5.3 Double Angle Expression (6.2/6.3) Double Angle Equations

5.3 Double Angle Formulas

$$\boxed{\sin 2A = 2\sin A \cos A}$$

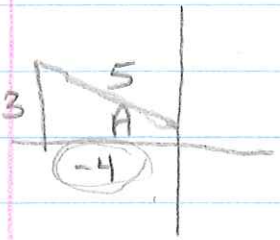
$$\boxed{\cos 2A} = \boxed{\cos^2 A - \sin^2 A} \rightarrow$$

$\hookrightarrow (1-\sin^2) - \sin^2 = \boxed{1 - 2\sin^2 A}$

$\cos^2 A - (1-\cos^2 A) \rightarrow \boxed{2\cos^2 A - 1}$

$$\boxed{\tan 2A = \frac{2\tan A}{1-\tan^2 A}}$$

ex 1) $\sin A = \frac{3}{5}$ in QII. Find $\sin 2A$



$\sin 2A = 2\sin A \cos A$

$2\left(\frac{3}{5}\right)\left(\frac{-4}{5}\right) = \frac{-24}{25}$

ex 2) $\sin A = \frac{1}{\sqrt{5}}$, find $\cos 2A$

$1 - 2\sin^2 A \rightarrow 1 - 2\left(\frac{1}{\sqrt{5}}\right)^2$

$1 - 2\left(\frac{1}{5}\right) = \boxed{\frac{3}{5}}$

Condense into graphable trig function.

ex 3) $y = 3 - 6\sin^2 x$

$y = 3(1 - 2\sin^2 x)$

$= 3\cos(2x)$ ✓

ex 4) Simplify $\frac{2\tan 15}{1 - \tan^2 15} = ?$ $\tan 2A$!!

$\tan(2 \cdot 15) = \tan 30 = \boxed{\frac{\sqrt{3}}{3}}$

Proofs! Prove

ex 5) $(\sin x + \cos x)^2 = 1 + \sin 2x$

$\sin^2 + 2\sin\cos + \cos^2$

$1 + 2\sin\cos$

$1 + \sin 2x$ ✓

ex 6) $\frac{2\cot x}{1+\cot^2 x} = \sin 2x$

$\frac{2\cot x}{\csc^2 x}$ $\frac{\cos}{\sin}$

$2\frac{\cos x \sin^2}{\sin x}$

$2\cos\sin$

$\sin 2x$ ✓

ex 7) $\frac{1-\cos 2x}{\sin 2x} = \tan x$

$\frac{1-(2\cos^2 x - 1)}{2\sin x\cos x}$

$\frac{1-2\cos^2 x+1}{2-2\cos^2 x}$

$\frac{2\sin x\cos}{}$

$\frac{2(1-\cos^2 x)}{2\sin x\cos x}$

$\frac{\sin^2 x}{\sin x\cos x}$

$\frac{\sin}{\cos} = \tan$

$\frac{1-(1-2\sin^2 x)}{2\sin x\cos x}$

$\frac{1-1+2\sin^2 x}{2\sin x\cos x}$

$\frac{\sin}{\cos}$

$\tan$

really a double angle

ex8) $\cos(4x) = 8\cos^4 x - 8\cos x^2 + 1$

$\cos(2 \cdot 2x)$ $\quad \cos 2A = 2(\cos^2 A) - 1$

$2(\cos^2(2x)) - 1$

$2[\cos(2x) \cdot \cos(2x)] - 1$

$2[(2\cos^2 x - 1)(2\cos^2 x - 1)] - 1$

$2[4\cos^4 x - 4\cos^2 x + 1] - 1$

$8\cos^4 x - 8\cos^2 x + 2 - 1$

$8\cos^4 x - 8\cos^2 x + 1$ ☺

We can solve equations!

We can also solve Equations!

ex 9) $\sin(2x) + \sqrt{2}\cos x = 0$

$2\sin x \cos x + \sqrt{2}\cos x = 0$

$\cos x(2\sin x + \sqrt{2}) = 0$

$\cos x = 0 \qquad \sin x = -\frac{\sqrt{2}}{2}$

$x = 90^\circ, 270^\circ, 225^\circ, 315^\circ$

ex 10) $\cos(2x) + 3\sin x - 2 = 0$

$1 - 2\sin^2 x + 3\sin x - 2 = 0$

blaah...

$x = 30^\circ, 90^\circ, 150^\circ$

ex 11) $(\sin x - \cos x)^2 = (\sqrt{2})^2$

$\sin^2 - 2\sin\cos + \cos^2 = 2$

$1 - 2\sin\cos = 2$

$-1 \qquad\qquad -1$

$-2\sin\cos = 1$

$2\sin\cos = -1$

$\sin(2x) = -1$

$\frac{2x}{2} = \frac{270}{2} + \frac{360k}{2}$

$x = 135° + 180°k$