

CHAPTER 5 Identities and Formulas

Problem Set 5.1

1. $\cos \theta \tan \theta = \cos \theta \cdot \frac{\sin \theta}{\cos \theta}$ Ratio identity
 $= \frac{\cos \theta \sin \theta}{\cos \theta}$ Multiply
 $= \sin \theta$ Reduce
3. $\csc \theta \tan \theta = \frac{1}{\sin \theta} \cdot \frac{\sin \theta}{\cos \theta}$ Ratio and reciprocal identities
 $= \frac{1}{\cos \theta}$ Reduce
 $= \sec \theta$ Reciprocal identity
5. $\frac{\tan A}{\sec A} = \frac{\frac{\sin A}{\cos A}}{\frac{1}{\cos A}}$ Ratio and reciprocal identities
 $= \frac{\sin A \cos A}{\cos A}$ Divide
 $= \sin A$ Reduce
7. $\sec \theta \cot \theta \sin \theta = \frac{1}{\cos \theta} \cdot \frac{\cos \theta}{\sin \theta} \cdot \sin \theta$ Ratio and reciprocal identities
 $= 1$ Reduce
9. $\cos x (\csc x + \tan x) = \cos x \csc x + \cos x \tan x$ Distributive property
 $= \cos x \cdot \frac{1}{\sin x} + \cos x \cdot \frac{\sin x}{\cos x}$ Reciprocal and ratio identities
 $= \frac{\cos x}{\sin x} + \frac{\cos x \sin x}{\cos x}$ Multiply
 $= \cot x + \sin x$ Ratio identity and reduce second fraction

11. $\cos x(\csc x - \sec x) = \cos x\left(\frac{1}{\sin x} - \frac{1}{\cos x}\right)$ Reciprocal identities
 $= \frac{\cos x}{\sin x} - 1$ Multiply
 $= \cot x - 1$ Ratio identity
13. $\cos^2 x(1 + \tan^2 x) = \cos^2 x(\sec^2 x)$ Pythagorean identity
 $= \cos^2 x\left(\frac{1}{\cos^2 x}\right)$ Reciprocal identity
 $= 1$ Multiply and reduce
15. $(1 - \sin x)(1 + \sin x) = 1 - \sin^2 x$ Multiply
 $= \cos^2 x$ Pythagorean identity
17. $\frac{\cos^4 t - \sin^4 t}{\sin^2 t} = \frac{(\cos^2 t + \sin^2 t)(\cos^2 t - \sin^2 t)}{\sin^2 t}$ Factor
 $= \frac{1(\cos^2 t - \sin^2 t)}{\sin^2 t}$ Pythagorean identity
 $= \frac{\cos^2 t}{\sin^2 t} - \frac{\sin^2 t}{\sin^2 t}$ Separate into 2 fractions
 $= \cot^2 t - 1$ Ratio identity and reduce second fraction
19. $\frac{\cos^2 \theta}{1 - \sin \theta} = \frac{1 - \sin^2 \theta}{1 - \sin \theta}$ Pythagorean identity
 $= \frac{(1 - \sin \theta)(1 + \sin \theta)}{1 - \sin \theta}$ Factor numerator
 $= 1 + \sin \theta$ Reduce
21. $\frac{1 - \sin^4 \theta}{1 + \sin^2 \theta} = \frac{(1 - \sin^2 \theta)(1 + \sin^2 \theta)}{1 + \sin^2 \theta}$ Factor
 $= 1 - \sin^2 \theta$ Reduce
 $= \cos^2 \theta$ Pythagorean identity
23. $\sec^2 \theta - \tan^2 \theta = (\tan^2 \theta + 1) - \tan^2 \theta$ Pythagorean identity
 $= 1$ Combine

$$\begin{aligned}
 25. \quad \sec^4 \theta - \tan^4 \theta &= (\sec^2 \theta - \tan^2 \theta)(\sec^2 \theta + \tan^2 \theta) \\
 &= 1(\sec^2 \theta + \tan^2 \theta) \\
 &= \frac{1}{\cos^2 \theta} + \frac{\sin^2 \theta}{\cos^2 \theta} \\
 &= \frac{1 + \sin^2 \theta}{\cos^2 \theta}
 \end{aligned}$$

