(1)

Use Reciprocal + Ratio Identities

For each of the following, write an algebraic proof.

Identities will always have the following two properties:

1) If you graph the left and right sides, you will obtain exactly the same graph.

2) If you plug in the same angle for x on both sides, you will obtain exactly the same number.

1) Prove: $\cot x \tan x = 1$

2) Prove: $\csc x \cos x = \cot x$

3) Prove: $\dfrac{\sin x}{\tan x} = \cos x$

4) Prove: $\dfrac{1}{\cot x \cos x \tan x} = \sec x$

TRIGONOMETRY II - LESSON TWO
PART I MULTIPLICATION & DIVISION IDENTITIES

5) Prove: $\dfrac{\tan x}{\csc x} = \dfrac{\sin^2 x}{\cos x}$

6) Prove: $\dfrac{\tan x}{\sec x} = \sin x$

7) Prove: $\dfrac{\cos^2 x}{\cot x} = \sin x \cos x$

8) Prove: $\dfrac{\sec x \csc x}{\cot x} = \sec^2 x$

9) Prove: $\dfrac{\sec x \csc x}{\csc^2 x} = \tan x$

10) Prove: $\dfrac{\tan^2 x \cos x}{2 \sec x} = \dfrac{1}{2} \sin^2 x$

1 key

1K

TRIGONOMETRY II - LESSON TWO

PART I MULTIPLICATION & DIVISION IDENTITIES

Answers

1) $\cot x \tan x$

$= \left(\frac{\cos x}{\sin x}\right)\left(\frac{\sin x}{\cos x}\right)$

$= 1$

2) $\csc x \cos x$

$= \left(\frac{1}{\sin x}\right)\cos x$

$= \frac{\cos x}{\sin x}$

$= \cot x$

3) $\frac{\sin x}{\tan x}$

$= \frac{\sin x}{\frac{\sin x}{\cos x}}$

$= \sin x \times \frac{\cos x}{\sin x}$

$= \cos x$

4) $\frac{1}{\cot x \cos x \tan x}$

$= \frac{1}{\frac{\cos x}{\sin x}\cos x \frac{\sin x}{\cos x}}$

$= \frac{1}{\cos x}$

$= \sec x$

5) $\frac{\tan x}{\csc x}$

$= \frac{\frac{\sin x}{\cos x}}{\frac{1}{\sin x}}$

$= \frac{\sin x}{\cos x} \times \frac{\sin x}{1}$

$= \frac{\sin^2 x}{\cos x}$

6) $\frac{\tan x}{\sec x}$

$= \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}}$

$= \frac{\sin x}{\cos x} \times \frac{\cos x}{1}$

$= \sin x$

7) $\frac{\cos^2 x}{\cot x}$

$= \frac{\cos^2 x}{\frac{\cos x}{\sin x}}$

$= \cos x \times \frac{\sin x}{\cos x}$

$= \cos x \sin x$

8) $\frac{\sec x \csc x}{\cot x}$

$= \frac{\frac{1}{\cos x}\frac{1}{\sin x}}{\frac{\cos x}{\sin x}}$

$= \frac{1}{\cos x \sin x} \times \frac{\sin x}{\cos x}$

$= \frac{1}{\cos^2 x}$

$= \sec^2 x$

9) $\frac{\sec x \csc x}{\csc^2 x}$

$= \frac{\sec x}{\csc x}$

$= \frac{\frac{1}{\cos x}}{\frac{1}{\sin x}}$

$= \frac{1}{\cos x} \times \frac{\sin x}{1}$

$= \frac{\sin x}{\cos x}$

$= \tan x$

10) $\frac{\tan^2 x \cos x}{2 \sec x}$

$= \frac{\frac{\sin^2 x}{\cos^2 x}\cos x}{\frac{2}{\cos x}}$

$= \frac{\sin^2 x}{\cos x} \times \frac{\cos x}{2}$

$= \frac{1}{2}\sin^2 x$

(2)

