$
$x \in \{-5\} \cup [-1, \infty)$

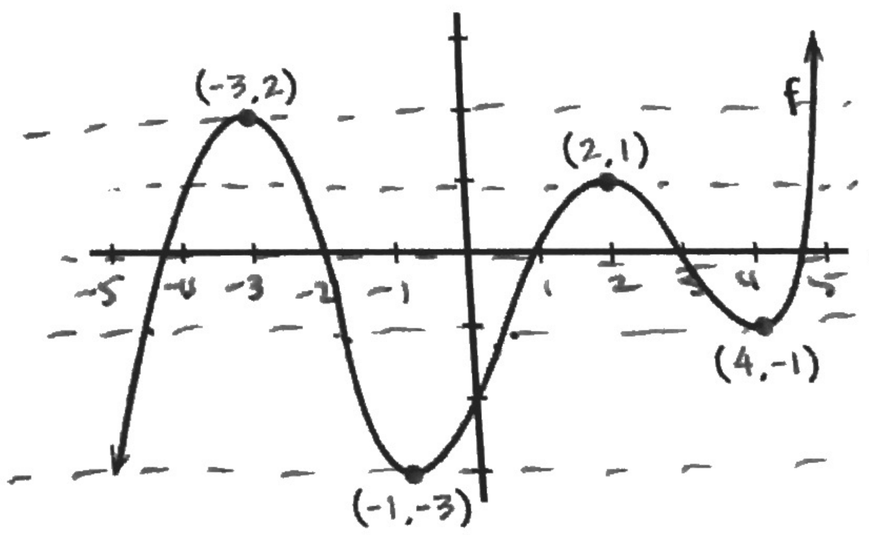
f(x) is ≥ 0 $x \leq -1$ or $x = 2$
$x \in (-\infty, -1] \cup \{2\}$



3. **Analyze the following function, f(x) =**

a. Degree: 5 Last term: −2

b. How MANY real or imaginary solutions exist for the following (remember what the total must be). State multiplicity (in a circle)
↑5



a. f(x) = 0
5 real: −4.3 (1), −2 (1), 0.9 (1), 2.9 (1), 4.5 (1)

b. f(x) = 2
3 real: −3 (2), ≈5 (1) and 2 complex

c. f(x) = −1
5 real: ≈−4.5 (1), −1.9 (1), 0.4 (1), 4 (2)

d. f(x) = 1
5 real: −3.9 (1), −2.2 (1), 2 (2), 4.6 (1)

e. f(x) = −3
3 real: −5 (1), −1 (2) and 2 complex

f. f(x) = 10
1 real (mult 1) 4 imaginary

4. Evaluate the given polynomials using sythetic substitution. Write the resulting points.

a. $y = -x^4 - 4x^3 + 3x^2 + 4x - 2$ find f(−5), f(−2) and f(1)

−5	−1	−4	3	4	−2
		5	−5	10	−70
	−1	1	−2	14	−72

−2	−1	−4	3	4	−2
		2	4	−14	20
	−1	−2	7	−10	18

1	−1	−4	3	4	−2
		−1	−5	−2	2
	−1	−5	−2	2	0

b. $y = 2x^3 - 5x^2 + 5$ find f(−2) and f(3)

−2	2	−5	0	5
		−4	18	−36
	2	−9	18	−31

3	2	−5	0	5
		6	3	9
	2	1	3	14

(−5, −72) (−2, 18) (1, 0)

(−2, −3)
(3, 14)

c. $y = -4x^3 + 2x^2 + 4x$ find f(−2), f(½), and f(2)

−2	−4	2	4	0
		8	−20	32
	−4	10	−16	32

½	−4	2	4	0
		−2	0	2
	−4	0	4	2

2	−4	2	4	0
		−8	−12	−16
	−4	−6	−8	−16

(−2, 32)
(½, 2)
(2, −16)

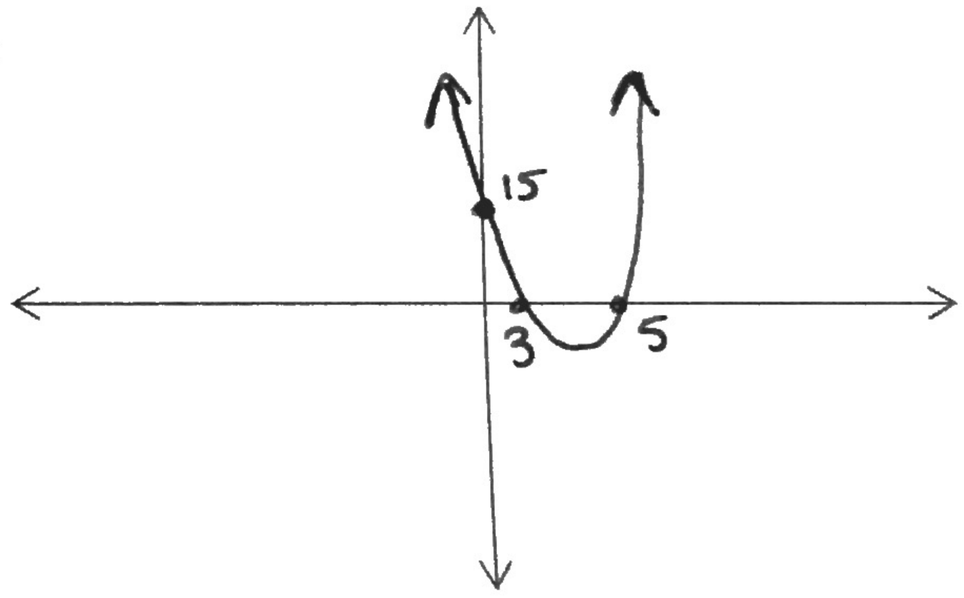
Sketching polynomial functions in intercept form: (EB = end behavior)