Factor

Pythagorean identity (problem #23)

Reciprocal and ratio identities

Add fractions

$$\begin{aligned}
 27. \quad \frac{\sin^2 \theta - \cos^2 \theta}{\sin \theta \cos \theta} &= \frac{\sin^2 \theta}{\sin \theta \cos \theta} - \frac{\cos^2 \theta}{\sin \theta \cos \theta} \\
 &= \frac{\sin \theta}{\cos \theta} - \frac{\cos \theta}{\sin \theta} \\
 &= \tan \theta - \cot \theta
 \end{aligned}$$

Separate into 2 fractions

Reduce

Ratio identities

$$\begin{aligned}
 29. \quad \csc B - \sin B &= \frac{1}{\sin B} - \sin B \\
 &= \frac{1}{\sin B} - \sin B \cdot \frac{\sin B}{\sin B} \\
 &= \frac{1 - \sin^2 B}{\sin B} \\
 &= \frac{\cos^2 B}{\sin B} \\
 &= \frac{\cos B}{\sin B} \cdot \cos B \\
 &= \cot B \cos B
 \end{aligned}$$

Reciprocal identity

LCD is $\sin B$

Subtract fractions

Pythagorean identity

Separate fraction

Ratio identity

$$\begin{aligned}
 31. \quad \cot \theta \cos \theta + \sin \theta &= \frac{\cos \theta}{\sin \theta} \cdot \cos \theta + \sin \theta \\
 &= \frac{\cos^2 \theta}{\sin \theta} + \sin \theta \cdot \frac{\sin \theta}{\sin \theta} \\
 &= \frac{\cos^2 \theta}{\sin \theta} + \frac{\sin^2 \theta}{\sin \theta} \\
 &= \frac{\cos^2 \theta + \sin^2 \theta}{\sin \theta} \\
 &= \frac{1}{\sin \theta} \\
 &= \csc \theta
 \end{aligned}$$

Ratio identity

LCD is $\sin \theta$

Multiply

Add fractions

Pythagorean identity

Reciprocal identity

$$\begin{aligned}
33. \quad \frac{\cos x}{1 + \sin x} + \frac{1 + \sin x}{\cos x} &= \frac{\cos x}{1 + \sin x} \cdot \frac{\cos x}{\cos x} + \frac{1 + \sin x}{\cos x} \cdot \frac{1 + \sin x}{1 + \sin x} && \text{LCD} \\
&= \frac{\cos^2 x}{\cos x(1 + \sin x)} + \frac{1 + 2\sin x + \sin^2 x}{\cos x(1 + \sin x)} && \text{Multiply fractions} \\
&= \frac{(\cos^2 x + \sin^2 x) + 1 + 2\sin x}{\cos x(1 + \sin x)} && \text{Add fractions} \\
&= \frac{1 + 1 + 2\sin x}{\cos x(1 + \sin x)} && \text{Pythagorean identity} \\
&= \frac{2 + 2\sin x}{\cos x(1 + \sin x)} && \text{Combine} \\
&= \frac{2(1 + \sin x)}{\cos x(1 + \sin x)} && \text{Factor out a 2} \\
&= \frac{2}{\cos x} && \text{Reduce} \\
&= 2\sec x && \text{Reciprocal identity}
\end{aligned}$$

$$\begin{aligned}
35. \quad \frac{1}{1 + \cos x} + \frac{1}{1 - \cos x} &= \frac{1}{1 + \cos x} \cdot \frac{1 - \cos x}{1 - \cos x} + \frac{1}{1 - \cos x} \cdot \frac{1 + \cos x}{1 + \cos x} && \text{LCD} \\
&= \frac{1 - \cos x}{1 - \cos^2 x} + \frac{1 + \cos x}{1 - \cos^2 x} && \text{Multiply} \\
&= \frac{1 - \cos x + 1 + \cos x}{1 - \cos^2 x} && \text{Add fractions} \\
&= \frac{2}{1 - \cos^2 x} && \text{Combine numerator} \\
&= \frac{2}{\sin^2 x} && \text{Pythagorean identity} \\
&= 2\csc^2 x && \text{Reciprocal identity}
\end{aligned}$$