Get Common Denominator

Questions: ***For each of the following, write an algebraic proof.***

1) $\sec x - \sin x = \dfrac{1-\sin x\cos x}{\cos x}$

2) $\sin x + \tan x \sin x = \dfrac{\sin x\cos x + \sin^2 x}{\cos x}$

3) $\sec^2 x + \cot x = \dfrac{\sin x + \cos^3 x}{\cos^2 x \sin x}$

4) $\csc^2 x - \tan x = \dfrac{\cos x - \sin^3 x}{\sin^2 x \cos x}$

5) $\csc x - \sec x = \dfrac{\cos x - \sin x}{\sin x \cos x}$

6) $\sec x - \tan x = \dfrac{1-\sin x}{\cos x}$

7) $\cos x + \tan x = \dfrac{\cos^2 x + \sin x}{\cos x}$

8) $\cot x + \sin x = \dfrac{\cos x + \sin^2 x}{\sin x}$

9) $1 + \tan x = \dfrac{\cos x + \sin x}{\cos x}$

10) $\csc x + 1 = \dfrac{1+\sin x}{\sin x}$

2 key

TRIGONOMETRY II - LESSON TWO

PART II ADDITION & SUBTRACTION IDENTITIES

1) $\sec x - \sin x$

$= \frac{1}{\cos x} - \frac{\sin x}{1}$

$= \frac{1}{\cos x} - \left(\frac{\sin x}{1}\right)\frac{\cos x}{\cos x}$

$= \frac{1}{\cos x} - \frac{\sin x \cos x}{\cos x}$

$= \frac{1 - \sin x \cos x}{\cos x}$

2) $\sin x + \tan x \sin x$

$= \sin x + \left(\frac{\sin x}{\cos x}\right)\sin x$

$= \frac{\sin x}{1} + \frac{\sin^2 x}{\cos x}$

$= \left(\frac{\sin x}{1}\right)\frac{\cos x}{\cos x} + \frac{\sin^2 x}{\cos x}$

$= \frac{\sin x \cos x}{\cos x} + \frac{\sin^2 x}{\cos x}$

$= \frac{\sin x \cos x + \sin^2 x}{\cos x}$

3) $\sec^2 x + \cot x$

$= \frac{1}{\cos^2 x} + \frac{\cos x}{\sin x}$

$= \frac{1}{\cos^2 x}\left(\frac{\sin x}{\sin x}\right) + \frac{\cos x}{\sin x}\left(\frac{\cos^2 x}{\cos^2 x}\right)$

$= \frac{\sin x}{\cos^2 x \sin x} + \frac{\cos^3 x}{\cos^2 x \sin x}$

$= \frac{\sin x + \cos^3 x}{\cos^2 x \sin x}$

4) $\csc^2 x - \tan x$

$= \frac{1}{\sin^2 x} - \frac{\sin x}{\cos x}$

$= \frac{1}{\sin^2 x}\left(\frac{\cos x}{\cos x}\right) - \frac{\sin x}{\cos x}\left(\frac{\sin^2 x}{\sin^2 x}\right)$

$= \frac{\cos x}{\sin^2 x \cos x} - \frac{\sin^3 x}{\sin^2 x \cos x}$

$= \frac{\cos x - \sin^3 x}{\sin^2 x \cos x}$

5) $\csc x - \sec x$

$= \frac{1}{\sin x} - \frac{1}{\cos x}$

$= \left(\frac{1}{\sin x}\right)\frac{\cos x}{\cos x} - \left(\frac{1}{\cos x}\right)\frac{\sin x}{\sin x}$

$= \frac{\cos x}{\sin x \cos x} - \frac{\sin x}{\sin x \cos x}$

$= \frac{\cos x - \sin x}{\sin x \cos x}$

6) $\sec x - \tan x$

$= \frac{1}{\cos x} - \frac{\sin x}{\cos x}$

$= \frac{1 - \sin x}{\cos x}$

7) $\cos x + \tan x$

$= \cos x + \frac{\sin x}{\cos x}$

$= \frac{\cos x}{1}\left(\frac{\cos x}{\cos x}\right) + \frac{\sin x}{\cos x}$

$= \frac{\cos^2 x}{\cos x} + \frac{\sin x}{\cos x}$

$= \frac{\cos^2 x + \sin x}{\cos x}$

8) $\cot x + \sin x$

$= \frac{\cos x}{\sin x} + \sin x$

$= \frac{\cos x}{\sin x} + \left(\frac{\sin x}{1}\right)\frac{\sin x}{\sin x}$

$= \frac{\cos x}{\sin x} + \left(\frac{\sin x}{1}\right)\frac{\sin x}{\sin x}$

$= \frac{\cos x}{\sin x} + \frac{\sin^2 x}{\sin x}$

$= \frac{\cos x + \sin^2 x}{\sin x}$

9) $1 + \tan x$

$= 1 + \frac{\sin x}{\cos x}$

$= \left(\frac{1}{1}\right)\frac{\cos x}{\cos x} + \frac{\sin x}{\cos x}$

$= \frac{\cos x}{\cos x} + \frac{\sin x}{\cos x}$

$= \frac{\cos x + \sin x}{\cos x}$

10) $\csc x + 1$

$= \frac{1}{\sin x} + \frac{\sin x}{\sin x}$

$= \frac{1 + \sin x}{\sin x}$

(3)

