EXAMPLE 1 Find the exact value for $\cos 75°$.

SOLUTION We write $75°$ as $45° + 30°$ and then apply the formula for $\cos(A + B)$.

$$\begin{aligned}\cos 75° &= \cos(45° + 30°)\\ &= \cos 45° \cos 30° - \sin 45° \sin 30°\\ &= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}\\ &= \frac{\sqrt{6} - \sqrt{2}}{4}\end{aligned}$$

NOTE If you completed the Chapter 2 Group Project, compare your value of $\cos 75°$ from the project with our result in Example 1. Convince yourself that the two values are the same.

EXAMPLE 2 Show that $\cos(x + 2\pi) = \cos x$.

SOLUTION Applying the formula for $\cos(A + B)$, we have

$$\begin{aligned}\cos(x + 2\pi) &= \cos x \cos 2\pi - \sin x \sin 2\pi\\ &= \cos x \cdot 1 - \sin x \cdot 0\\ &= \cos x\end{aligned}$$

Notice that this is not a new relationship. We already know that if two angles are coterminal, then their cosines are equal— and $x + 2\pi$ and x are coterminal. What we have done here is shown this to be true with a formula instead of the definition of cosine.

EXAMPLE 3 Write $\cos 3x \cos 2x - \sin 3x \sin 2x$ as a single cosine.

SOLUTION We apply the formula for $\cos(A + B)$ in the reverse direction from the way we applied it in the first two examples.

$$\begin{aligned}\cos 3x \cos 2x - \sin 3x \sin 2x &= \cos(3x + 2x)\\ &= \cos 5x\end{aligned}$$

Here is the derivation of the formula for $\cos(A - B)$. It involves the formula for $\cos(A + B)$ and the formulas for even and odd functions.

$\cos(A - B) = \cos[A + (-B)]$	Write $A - B$ as a sum
$= \cos A \cos(-B) - \sin A \sin(-B)$	Sum formula
$= \cos A \cos B - \sin A(-\sin B)$	Cosine is an even function, sine is odd
$= \cos A \cos B + \sin A \sin B$	

The only difference in the formulas for the expansion of $\cos(A + B)$ and $\cos(A - B)$ is the sign between the two terms. Here are both formulas again.

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$
$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

Again, both formulas are important and should be memorized.

EXAMPLE 4 Show that $\cos(90° - A) = \sin A$.

SOLUTION We will need this formula when we derive the formula for $\sin(A + B)$.

$$\begin{aligned}\cos(90° - A) &= \cos 90° \cos A + \sin 90° \sin A \\ &= 0 \cdot \cos A + 1 \cdot \sin A \\ &= \sin A\end{aligned}$$

Note that the formula we just derived is not a new formula. The angles $90° - A$ and A are complementary angles, and we already know the sine of an angle is always equal to the cosine of its complement. We could also state it this way:

$$\sin(90° - A) = \cos A$$

We can use this information to derive the formula for $\sin(A + B)$. To understand this derivation, you must recognize that $A + B$ and $90° - (A + B)$ are complementary angles.

$\sin(A + B) = \cos[90° - (A + B)]$	The sine of an angle is the cosine of its complement
$= \cos[90° - A - B]$	Remove parentheses
$= \cos[(90° - A) - B]$	Regroup within brackets

Now we expand using the formula for the cosine of a difference.

$$\begin{aligned}&= \cos(90° - A)\cos B + \sin(90° - A)\sin B \\ &= \sin A \cos B + \cos A \sin B\end{aligned}$$

This gives us an expansion formula for $\sin(A + B)$.

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

This is the formula for the sine of a sum. To find the formula for $\sin(A - B)$, we write $A - B$ as $A + (-B)$ and proceed as follows:

$$\begin{aligned}\sin(A - B) &= \sin[A + (-B)] \\ &= \sin A \cos(-B) + \cos A \sin(-B) \\ &= \sin A \cos B - \cos A \sin B\end{aligned}$$

This gives us the formula for the sine of a difference.

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

EXAMPLE 5 Graph $y = 4 \sin 5x \cos 3x - 4 \cos 5x \sin 3x$ from $x = 0$ to $x = 2\pi$.

SOLUTION To write the equation in the form $y = A \sin Bx$, we factor 4 from each term on the right and then apply the formula for $\sin (A - B)$ to the remaining expression to write it as a single trigonometric function.

$$\begin{aligned} y &= 4 \sin 5x \cos 3x - 4 \cos 5x \sin 3x \\ &= 4 (\sin 5x \cos 3x - \cos 5x \sin 3x) \\ &= 4 \sin (5x - 3x) \\ &= 4 \sin 2x \end{aligned}$$

The graph of $y = 4 \sin 2x$ will have an amplitude of 4 and a period of $2\pi/2 = \pi$. The graph is shown in Figure 2.

