4B :2

Do: pg 313 # 10-31

Do pg 316 # 7-15

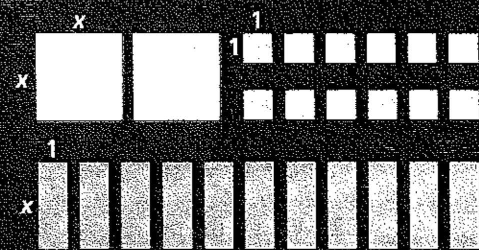
5-3 PART 1 Solving Polynomial Equations

Objective To solve polynomial equations by factoring

I count 2 pieces with area x^2, 11 with area x, and 12 with area 1. The rectangle would have the same total area.

SOLVE IT! Getting Ready!

Can you arrange all of these pieces to make a rectangle with no pieces overlapping and no gaps? If you can, make a sketch. If you cannot, explain why.



Lesson Vocabulary
- sum of cubes
- difference of cubes

Factoring a polynomial like $ax^2 + bx + c$ can help you solve a polynomial equation like $ax^2 + bx + c = 0$.

Focus Question How can you use factoring to solve a polynomial equation?

To solve a polynomial equation by factoring:

1. Write the equation in the form $P(x) = 0$ for some polynomial function P.
2. Factor $P(x)$. Use the Zero-Product Property to find the roots.

Problem 1 Solving Polynomial Equations Using Factors

What are the real and imaginary solutions of each polynomial equation?

Plan

What does it mean if x is a common factor of the terms in $P(x)$? You can write $P(x)$ as $xQ(x)$, and 0 will be a solution.

A $2x^3 - 5x^2 = 3x$

Rewrite in the form $P(x) = 0$.	$2x^3 - 5x^2 - 3x = 0$
Factor out the GCF, x.	$x(2x^2 - 5x - 3) = 0$
Factor $2x^2 - 5x - 3$.	$x(2x + 1)(x - 3) = 0$
Use the Zero-Product Property.	$x = 0$ or $2x + 1 = 0$ or $x - 3 = 0$
Solve each equation for x.	$x = 0$ $\quad x = -\frac{1}{2}$ $\quad x = 3$

The solutions are 0, $-\frac{1}{2}$, and 3.

B $3x^4 + 12x^2 = 6x^3$

Rewrite in the form $P(x) = 0$.	$3x^4 - 6x^3 + 12x^2 = 0$
Multiply each side by $\frac{1}{3}$ to simplify.	$x^4 - 2x^3 + 4x^2 = 0$
Factor out the GCF, x^2.	$x^2(x^2 - 2x + 4) = 0$
Use the Zero-Product Property.	$x^2 = 0$ or $x^2 - 2x + 4 = 0$

Solve each equation.

Find square roots to solve $x^2 = 0$.

Use the Quadratic Formula to solve $x^2 - 2x + 4 = 0$. Substitute $a = 1, b = -2, c = 4$.

$x = 0$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(4)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{-12}}{2} = \frac{2 \pm 2i\sqrt{3}}{2} = 1 \pm i\sqrt{3}$$

The solutions are $0, 1 + i\sqrt{3}$, and $1 - i\sqrt{3}$.

Think

How will the solution be similar to the solution of the equation in part (a)?
Both equations have 0 as a solution, but here it will have a multiplicity of 2.

Got It? 1. What are the real or imaginary solutions of each equation?

a. $(x^2 - 1)(x^2 + 4) = 0$ **b.** $x^5 + 4x^3 = 5x^4 - 2x^3$

Hint

You used many of these factoring techniques when you solved quadratic equations in Chapter 4.

take note

Concept Summary Polynomial Factoring Techniques

Techniques	Examples
Factoring out the GCF Factor out the greatest common factor of all the terms.	$15x^4 - 20x^3 + 35x^2$ $= 5x^2(3x^2 - 4x + 7)$
Quadratic Trinomials For $ax^2 + bx + c$, find factors with product ac and sum b.	$6x^2 + 11x - 10$ $= (3x - 2)(2x + 5)$
Perfect Square Trinomials $a^2 + 2ab + b^2 = (a + b)^2$ $a^2 - 2ab + b^2 = (a - b)^2$	$x^2 + 10x + 25 = (x + 5)^2$ $x^2 - 10x + 25 = (x - 5)^2$
Difference of Squares $a^2 - b^2 = (a + b)(a - b)$	$4x^2 - 15 = (2x + \sqrt{15})(2x - \sqrt{15})$
Factoring by Grouping $ax + ay + bx + by$ $= a(x + y) + b(x + y)$ $= (a + b)(x + y)$	$x^3 + 2x^2 - 3x - 6$ $= x^2(x + 2) + (-3)(x + 2)$ $= (x^2 - 3)(x + 2)$
Sum or Difference of Cubes $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$	$8x^3 + 1 = (2x + 1)(4x^2 - 2x + 1)$ $8x^3 - 1 = (2x - 1)(4x^2 + 2x + 1)$

The sum and difference of cubes are useful factoring techniques.