Use Pythagorean identities (I)

Questions: ***Use the special identities to do each of the following proofs.***

1) $\sec x - \tan x \sin x = \cos x$

2) $\cos x + \tan x \sin x = \sec x$

3) $\tan x + \cot x = \sec x \csc x$

4) $1 + \tan^2 x = \sec^2 x$

5) $\sec x - \cos x = \tan x \sin x$

6) $\sin x + \cot x \cos x = \csc x$

TRIGONOMETRY II - LESSON TWO
PART III THREE SPECIAL IDENTITIES

7) $\sec^2 x - 1 = \sin^2 x \sec^2 x$

8) $1 - \csc^2 x = -\cot^2 x$

9) $\csc x - \sin x = \cos x \cot x$

10) $1 - \sec^2 x = -\tan^2 x$

(3 key)

TRIGONOMETRY II - LESSON TWO

PART III THREE SPECIAL IDENTITIES

1)

$$\begin{aligned}&\sec x - \tan x \sin x\\ &= \frac{1}{\cos x} - \left(\frac{\sin x}{\cos x}\right)\sin x\\ &= \frac{1}{\cos x} - \frac{\sin^2 x}{\cos x}\\ &= \frac{1-\sin^2 x}{\cos x}\\ &= \frac{\cos^2 x}{\cancel{\cos x}}\\ &= \cos x\end{aligned}$$

2)

$$\begin{aligned}&\cos x + \tan x \sin x\\ &= \cos x + \left(\frac{\sin x}{\cos x}\right)\sin x\\ &= \frac{\cos x}{1} + \frac{\sin^2 x}{\cos x}\\ &= \left(\frac{\cos x}{1}\right)\frac{\cos x}{\cos x} + \frac{\sin^2 x}{\cos x}\\ &= \frac{\cos^2 x}{\cos x} + \frac{\sin^2 x}{\cos x}\\ &= \frac{1}{\cos x}\\ &= \sec x\end{aligned}$$

3)

$$\begin{aligned}&\tan x + \cot x\\ &= \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\\ &= \left(\frac{\sin x}{\cos x}\right)\frac{\sin x}{\sin x} + \left(\frac{\cos x}{\sin x}\right)\frac{\cos x}{\cos x}\\ &= \frac{\sin^2 x}{\sin x \cos x} + \frac{\cos^2 x}{\sin x \cos x}\\ &= \frac{1}{\sin x \cos x}\\ &= \sec x \csc x\end{aligned}$$

4)

$$\begin{aligned}&1 + \tan^2 x\\ &= 1 + \frac{\sin^2 x}{\cos^2 x}\\ &= \left(\frac{1}{1}\right)\frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}\\ &= \frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}\\ &= \frac{1}{\cos^2 x}\\ &= \sec^2 x\end{aligned}$$

5)

$$\begin{aligned}&\sec x - \cos x\\ &= \frac{1}{\cos x} - \cos x\\ &= \frac{1}{\cos x} - \left(\frac{\cos x}{1}\right)\frac{\cos x}{\cos x}\\ &= \frac{1}{\cos x} - \frac{\cos^2 x}{\cos x}\\ &= \frac{1-\cos^2 x}{\cos x}\\ &= \frac{\sin^2 x}{\cos x}\\ &= \frac{\sin x}{\cos x}\sin x\\ &= \tan x \sin x\end{aligned}$$

6)

$$\begin{aligned}&\sin x + \cot x \cos x\\ &= \sin x + \left(\frac{\cos x}{\sin x}\right)\cos x\\ &= \sin x + \frac{\cos^2 x}{\sin x}\\ &= \left(\frac{\sin x}{1}\right)\frac{\sin x}{\sin x} + \frac{\cos^2 x}{\sin x}\\ &= \frac{\sin^2 x}{\sin x} + \frac{\cos^2 x}{\sin x}\\ &= \frac{1}{\sin x}\\ &= \csc x\end{aligned}$$

