Name ______________________ Class ____________ Date ____________

5-5 Reteaching Answer key

Theorems About Roots of Polynomials Equations

Problem

What are the rational roots of $6x^4 + 29x^3 + 40x^2 + 7x - 12 = 0$?

Step 1 Determine the factors of the constant term and the factors of the leading coefficient.

constant term: 12 — factors: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

leading coefficient: 6 — factors: $\pm1, \pm2, \pm3, \pm6$

Step 2 Find all the possible roots by dividing the factors of the constant term by the factors of the leading coefficient. $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12, \pm\frac{1}{6}, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}$

Step 3 Substitute each possible root into the polynomial until you find one that causes the polynomial to equal zero. This is one rational root.

Test $-\frac{3}{2}$: $6\left(-\frac{3}{2}\right)^4 + 29\left(-\frac{3}{2}\right)^3 + 40\left(-\frac{3}{2}\right)^2 + 7\left(-\frac{3}{2}\right) - 12 = 0$ $\quad -\frac{3}{2}$ is a rational root.

Step 4 Factor the polynomial by synthetic division using the first rational root as the divisor.

$$\begin{array}{r|rrrrr} -\frac{3}{2} & 6 & 29 & 40 & 7 & -12 \\ & & -9 & -30 & -15 & 12 \\ \hline & 6 & 20 & 10 & -8 & 0 \end{array}$$

Step 5 If the dividend is a second-degree polynomial, factor to find any additional rational roots. If the dividend does not factor, there are no additional rational roots. If the dividend is greater than a second-degree polynomial, repeat Steps 1–4 until the dividend is a second-degree polynomial.

$$\begin{array}{r|rrrr} -\frac{4}{3} & 6 & 20 & 10 & -8 \\ & & -8 & -16 & 8 \\ \hline & 6 & 12 & -6 & 0 \end{array}$$

$6x^2 + 12x - 6 = 0$ does not factor. The rational roots of $6x^4 + 29x^3 + 40x^2 + 7x - 12$ are $-\frac{3}{2}$ and $-\frac{4}{3}$.

Exercises

Find all rational roots for $P(x) = 0$.

1. $P(x) = x^3 - x^2 - 8x + 12$

   $x = 2, -3$

2. $P(x) = x^4 - 49x^2$

   $x = 0, 7, -7$

3. $P(x) = 2x^3 - 7x^2 - 21x + 54$

   $x = -3, 2, 9/2$

4. $P(x) = x^4 - 2x^3 - 3$