Here's Why It Works Factoring $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$:

Add the two pairs of additive inverses as shown.	$a^3 + b^3 = a^3 + a^2b - a^2b - ab^2 + ab^2 + b^3$
Factor out a^2, $-ab$, and b^2.	$= a^2(a + b) - ab(a + b) + b^2(a + b)$
Factor out $(a + b)$.	$= (a + b)(a^2 - ab + b^2)$

For $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$, you can follow steps similar to those above, or you can factor $a^3 - b^3$ as the sum of cubes $a^3 + (-b)^3$.

Problem 2 Solving Polynomial Equations by Factoring

What are the real or imaginary solutions of each polynomial equation?

Think

How can you write the polynomial in quadratic form?
Write in terms of x^2: $(x^2)^2 - 3(x^2) - 4 = 0$, which shows the factorable quadratic form $a^2 - 3a - 4 = 0$.

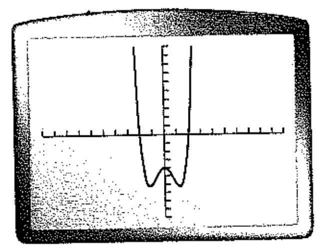
A $x^4 - 3x^2 = 4$

Rewrite in the form $P(x) = 0$.	$x^4 - 3x^2 - 4 = 0$
Let $a = x^2$.	$a^2 - 3a - 4 = 0$
Factor.	$(a - 4)(a + 1) = 0$
Replace a with x^2.	$(x^2 - 4)(x^2 + 1) = 0$
Factor $x^2 - 4$ as a difference of squares.	$(x + 2)(x - 2)(x^2 + 1) = 0$

It follows from the Zero-Product Property that $x = -2$, $x = 2$, or $x^2 = -1$. Solving $x^2 = -1$ yields two imaginary roots: $x = i$ or $x = -i$.

Check Graph the related function $y = x^4 - 3x^2 - 4$.

The graph shows real zeros at $x = -2$ and $x = 2$.
It also shows three turning points.



B $x^3 = 1$

Rewrite in the form $P(x) = 0$.	$x^3 - 1 = 0$
Factor the difference of cubes.	$(x - 1)(x^2 + x + 1) = 0$

It follows from the Zero-Product Property that $x = 1$ or $x^2 + x + 1 = 0$.
Use the Quadratic Formula to solve $x^2 + x + 1 = 0$.

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)} = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$$

The three solutions of $x^3 = 1$ are 1, $-\frac{1}{2} + i\frac{\sqrt{3}}{2}$, and $-\frac{1}{2} - i\frac{\sqrt{3}}{2}$.

Hint
For part (c), rewrite the equation in the form $P(x) = 0$ and let $y = x^2$.

Got It? 2. What are the real or imaginary solutions of each polynomial equation?

a. $x^4 = 16$ **b.** $x^3 - 8x = 2x^2$ **c.** $x^4 - 10 = 3x^2$

Focus Question How can you use factoring to solve a polynomial equation?
Answer Write the equation in the form $P(x) = 0$, where $P(x)$ is a polynomial function. Find roots using factoring techniques, the Zero-Product Property, and the Quadratic Formula.

Lesson Check

Do you know HOW?

Factor each polynomial.

1. $x^2 - 3x - 18$
2. $x^3 - 27$
3. $x^3 + 3x^2 + 4x + 12$
4. $x^4 - 2x^2 - 8$

Solve each equation by factoring.

5. $2x^2 + 7x - 4 = 0$
6. $2x^3 + 2x^2 - 4x = 0$

Do you UNDERSTAND?