7)

$$\begin{aligned}&\sec^2 x - 1\\ &= \frac{1}{\cos^2 x} - 1\\ &= \frac{1}{\cos^2 x} - \left(\frac{1}{1}\right)\frac{\cos^2 x}{\cos^2 x}\\ &= \frac{1}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x}\\ &= \frac{1-\cos^2 x}{\cos^2 x}\\ &= \frac{\sin^2 x}{\cos^2 x}\\ &= \sin^2 x\left(\frac{1}{\cos^2 x}\right)\\ &= \sin^2 x \sec^2 x\end{aligned}$$

8)

$$\begin{aligned}&1 - \csc^2 x\\ &= 1 - \frac{1}{\sin^2 x}\\ &= \left(\frac{1}{1}\right)\frac{\sin^2 x}{\sin^2 x} - \frac{1}{\sin^2 x}\\ &= \frac{\sin^2 x}{\sin^2 x} - \frac{1}{\sin^2 x}\\ &= \frac{\sin^2 x - 1}{\sin^2 x}\\ &= \frac{-\cos^2 x}{\sin^2 x}\\ &= -\cot^2 x\end{aligned}$$

9)

$$\begin{aligned}&\csc x - \sin x\\ &= \frac{1}{\sin x} - \sin x\\ &= \frac{1}{\sin x} - \left(\frac{\sin x}{1}\right)\frac{\sin x}{\sin x}\\ &= \frac{1}{\sin x} - \frac{\sin^2 x}{\sin x}\\ &= \frac{1-\sin^2 x}{\sin x}\\ &= \frac{\cos^2 x}{\sin x}\\ &= \left(\frac{\cos x}{\sin x}\right)\cos x\\ &= \cot x \cos x\end{aligned}$$

10)

$$\begin{aligned}&1 - \sec^2 x\\ &= 1 - \frac{1}{\cos^2 x}\\ &= \left(\frac{1}{1}\right)\frac{\cos^2 x}{\cos^2 x} - \frac{1}{\cos^2 x}\\ &= \frac{\cos^2 x}{\cos^2 x} - \frac{1}{\cos^2 x}\\ &= \frac{\cos^2 x - 1}{\cos^2 x}\\ &= \frac{-\sin^2 x}{\cos^2 x}\\ &= -\tan^2 x\end{aligned}$$

(4)

use Pythagorean identities (II)

1) $\frac{3\tan x}{1+\tan^2 x} = 3\sin x\cos x$

2) $\frac{1}{1+\cot^2 x} = \sin^2 x$

3) $\sec^2 x - \cos^2 x - \sin^2 x = \tan^2 x$

4) $(\sin x + \cos x)^2 + (\sin x - \cos x)^2 = 2$

5) $(1+\sin x)^2 + \cos^2 x = 2(1+\sin x)$

6) $\sin^4 x - \cos^4 x = 2\sin^2 x - 1$

TRIGONOMETRY II - LESSON TWO
PART V OTHER PROOFS

7) $(\tan x - 1)^2 = \sec^2 x - 2\tan x$

8) $(1-\sec^2 x)(1-\sin^2 x) = -\sin^2 x$

9) $\csc x - \csc^3 x = \frac{-\cos^2 x}{\sin^3 x}$

10) $\csc^4 x - 1 = \frac{\cos^2 x(1+\sin^2 x)}{\sin^4 x}$

11) $\tan^2 x - \cot^2 x = \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x}$

4 key

TRIGONOMETRY II - LESSON TWO

PART V OTHER PROOFS

1)
$$\frac{3\tan x}{1+\tan^2 x}$$
$$=\frac{3\tan x}{\sec^2 x}$$
$$=\frac{3\dfrac{\sin x}{\cos x}}{\dfrac{1}{\cos^2 x}}$$
$$=3\frac{\sin x}{\cancel{\cos x}}\times\frac{\cos^{\cancel{2}} x}{1}$$
$$=3\sin x\cos x$$

2)
$$\frac{1}{1+\cot^2 x}$$
$$=\frac{1}{\csc^2 x}$$
$$=\frac{1}{\dfrac{1}{\sin^2 x}}$$
$$=1\times\frac{\sin^2 x}{1}$$
$$=\sin^2 x$$

3)
$$\sec^2 x-\cos^2 x-\sin^2 x$$
$$\sec^2 x-(\cos^2 x+\sin^2 x)$$
$$\sec^2 x-1$$
$$\tan^2 x$$

4)
$$(\sin x+\cos x)^2+(\sin x-\cos x)^2$$
$$=\sin^2 x+2\sin x\cos x+\cos^2 x+\sin^2 x-2\sin x\cos x+\cos^2 x$$
$$=2\sin^2 x+2\cos^2 x$$
$$=2(\sin^2 x+\cos^2 x)$$
$$=2$$