$$\begin{aligned}
37. \quad \frac{1 - \sec x}{1 + \sec x} &= \frac{1 - \frac{1}{\cos x}}{1 + \frac{1}{\cos x}} && \text{Reciprocal identity} \\
&= \frac{\cos x \left(1 - \frac{1}{\cos x}\right)}{\cos x \left(1 + \frac{1}{\cos x}\right)} && \text{Multiply numerator and denominator by LCD} \\
&= \frac{\cos x - 1}{\cos x + 1} && \text{Distributive property}
\end{aligned}$$

$$\begin{aligned}
 39. \quad \frac{\cos t}{1 + \sin t} &= \frac{\cos t}{1 + \sin t} \cdot \frac{1 - \sin t}{1 - \sin t} && \text{Multiply numerator and denominator by } 1 - \sin t \\
 &= \frac{\cos t(1 - \sin t)}{1 - \sin^2 t} && \text{Multiply fractions} \\
 &= \frac{\cos t(1 - \sin t)}{\cos^2 t} && \text{Pythagorean identity} \\
 &= \frac{1 - \sin^2 t}{\cos t} && \text{Reduce}
 \end{aligned}$$

$$\begin{aligned}
 41. \quad \frac{1 - \sin t}{1 + \sin t} &= \frac{1 - \sin t}{1 + \sin t} \cdot \frac{1 - \sin t}{1 - \sin t} && \text{Multiply numerator and denominator by } 1 - \sin t \\
 &= \frac{(1 - \sin t)^2}{1 - \sin^2 t} && \text{Multiply fractions} \\
 &= \frac{(1 - \sin t)^2}{\cos^2 t} && \text{Pythagorean identity}
 \end{aligned}$$

$$\begin{aligned}
 43. \quad \frac{\sec \theta + 1}{\tan \theta} &= \frac{\sec \theta + 1}{\tan \theta} \cdot \frac{\sec \theta - 1}{\sec \theta - 1} && \text{Multiply numerator and denominator by } \sec \theta - 1 \\
 &= \frac{\sec^2 \theta - 1}{\tan \theta(\sec \theta - 1)} && \text{Multiply fractions} \\
 &= \frac{\tan^2 \theta}{\tan \theta(\sec \theta - 1)} && \text{Pythagorean identity} \\
 &= \frac{\tan \theta}{\sec \theta - 1} && \text{Reduce}
 \end{aligned}$$

$$\begin{aligned}
 45. \quad (\sec x - \tan x)^2 &= \left(\frac{1}{\cos x} - \frac{\sin x}{\cos x} \right)^2 && \text{Reciprocal and ratio identities} \\
 &= \left(\frac{1 - \sin x}{\cos x} \right)^2 && \text{Subtract fractions} \\
 &= \frac{(1 - \sin x)^2}{\cos^2 x} && \text{Property of exponents} \\
 &= \frac{(1 - \sin x)^2}{1 - \sin^2 x} && \text{Pythagorean identity} \\
 &= \frac{(1 - \sin x)(1 - \sin x)}{(1 - \sin x)(1 + \sin x)} && \text{Factor} \\
 &= \frac{1 - \sin x}{1 + \sin x} && \text{Reduce}
 \end{aligned}$$

$$= \frac{\sin^2 B(\cos^2 B - 1)}{\cos^2 B - 1}$$

Factor

$$= \sin^2 B$$

Reduce

55.
$$\frac{\sec^4 y - \tan^4 y}{\sec^2 y + \tan^2 y} = \frac{(\sec^2 y - \tan^2 y)(\sec^2 y + \tan^2 y)}{\sec^2 y + \tan^2 y}$$

Factor

$$= \sec^2 y - \tan^2 y$$

Reduce

$$= 1$$

Pythagorean identity

57.
$$\frac{\sin^3 A - 8}{\sin A - 2} = \frac{(\sin A - 2)(\sin^2 A + 2\sin A + 4)}{\sin A - 2}$$

Factor as difference of 2 cubes

$$= \sin^2 A + 2\sin A + 4$$

Reduce

59.
$$\frac{1 - \tan^3 t}{1 - \tan t} = \frac{(1 - \tan t)(1 + \tan t + \tan^2 t)}{1 - \tan t}$$