7. **Vocabulary** Identify each expression as a sum of cubes, difference of cubes, or difference of squares.
   a. $x^2 - 64$
   b. $x^3 + 8$
   c. $x^3 - 125$
   d. $x^2 - 81$
8. **Reasoning** Show two different ways to find the real roots of the polynomial equation $0 = x^6 - x^2$. Show your steps.

Practice and Problem-Solving Exercises

A Practice Find the real or imaginary solutions of each equation by factoring.

See Problems 1 and 2.

9. $x^3 + 64 = 0$

Guided Practice To start, write $x^3 + 64$ as a sum of cubes and factor.

$$x^3 + 4^3 = 0$$
$$(x + 4)(x^2 - 4x + 16) = 0$$

10. $x^3 - 1000 = 0$
11. $125x^3 - 27 = 0$
12. $64x^3 - 1 = 0$
13. $x^3 + 2x^2 + 5x + 10 = 0$
14. $6x^2 + 13x - 5 = 0$
15. $0 = x^3 - 27$
16. $0 = x^3 - 64$
17. $8x^3 = 1$
18. $64x^3 = -8$
19. $x^3 = 8x - 2x^2$
20. $x^4 - 10x^2 = -9$
21. $x^4 - 8x^2 = -16$
22. $x^4 - 12x^2 = 64$
23. $x^4 + 7x^2 = 18$
24. $x^4 + 4x^2 = 12$

B Apply Solve each equation.

25. $125x^3 + 216 = 0$
26. $81x^3 - 192 = 0$
27. $x^4 - 64 = 0$
28. $27 = -x^4 - 12x^2$
29. $x^5 - 5x^3 + 4x = 0$
30. $5x^3 = 5x^2 + 12x$
31. What are the solutions of $2x^3 - 5x^2 = 12x$?
   (A) $-4, -\frac{3}{2}$, and 0
   (B) -4, 0, and $\frac{3}{2}$
   (C) $-\frac{3}{2}$, 0, and 4
   (D) 0, $\frac{3}{2}$, and 4
32. **Writing** Show how you can rewrite $\frac{m^3}{n^6} + \frac{1}{8}$ as a sum of two cubes.

Lesson Check

Do you know HOW?

Find the real solutions of each equation using a graphing calculator.

1. $x^3 + 13x = 10x^2$
2. $x^3 - 6x^2 + 6x = 0$
3. $12x^3 = 60x^2 + 75x$

Do you UNDERSTAND?

4. **Reasoning** Which method of solving polynomial equations does not identify the imaginary roots? Explain.
5. **Error Analysis** Your friend solved the equation $2x^3 + 6 = 2 - x$ by graphing $y_1 = 2x^3 + 6$ and $y_2 = 2 - x$ and got $x = 3.13$ for the solution. What error did she make?

Practice and Problem-Solving Exercises

A Practice

Find the real solutions of each equation by graphing. See Problem 3.

Guided Practice

6. $x^3 - 4x^2 - 7x = -10$

To start, rewrite the equation with one side equal to zero.

$x^3 - 4x^2 - 7x + 10 = 0$

7. $3x^3 - 6x^2 - 9x = 0$
8. $4x^3 - 8x^2 + 4x = 0$
9. $6x^2 = 48x$
10. $x^3 + 3x^2 + 2x = 0$
11. $2x^3 + 5x^2 = 7x$
12. $4x^3 = 4x^2 + 3x$
13. $2x^4 - 5x^3 - 3x^2 = 0$
14. $x^2 - 8x + 7 = 0$
15. $x^3 - x^2 - 16x = 20$

Graphing Calculator **Write an equation to model each situation. Then solve each equation by graphing.** See Problem 4.

16. The Johnson twins were born two years after their older sister. This year, the product of the three siblings' ages is exactly 4558 more than the sum of their ages. How old are the twins?

Guided Practice

To start, relate the sum and product of the siblings' ages.

(Product of ages) = (Sum of ages) + 4558

Then define variables.

Let x = the twins' ages.
The sister's age is $x + 2$.

17. The product of three consecutive integers is 210. What are the numbers?

B Apply

Solve each equation.

18. $-2x^4 + 100 = 0$
19. $x^4 - 100 = 0$
20. $7x^3 = 5x^2 + 12x$
21. $x^3 + x = x^2 + 1$
22. $x^3 + x^2 + x + 1 = 0$
23. $x^3 + 1 = x^2 + x$
24. **Error Analysis** A student says that 1, 2, 3, and 4 are the zeros of a cubic polynomial function. Explain why the student is mistaken.