5)
$$(1+\sin x)^2+\cos^2 x$$
$$=1+2\sin x+\sin^2 x+\cos^2 x$$
$$=1+2\sin x+1$$
$$=2+2\sin x$$
$$=2(1+\sin x)$$

6)
$$\sin^4 x-\cos^4 x$$
$$=(\sin^2 x-\cos^2 x)(\sin^2 x+\cos^2 x)$$
$$=(\sin^2 x-\cos^2 x)$$
$$=\sin^2 x-(1-\sin^2 x)$$
$$=\sin^2 x-1+\sin^2 x$$
$$=2\sin^2 x-1$$

7)
$$(\tan x-1)^2$$
$$=(\tan x-1)(\tan x-1)$$
$$=\tan^2 x-2\tan x+1$$
$$=\tan^2 x+1-2\tan x$$
$$=\sec^2 x-2\tan x$$

8)
$$(1-\sec^2 x)(1-\sin^2 x)$$
$$=(-\tan^2 x)(\cos^2 x)$$
$$=-\frac{\sin^2 x}{\cos^2 x}\cos^2 x$$
$$=-\sin^2 x$$

9)
$$\csc x-\csc^3 x$$
$$=\csc x(1-\csc^2 x)$$
$$=\csc x(-\cot^2 x)$$
$$=\frac{1}{\sin x}\left(-\frac{\cos^2 x}{\sin^2 x}\right)$$
$$=-\frac{\cos^2 x}{\sin^3 x}$$

10)
$$\csc^4 x-1$$
$$=(\csc^2 x-1)(\csc^2 x+1)$$
$$=\cot^2 x(\csc^2 x+1)$$
$$=\frac{\cos^2 x}{\sin^2 x}\left(\frac{1}{\sin^2 x}+1\right)$$
$$=\frac{\cos^2 x}{\sin^4 x}+\frac{\cos^2 x}{\sin^2 x}$$
$$=\frac{\cos^2 x}{\sin^4 x}+\frac{\cos^2 x}{\sin^2 x}\left(\frac{\sin^2 x}{\sin^2 x}\right)$$
$$=\frac{\cos^2 x}{\sin^4 x}+\frac{\cos^2 x\sin^2 x}{\sin^4 x}$$
$$=\frac{\cos^2 x+\cos^2 x\sin^2 x}{\sin^4 x}$$
$$=\frac{\cos^2 x(1+\sin^2 x)}{\sin^4 x}$$

11)
$$\tan^2 x-\cot^2 x$$
$$=\frac{\sin^2 x}{\cos^2 x}-\frac{\cos^2 x}{\sin^2 x}$$
$$=\frac{\sin^2 x}{\cos^2 x}\left(\frac{\sin^2 x}{\sin^2 x}\right)-\frac{\cos^2 x}{\sin^2 x}\left(\frac{\cos^2 x}{\cos^2 x}\right)$$
$$=\frac{\sin^4 x}{\cos^2 x\sin^2 x}-\frac{\cos^4 x}{\sin^2 x\cos^2 x}$$
$$=\frac{\sin^4 x-\cos^4 x}{\cos^2 x\sin^2 x}$$
$$=\frac{(\sin^2 x-\cos^2 x)(\sin^2 x+\cos^2 x)}{\cos^2 x\sin^2 x}$$
$$=\frac{(\sin^2 x-\cos^2 x)}{\cos^2 x\sin^2 x}$$

TRIGONOMETRY II - LESSON TWO

PART VI CONJUGATES

5

Sometimes you will get identities that can't be broken down any further. In these cases, you can multiply numerator & denominator by the conjugate. This will convert the fraction into something that will give you identities to work with.

The conjugate is obtained by taking a binomial from the original expression and changing the sign in the middle.