1. $f(x) = (x - 3)(x - 5)$ EB: "up/up"

Leading Term: x^2 Last Term: 15

x–int: (3, 0) (5, 0)

y–int: (0, 15)

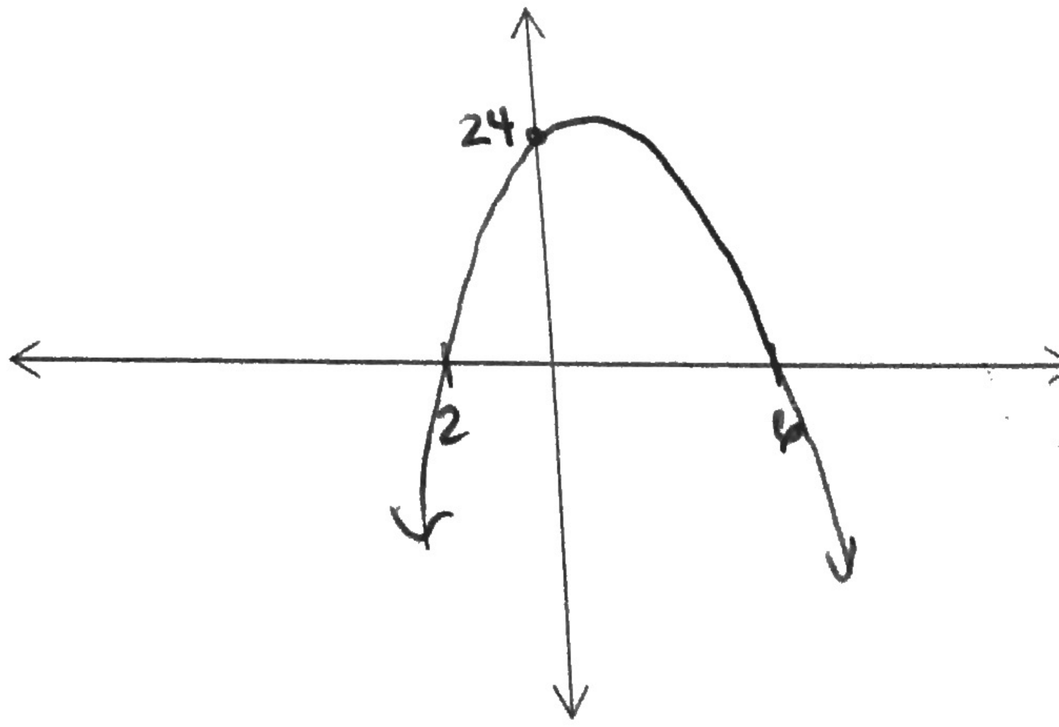


2. $f(x) = -2(x + 2)(x - 6)$ EB: "d/d"

Leading Term: $-2x^2$ Last Term: 24

x–int: (−2, 0) (6, 0)

y–int: (0, 24)

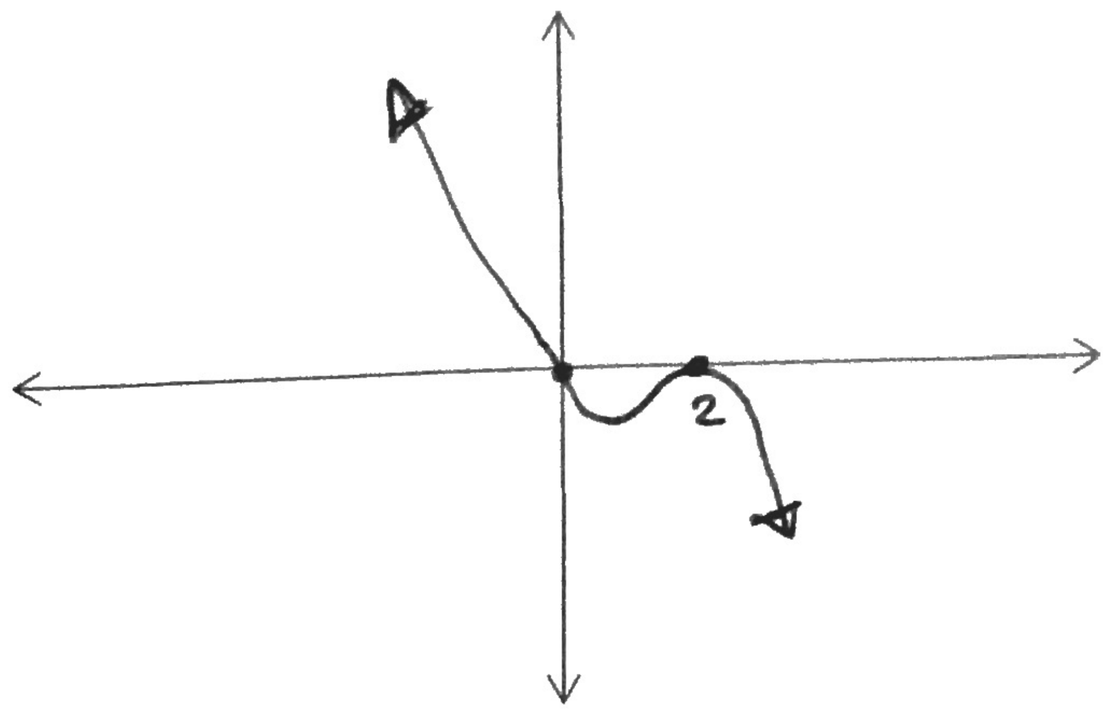


3. $f(x) = -3x(x-2)(x-2)$ EB: u/d

Leading Term: $-3x^3$ Last Term: $-12x$

x-int: (1) (0,0) (2) (2,0)

y-int: (0,0)