Factor

$$= (1 + \tan^2 t) + \tan t$$

Reduce and regroup numerator

$$= \sec^2 t + \tan t$$

Pythagorean identity

61.
$$\frac{\sin^2 x + \sin x \cos x}{\cos x - 2\cos^3 x} = \frac{\sin x(\sin x + \cos x)}{\cos x(1 - 2\cos^2 x)}$$

Factor

$$= \frac{\sin x(\sin x + \cos x)}{\cos x[(1 - \cos^2 x) - \cos^2 x]}$$

Regrouping denominator

$$= \frac{\sin x(\sin x + \cos x)}{\cos x(\sin^2 x - \cos^2 x)}$$

Pythagorean identity

$$= \frac{\sin x(\sin x + \cos x)}{\cos x(\sin x + \cos x)(\sin x - \cos x)}$$

Factor denominator

$$= \frac{\sin x}{\cos x(\sin x - \cos x)}$$

Reduce

$$= \frac{\tan x}{\sin x - \cos x}$$

Ratio identity

63.
$$(\tan \theta + \cot \theta)^2 = \tan^2 \theta + 2 \tan \theta \cot \theta + \cot^2 \theta$$

Multiply

$$= \sec^2 \theta - 1 + 2 \tan \theta \cdot \frac{1}{\tan \theta} + \csc^2 \theta - 1$$

Reciprocal and Pythagorean identities

$$= \sec^2 \theta + \csc^2 \theta + 2 - 2$$

Reduce and combine

$$= \sec^2 \theta + \csc^2 \theta$$

Combine

$$\begin{aligned}
 47. \quad \sec x + \tan x &= \frac{\sec x + \tan x}{1} \cdot \frac{\sec x - \tan x}{\sec x - \tan x} && \text{Multiply numerator and denominator by } \sec x - \tan x \\
 &= \frac{\sec^2 x - \tan^2 x}{\sec x - \tan x} && \text{Multiply fractions} \\
 &= \frac{1}{\sec x - \tan x} && \text{Pythagorean identity}
 \end{aligned}$$

$$\begin{aligned}
 49. \quad \frac{\sin x + 1}{\cos x + \cot x} &= \frac{\sin x + 1}{\cos x + \frac{\cos x}{\sin x}} && \text{Ratio identity} \\
 &= \frac{\sin x}{\sin x} \cdot \frac{(\sin x + 1)}{\left(\cos x + \frac{\cos x}{\sin x}\right)} && \text{Multiply numerator and denominator by LCD} \\
 &= \frac{\sin x(\sin x + 1)}{\sin x \cos x + \cos x} && \text{Distributive property} \\
 &= \frac{\sin x(\sin x + 1)}{\cos x(\sin x + 1)} && \text{Factor} \\
 &= \frac{\sin x}{\cos x} && \text{Reduce} \\
 &= \tan x && \text{Ratio identity}
 \end{aligned}$$

$$\begin{aligned}
 51. \quad \sin^4 A - \cos^4 A &= (\sin^2 A + \cos^2 A)(\sin^2 A - \cos^2 A) && \text{Factor} \\
 &= 1(\sin^2 A - \cos^2 A) && \text{Pythagorean identity} \\
 &= 1 - \cos^2 A - \cos^2 A && \text{Pythagorean identity} \\
 &= 1 - 2\cos^2 A && \text{Combine}
 \end{aligned}$$

$$\begin{aligned}
 53. \quad \frac{\sin^2 B - \tan^2 B}{1 - \sec^2 B} &= \frac{\sin^2 B - \frac{\sin^2 B}{\cos^2 B}}{1 - \frac{1}{\cos^2 B}} && \text{Ratio identity and reciprocal identity} \\
 &= \frac{\cos^2 B \left(\sin^2 B - \frac{\sin^2 B}{\cos^2 B} \right)}{\cos^2 B \left(1 - \frac{1}{\cos^2 B} \right)} && \text{Multiply numerator and denominator by LCD} \\
 &= \frac{\cos^2 B \sin^2 B - \sin^2 B}{\cos^2 B - 1} && \text{Distributive property}
 \end{aligned}$$

This problem is continued on the next page