Example 1: Prove $\frac{1+\cos x}{\sin x}=\frac{\sin x}{1-\cos x}$

$\frac{1+\cos x}{\sin x}$ has the conjugate: **1 - cos x**

$$\frac{1+\cos x}{\sin x}$$

$$=\frac{1+\cos x}{\sin x}\left(\frac{1-\cos x}{1-\cos x}\right)$$

$$=\frac{1-\cos^2 x}{\sin x(1-\cos x)}$$

$$=\frac{\sin^2 x}{\sin x(1-\cos x)}$$

$$=\frac{\sin x}{1-\cos x}$$

Prove each of the following identities:

1) $\frac{\cos x}{1-\sin x}=\frac{1+\sin x}{\cos x}$ **2)** $\frac{1}{1-\sin x}=\frac{1+\sin x}{\cos^2 x}$ **3)** $\frac{1-\cos x}{\sin x}=\frac{\sin x}{1+\cos x}$ **4)** $\frac{1-\sin x}{\cos x}=\frac{\cos x}{1+\sin x}$

5 key

TRIGONOMETRY II - LESSON TWO

PART VI CONJUGATES

1) $\dfrac{\cos x}{1-\sin x}$

$$= \frac{\cos x}{1-\sin x}\left(\frac{1+\sin x}{1+\sin x}\right)$$

$$= \frac{\cos x(1+\sin x)}{1-\sin^2 x}$$

$$= \frac{\cancel{\cos x}(1+\sin x)}{\cos^{\cancel{2}} x}$$

$$= \frac{(1+\sin x)}{\cos x}$$

2) $\dfrac{1}{1-\sin x}$

$$= \frac{1}{1-\sin x}\left(\frac{1+\sin x}{1+\sin x}\right)$$

$$= \frac{1+\sin x}{1-\sin^2 x}$$

$$= \frac{1+\sin x}{\cos^2 x}$$

3) $\dfrac{1-\cos x}{\sin x}$

$$= \frac{1-\cos x}{\sin x}\left(\frac{1+\cos x}{1+\cos x}\right)$$

$$= \frac{1-\cos^2 x}{\sin x(1+\cos x)}$$

$$= \frac{\sin^2 x}{\sin x(1+\cos x)}$$

$$= \frac{\sin x}{1+\cos x}$$

4) $\dfrac{1-\sin x}{\cos x}$

$$= \frac{1-\sin x}{\cos x}\left(\frac{1+\sin x}{1+\sin x}\right)$$

$$= \frac{1-\sin^2 x}{\cos x(1+\sin x)}$$

$$= \frac{\cos^{\cancel{2}} x}{\cancel{\cos x}(1+\sin x)}$$

$$= \frac{\cos x}{1+\sin x}$$

(6)

Compound Fractions

Questions: ***Prove each of the following:***

1) $\frac{\sec x}{\cot x + \tan x} = \sin x$

2) $\frac{\sin x + \tan x}{\cos x + 1} = \tan x$

3) $\frac{\cos x - \csc x}{\sin x - \sec x} = \cot x$

4) $\frac{\sin x + \cos x}{\sec x + \csc x} = \sin x \cos x$

5) $\frac{\tan x - \sin x}{\tan x \sin x} = \frac{1 - \cos x}{\sin x}$

6) $\frac{1 + \cos x}{\tan x + \sin x} = \cot x$

TRIGONOMETRY II - LESSON TWO
PART IV COMPOUND FRACTIONS & SPECIAL IDENTITIES

7) $\frac{1 + \tan^2 x}{1 + \cot^2 x} = \tan^2 x$

8) $\frac{1}{\sec^2 x} + \frac{1}{\csc^2 x} = 1$

9) $\frac{1 + \tan x}{1 + \cot x} = \tan x$

10) $\frac{\cos x}{\sec x - 1} + \frac{\cos x}{\sec x + 1} = 2\cot^2 x$

11) $\frac{\tan x}{1 + \tan x} = \frac{\sin x}{\sin x + \cos x}$

12) $\frac{\sin^2 x}{1 - \sin x} + \frac{\sin^2 x}{1 + \sin x} = 2\tan^2 x$

1) $\dfrac{\sec x}{\cot x+\tan x}$

$=\dfrac{\frac{1}{\cos x}}{\frac{\cos x}{\sin x}+\frac{\sin x}{\cos x}}$

$=\dfrac{\frac{1}{\cos x}}{\left(\frac{\cos x}{\sin x}\right)\frac{\cos x}{\cos x}+\left(\frac{\sin x}{\cos x}\right)\frac{\sin x}{\sin x}}$

$=\dfrac{\frac{1}{\cos x}}{\frac{\cos^2 x}{\sin x\cos x}+\frac{\sin^2 x}{\sin x\cos x}}$