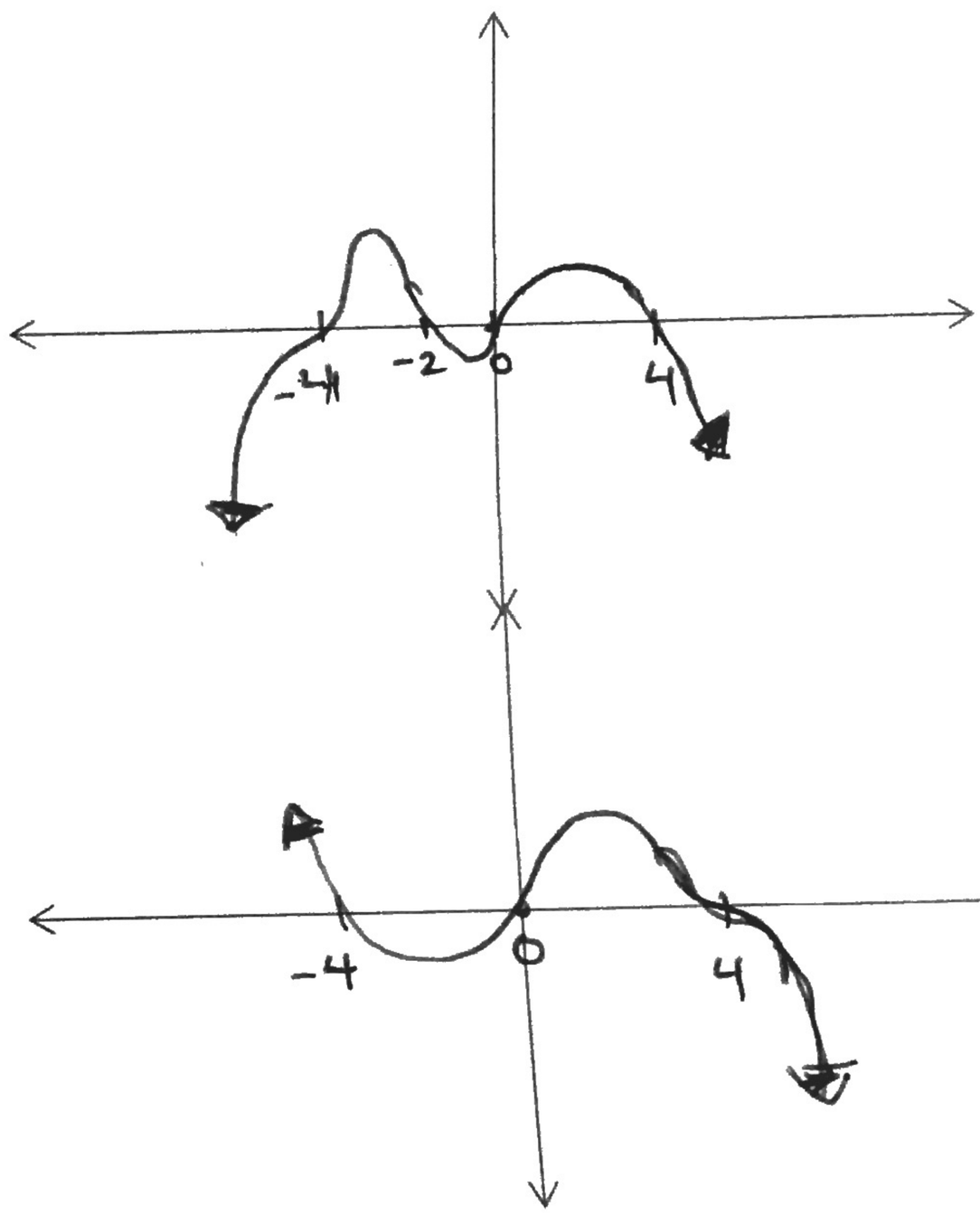


4. $f(x) = -\frac{1}{2}x(x+4)^3(x+2)(x-4)$ EB: d/d

Leading Term: $-\frac{1}{2}x^6$ Last Term: $256x$

x-int: (1) (0,0) (3) (-4,0) (1) (-2,0) (1) (4,0)

y-int: (0,0)



5. $f(x) = -x(x+4)(x-4)^3$ EB: u/d

Leading Term: $-5x^5$ Last Term: $256x$

x-int: (1) (0,0) (1) (-4,0) (3) (4,0)

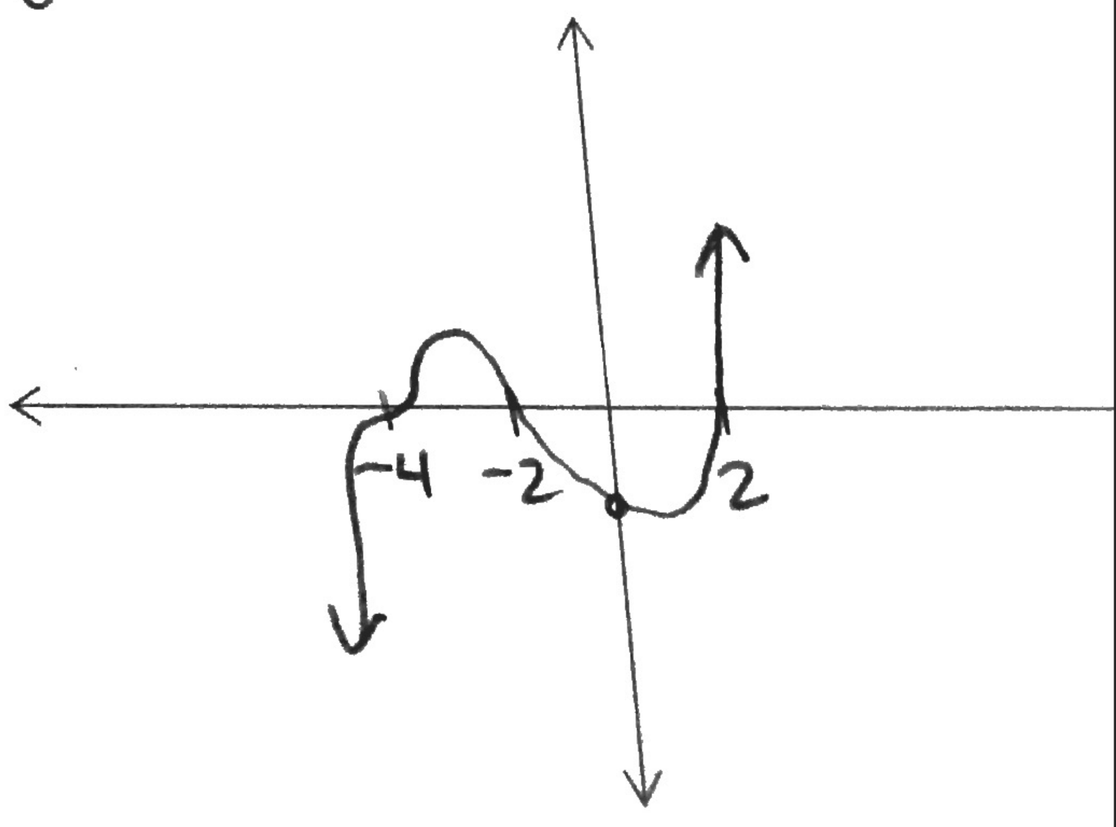
y-int: (0,0)

6. $f(x) = \frac{1}{10}(x+4)^3(x-2)(x+2)$ EB d/u

Leading Term: $\frac{1}{10}x^5$ Last Term: -25.6

x-int: (3) (-4,0) (1) (2,0) (1) (-2,0)

y-int: (0, -25.6)

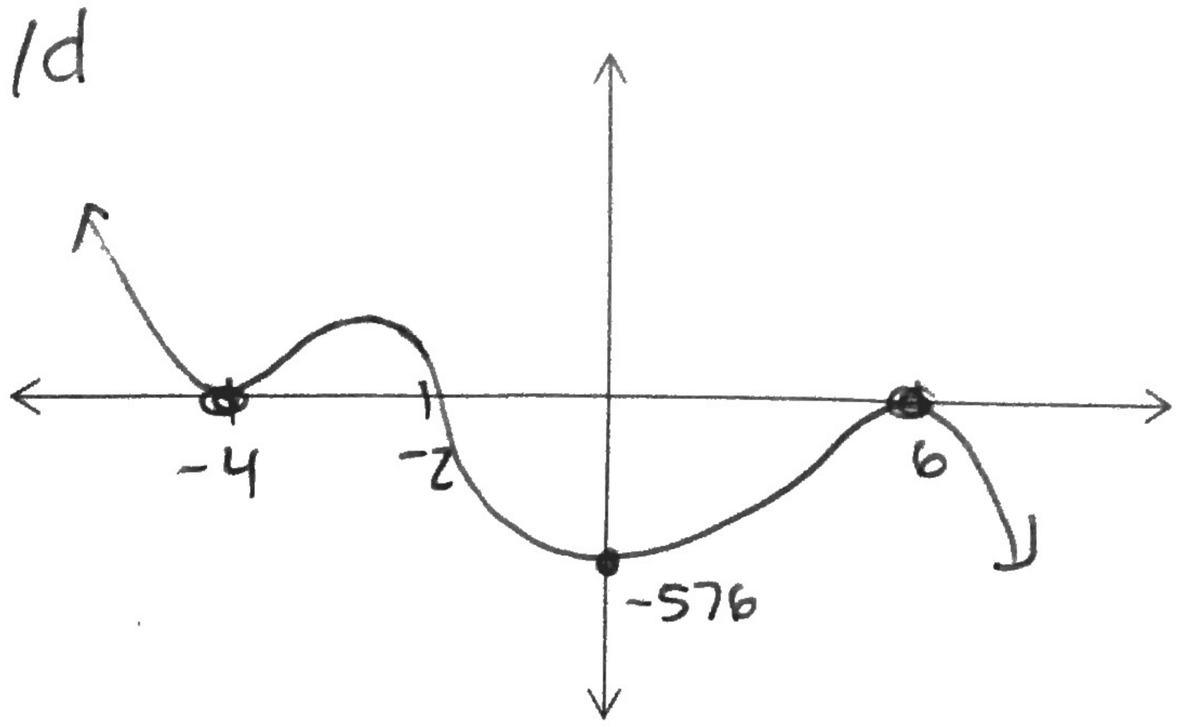


7. $f(x) = -\frac{1}{2}(x+4)^2(x+2)(x-6)^2$ EB: ∪/d

Leading Term: $-\frac{1}{2}x^5$ Last Term: -576

x–int: (-4,0) (-2,0) (6,0) (2) (1) (2)

y–int: (0,-576)