$=\dfrac{\frac{1}{\cos x}}{\frac{\cos^2 x+\sin^2 x}{\sin x\cos x}}$

$=\dfrac{\frac{1}{\cos x}}{\frac{1}{\sin x\cos x}}$

$=\dfrac{1}{\cancel{\cos x}}\times\dfrac{\sin x\cancel{\cos x}}{1}$

$=\sin x$

2) $\dfrac{\sin x+\tan x}{\cos x+1}$

$=\dfrac{\sin x+\frac{\sin x}{\cos x}}{\cos x+1}$

$=\dfrac{\left(\frac{\sin x}{1}\right)\frac{\cos x}{\cos x}+\frac{\sin x}{\cos x}}{\cos x+1}$

$=\dfrac{\frac{\sin x\cos x}{\cos x}+\frac{\sin x}{\cos x}}{\cos x+1}$

$=\dfrac{\frac{\sin x\cos x+\sin x}{\cos x}}{\cos x+1}$

$=\dfrac{\sin x\cos x+\sin x}{\cos x}\times\dfrac{1}{\cos x+1}$

$=\dfrac{\sin x\cancel{(\cos x+1)}}{\cos x}\times\dfrac{1}{\cancel{\cos x+1}}$

$=\dfrac{\sin x}{\cos x}$

$=\tan x$

3) $\dfrac{\cos x-\csc x}{\sin x-\sec x}$

$=\dfrac{\cos x-\frac{1}{\sin x}}{\sin x-\frac{1}{\cos x}}$

$=\dfrac{\left(\frac{\cos x}{1}\right)\frac{\sin x}{\sin x}-\frac{1}{\sin x}}{\left(\frac{\sin x}{1}\right)\frac{\cos x}{\cos x}-\frac{1}{\cos x}}$

$=\dfrac{\frac{\cos x\sin x-1}{\sin x}}{\frac{\sin x\cos x-1}{\cos x}}$

$=\dfrac{\cos x\sin x-1}{\sin x}\times\dfrac{\cos x}{\cos x\sin x-1}$

$=\dfrac{\cos x}{\sin x}$

$=\cot x$

4) $\dfrac{\sin x+\cos x}{\sec x+\csc x}$

$=\dfrac{\sin x+\cos x}{\frac{1}{\cos x}+\frac{1}{\sin x}}$

$=\dfrac{\sin x+\cos x}{\left(\frac{1}{\cos x}\right)\frac{\sin x}{\sin x}+\left(\frac{1}{\sin x}\right)\frac{\cos x}{\cos x}}$

$=\dfrac{\sin x+\cos x}{\frac{\sin x}{\cos x\sin x}+\frac{\cos x}{\cos x\sin x}}$

$=\dfrac{\sin x+\cos x}{\frac{\sin x+\cos x}{\cos x\sin x}}$

$=\dfrac{\cancel{\sin x+\cos x}}{1}\times\dfrac{\cos x\sin x}{\cancel{\sin x+\cos x}}$

$=\cos x\sin x$

5) $\dfrac{\tan x-\sin x}{\tan x\sin x}$

$=\dfrac{\frac{\sin x}{\cos x}-\sin x}{\left(\frac{\sin x}{\cos x}\right)\sin x}$

$=\dfrac{\frac{\sin x}{\cos x}-\left(\frac{\sin x}{1}\right)\frac{\cos x}{\cos x}}{\frac{\sin^2 x}{\cos x}}$

$=\dfrac{\frac{\sin x}{\cos x}-\frac{\sin x\cos x}{\cos x}}{\frac{\sin^2 x}{\cos x}}$

$=\dfrac{\frac{\sin x-\sin x\cos x}{\cos x}}{\frac{\sin^2 x}{\cos x}}$

$=\dfrac{\sin x-\sin x\cos x}{\cos x}\times\dfrac{\cos x}{\sin^2 x}$

$=\dfrac{\cancel{\sin x}(1-\cos x)}{\cancel{\cos x}}\times\dfrac{\cancel{\cos x}}{\sin^{\cancel{2}} x}$

$=\dfrac{1-\cos x}{\sin x}$

6) $\dfrac{1+\cos x}{\tan x+\sin x}$

$=\dfrac{1+\cos x}{\frac{\sin x}{\cos x}+\sin x}$

$=\dfrac{1+\cos x}{\frac{\sin x}{\cos x}+\left(\frac{\sin x}{1}\right)\frac{\cos x}{\cos x}}$