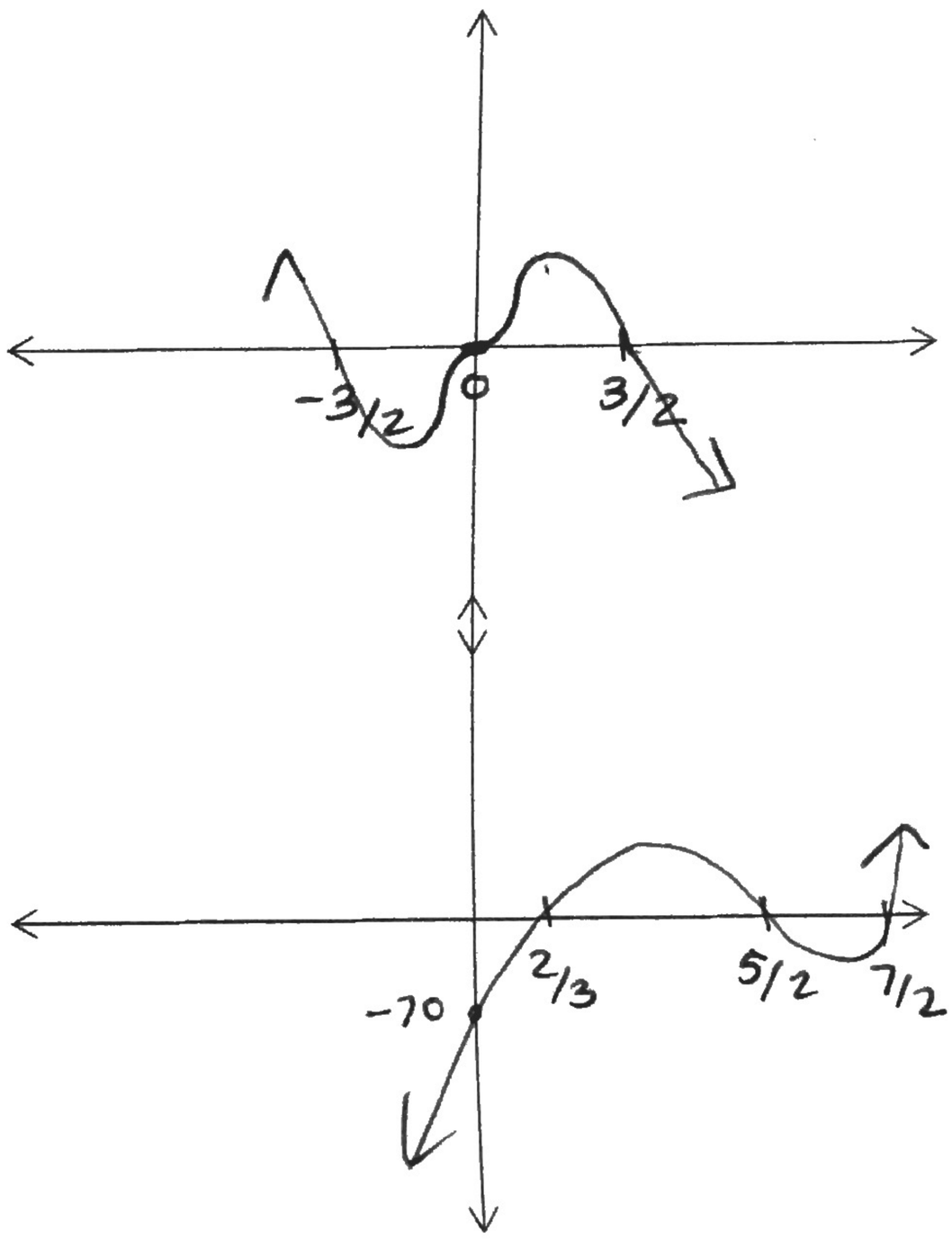


8. $f(x) = -4x^3(2x+3)(2x-3)$ EB ∪/d

Leading Term: $-16x^5$ Last Term: 36

x–int: (-3/2,0) (0,0) (3/2,0) (3)

y–int: (0,0)



9. $f(x) = (3x-2)(2x-7)(2x-5)$ EB: d/∪

Leading Term: $12x^3$ Last Term: -70

x–int: (2/3,0) (7/2,0) (5/2,0) .66 3.5 2.5

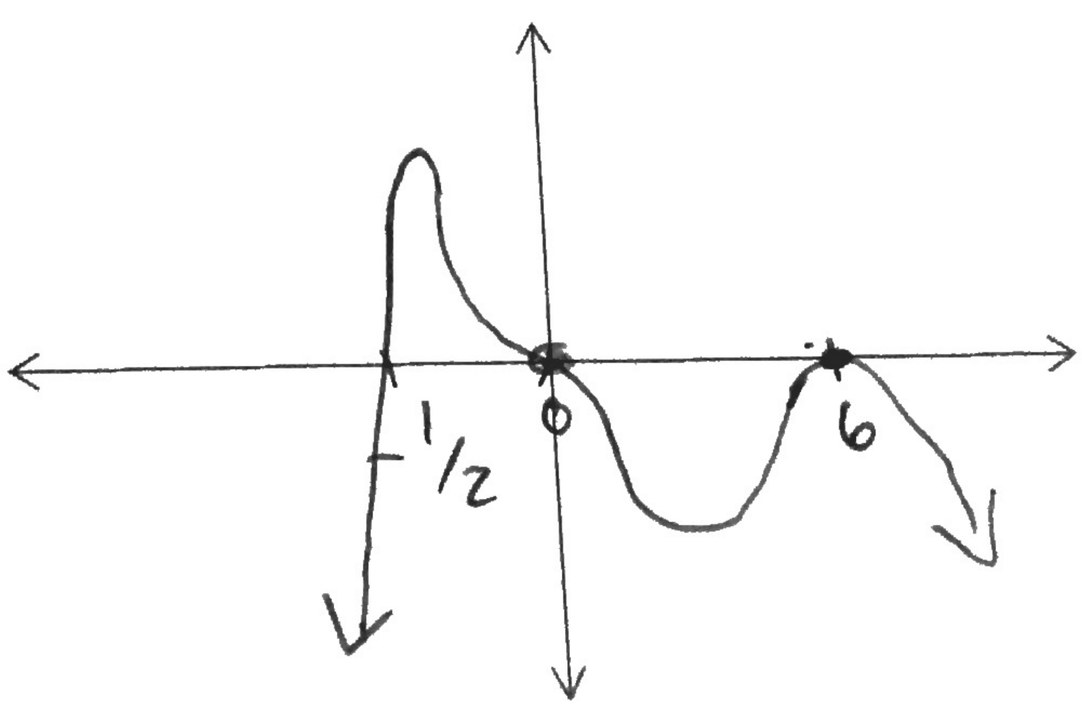
y–int: (0,-70)

10. $f(x) = -x^3(x-6)^4(2x+1)$ EB: d/d

Leading Term: $-2x^8$ Last Term: $-1296x$

x–int: (0,0) (6,0) (-1/2,0) (3) (4)

y–int: (0,0)



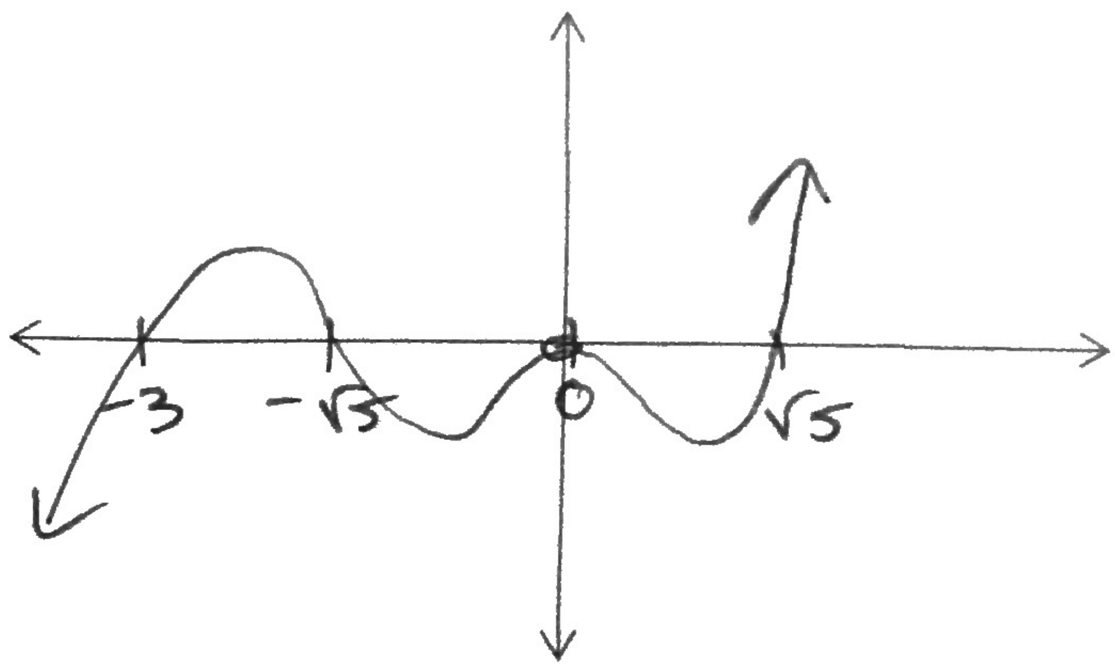
The following intercepts will be IRRATIONAL. That's okay. Proceed as usual.