$=\dfrac{1+\cos x}{\frac{\sin x}{\cos x}+\frac{\sin x\cos x}{\cos x}}$

$=\dfrac{1+\cos x}{\frac{\sin x+\sin x\cos x}{\cos x}}$

$=\dfrac{1+\cos x}{1}\times\dfrac{\cos x}{\sin x+\sin x\cos x}$

$=\dfrac{\cancel{1+\cos x}}{1}\times\dfrac{\cos x}{\sin x\cancel{(1+\cos x)}}$

$=\dfrac{\cos x}{\sin x}$

$=\cot x$

TRIGONOMETRY II - LESSON TWO
PART IV COMPOUND FRACTIONS & SPECIAL IDENTITIES

7) $\dfrac{1+\tan^2 x}{1+\cot^2 x}$

$=\dfrac{\sec^2 x}{\csc^2 x}$

$=\dfrac{1}{\frac{\cos^2 x}{\sin^2 x}}$

$=\dfrac{1}{\cos^2 x}\bullet\sin^2 x$

$=\dfrac{\sin^2 x}{\cos^2 x}$

$=\tan^2 x$

8)

$\dfrac{1}{\sec^2 x}+\dfrac{1}{\csc^2 x}$

$=\cos^2 x+\sin^2 x$

$=1$

9) $\dfrac{1+\tan x}{1+\cot x}$

$=\dfrac{1+\frac{\sin x}{\cos x}}{1+\frac{\cos x}{\sin x}}$

$=\dfrac{\frac{\cos x}{\cos x}+\frac{\sin x}{\cos x}}{\frac{\sin x}{\sin x}+\frac{\cos x}{\sin x}}$

$=\dfrac{\frac{\cos x+\sin x}{\cos x}}{\frac{\sin x+\cos x}{\sin x}}$

$=\dfrac{\cancel{\cos x+\sin x}}{\cos x}\times\dfrac{\sin x}{\cancel{\sin x+\cos x}}$

$=\dfrac{\sin x}{\cos x}$

$=\tan x$

10) $\dfrac{\cos x}{\sec x-1}+\dfrac{\cos x}{\sec x+1}$

$=\dfrac{\cos x}{\sec x-1}\left(\dfrac{\sec x+1}{\sec x+1}\right)+\dfrac{\cos x}{\sec x+1}\left(\dfrac{\sec x-1}{\sec x-1}\right)$

$=\dfrac{\cos x\sec x+\cos x}{(\sec x-1)(\sec x+1)}+\dfrac{\cos x\sec x-\cos x}{(\sec x+1)(\sec x-1)}$

$=\dfrac{\cos x\sec x+\cos x+\cos x\sec x-\cos x}{(\sec x-1)(\sec x+1)}$

$=\dfrac{2\cos x\sec x}{(\sec x-1)(\sec x+1)}$

$=\dfrac{2\cos x\sec x}{(\sec^2 x-1)}$

$=\dfrac{2\cos x\sec x}{\tan^2 x}$

$=\dfrac{2\cancel{\cos x}\frac{1}{\cancel{\cos x}}}{\tan^2 x}$

$=\dfrac{2}{\tan^2 x}$

$=2\cot^2 x$

11)

$\dfrac{\tan x}{1+\tan x}$

$=\dfrac{\frac{\sin x}{\cos x}}{1+\frac{\sin x}{\cos x}}$

$=\dfrac{\frac{\sin x}{\cos x}}{\frac{\cos x}{\cos x}+\frac{\sin x}{\cos x}}$

$=\dfrac{\frac{\sin x}{\cos x}}{\frac{\cos x+\sin x}{\cos x}}$

$=\dfrac{\sin x}{\cancel{\cos x}}\times\dfrac{\cancel{\cos x}}{\cos x+\sin x}$

$=\dfrac{\sin x}{\cos x+\sin x}$

12)

$\dfrac{\sin^2 x}{1-\sin x}+\dfrac{\sin^2 x}{1+\sin x}$

$=\dfrac{\sin^2 x}{1-\sin x}\left(\dfrac{1+\sin x}{1+\sin x}\right)+\dfrac{\sin^2 x}{1+\sin x}\left(\dfrac{1-\sin x}{1-\sin x}\right)$

$=\dfrac{\sin^2 x+\sin^3 x}{(1-\sin x)(1+\sin x)}+\dfrac{\sin^2 x-\sin^3 x}{(1+\sin x)(1-\sin x)}$

$=\dfrac{\sin^2 x+\sin^3 x+\sin^2 x-\sin^3 x}{(1-\sin x)(1+\sin x)}$

$=\dfrac{2\sin^2 x}{(1-\sin x)(1+\sin x)}$

$=\dfrac{2\sin^2 x}{(1-\sin^2 x)}$

$=\dfrac{2\sin^2 x}{\cos^2 x}$

$=2\tan^2 x$