11. $f(x) = x^2(x^2 - 5)(x + 3)$

Leading Term: x^5 Last Term: $-15x^2$

x-int: (2) $(0,0)(-\sqrt{5},0)(\sqrt{5},0)(-3,0)$

y-int: $(0,0)$

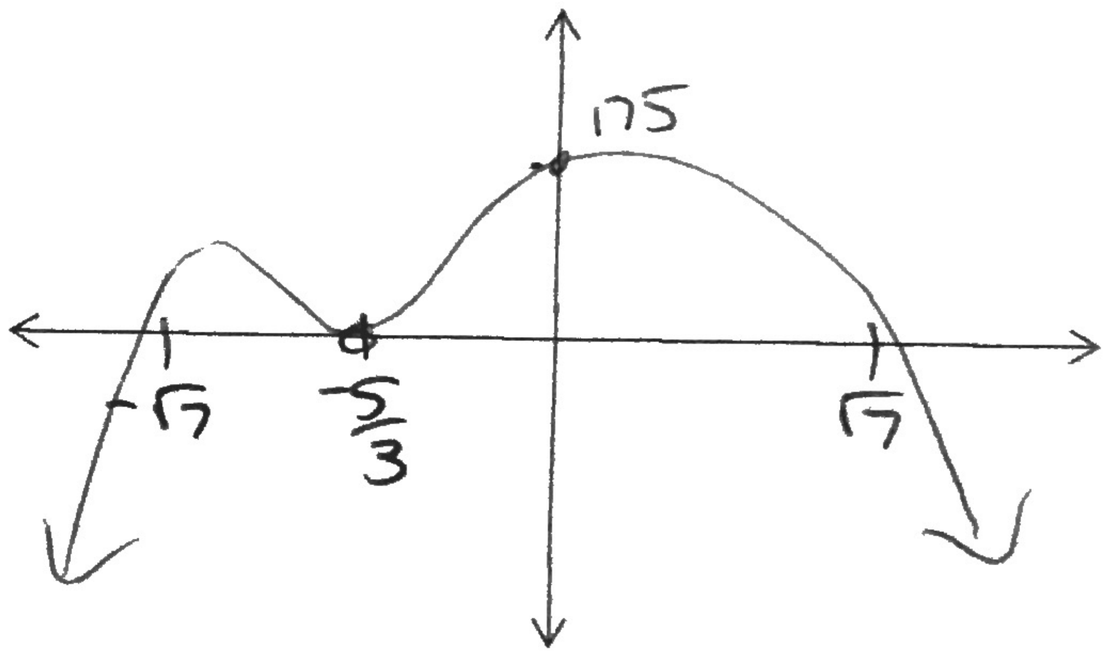


12. $f(x) = -(x^2 - 7)(3x + 5)^2$

Leading Term: $-9x^4$ Last Term: 175

x-int: (2) $(\sqrt{7},0)(-\sqrt{7},0)(-5/3,0)$

y-int: $(0,175)$

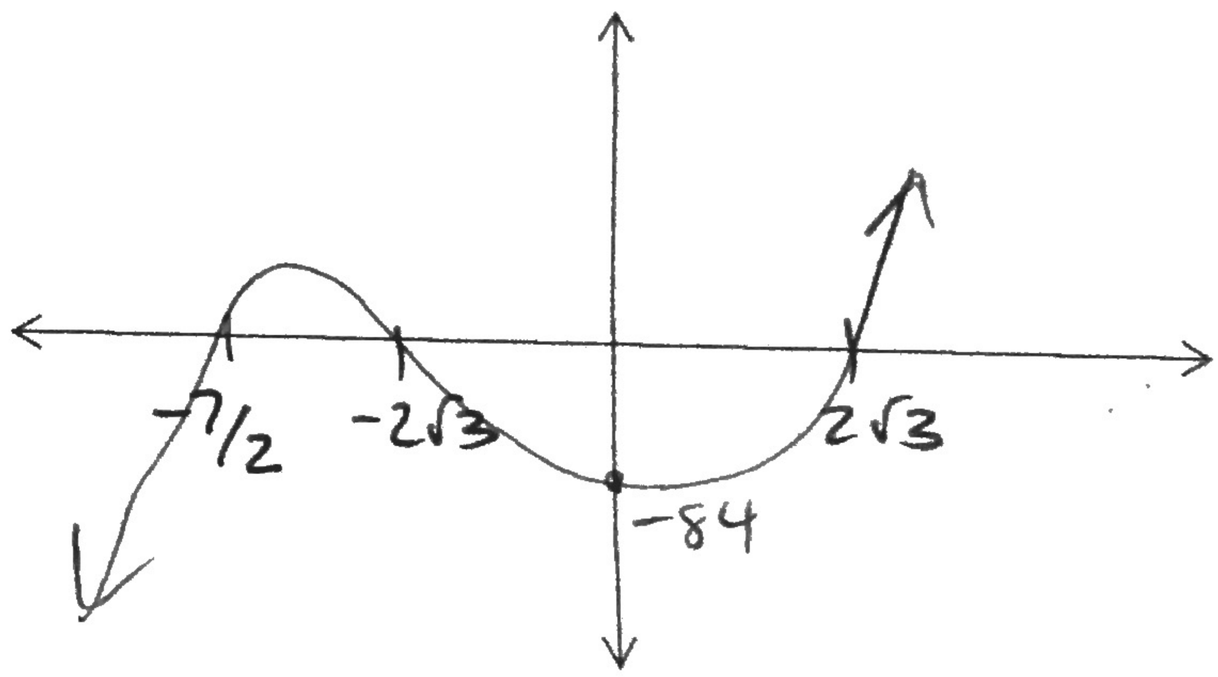


13. $f(x) = (x^2 - 12)(2x + 7)$

Leading Term: $2x^3$ Last Term: -84

x-int: $(-2\sqrt{3},0)(2\sqrt{3},0)(-7/2,0)$

y-int: $(0,-84)$

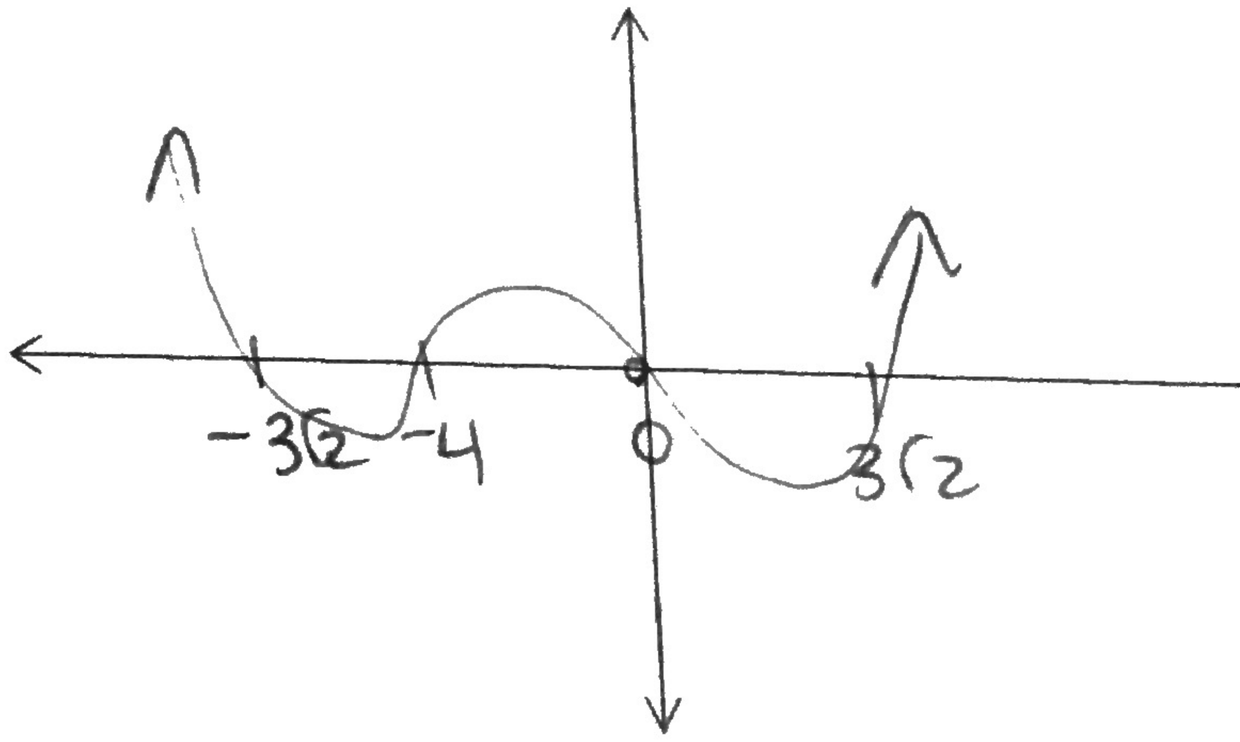


14. $f(x) = x(x^2 - 18)(x + 4)$

Leading Term: x^4 Last Term: $-72x$

x-int: $(0,0)(-3\sqrt{2},0)(3\sqrt{2},0)(-4,0)$

y-int: $(0,0)$



I'll get better w/ giving you room!

YOU will NOT have enough room to do your work here. Get out a separate sheet of paper.

15. Write a polynomial function in intercept form with the given zeros. Then, multiply to get standard form.

a. $x = -2, 1, 4$ $(x+2)(x-1)(x-4)$ $(x+2)(x^2-5x+4)$

$y = x^3 - 3x^2 - 6x + 8$

$x^3 - 5x^2 + 4x$
$+ 2x^2 - 10x + 8$

b. $x = 3, 3, 0$ $x(x+3)(x+3)$ $x(x^2+6x+9)$

$y = x^3 + 6x^2 + 9x$

c. $3/4, -1/8, 0$ $x(4x-3)(8x+1)$

$32x^2 - 24x + 4x - 3$

$y = 32x^3 - 20x^2 - 3x$

d. $x = 1/7, -2/3, 1, 0, 0$ $x^2(x-1)(3x+2)(7x-1)$ $(x-1)[21x^2+11x-2]$

$y = 21x^5 - 10x^4 - 13x^3 + 2x^2$

$21x^3 + 11x^2 - 2x$
$- 21x^2 - 11x + 2$

e. $x = \sqrt{3}, -\sqrt{3}, -5$ $(x+5)(x^2-3)$

$y = x^3 + 5x^2 - 3x - 15$

f. $x = \pm\sqrt{2}, \pm\sqrt{5}$ $(x^2-2)(x^2-5)$

$y = x^4 - 7x^2 + 10$

g. $x = \pm 2i, \pm 4$ $(x^2+4)(x^2-16)$

$y = x^4 - 12x^2 - 64$

h. $x = \pm 5, \pm i\sqrt{5}$ $(x^2-25)(x^2+5)$

$y = x^4 - 20x^2 - 125$

16. Find the error: On your homework, you were asked to write a standard form polynomial function from the given zeros $x = 3, 0, -9, 1$. You write $y = x^4 + 5x^3 - 33x^2 + 27x$. Your friend writes down $y = x^3 + 5x^2 - 33x + 27$. Who is correct? What mistake was made?

Me! Mine ends in a 27x which means $f(0)=0$, and that was a condition!

17. Write three (3) different polynomials in intercept form that have the following properties:

a. *Bounces at 0 and 4.*

$y = x^2(x-4)^2$

$y = x^4(x-4)^2$

$y = -2x^4(x-4)^4$

b. *Is "down / up" ; x- int: -4, 3, and 9*

$y = (x+4)(x-3)(x-9)$

$y = (x+4)^3(x-3)(x-9)$

$y = (x+4)(x-3)(x-9)^3$

c. Is "up / up"; x–int: -2, 2 and 5.

$y = (x-2)(x+2)(x-5)^2$

$y = (x-2)(x+2)^2(x-5)$

$y = (x-2)^2(x+2)(x-5)$

Answers may vary

d. Is "down / down" x–int: -10, 0, and 5

$y = -x^2(x+10)(x-5)$

$y = -x(x+10)^2(x-5)$

$y = -x(x+10)(x-5)^2$