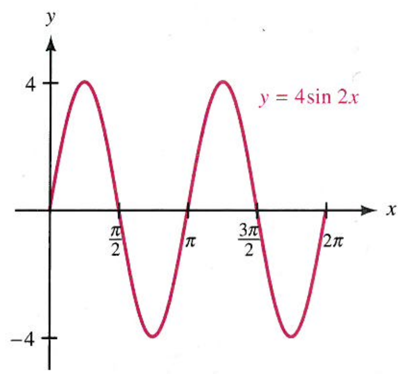


Figure 2

EXAMPLE 6 Find the exact value of $\sin \frac{\pi}{12}$.

SOLUTION We have to write $\pi/12$ in terms of two numbers the exact values of which are known. The numbers $\pi/3$ and $\pi/4$ will work since their difference is $\pi/12$.

$$\begin{aligned} \sin \frac{\pi}{12} &= \sin \left(\frac{\pi}{3} - \frac{\pi}{4} \right) \\ &= \sin \frac{\pi}{3} \cos \frac{\pi}{4} - \cos \frac{\pi}{3} \sin \frac{\pi}{4} \\ &= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{6} - \sqrt{2}}{4} \end{aligned}$$

This is the same answer we obtained in Example 1 when we found the exact value of $\cos 75°$. It should be, though, because $\pi/12 = 15°$, which is the complement of $75°$, and the cosine of an angle is equal to the sine of its complement.

EXAMPLE 7 If $\sin A = \frac{3}{5}$ with A in QI and $\cos B = -\frac{5}{13}$ with B in QIII, find $\sin(A + B)$, $\cos(A + B)$, and $\tan(A + B)$.

SOLUTION We have $\sin A$ and $\cos B$. We need to find $\cos A$ and $\sin B$ before we can apply any of our formulas. Some equivalent forms of our first Pythagorean identity will help here.

If $\sin A = \frac{3}{5}$ with A in QI, then

$$\cos A = \sqrt{1 - \sin^2 A} = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \frac{4}{5}$$

If $\cos B = -\frac{5}{13}$ with B in QIII, then

$$\sin B = -\sqrt{1 - \left(-\frac{5}{13}\right)^2} = -\frac{12}{13}$$

We have

$$\sin A = \frac{3}{5} \qquad \sin B = -\frac{12}{13}$$

$$\cos A = \frac{4}{5} \qquad \cos B = -\frac{5}{13}$$

Therefore,

$$\begin{aligned}\sin(A + B) &= \sin A \cos B + \cos A \sin B \\ &= \frac{3}{5}\left(-\frac{5}{13}\right) + \frac{4}{5}\left(-\frac{12}{13}\right) \\ &= -\frac{63}{65}\end{aligned}$$

$$\begin{aligned}\cos(A + B) &= \cos A \cos B - \sin A \text{ in } B \\ &= \frac{4}{5}\left(-\frac{5}{13}\right) - \frac{3}{5}\left(-\frac{12}{13}\right) \\ &= \frac{16}{65}\end{aligned}$$

$$\begin{aligned}\tan(A + B) &= \frac{\sin(A + B)}{\cos(A + B)} \\ &= \frac{-63/65}{16/65} \\ &= -\frac{63}{16}\end{aligned}$$

Notice also that $A + B$ must terminate in quadrant IV because

$$\sin(A + B) < 0 \qquad \text{and} \qquad \cos(A + B) > 0$$

While working through the last part of Example 7, you may have wondered if there is a separate formula for tan $(A + B)$. (More likely, you are hoping there isn't.) There is, and it is derived from the formulas we already have.

$$\tan(A + B) = \frac{\sin(A + B)}{\cos(A + B)}$$

$$= \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B}$$

To be able to write this last line in terms of tangents only, we must divide numerator and denominator by $\cos A \cos B$.

$$= \frac{\dfrac{\sin A \cos B}{\cos A \cos B} + \dfrac{\cos A \sin B}{\cos A \cos B}}{\dfrac{\cos A \cos B}{\cos A \cos B} - \dfrac{\sin A \sin B}{\cos A \cos B}}$$

$$= \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

The formula for tan $(A + B)$ is

$$\boxed{\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}}$$

Since tangent is an odd function, the formula for tan $(A - B)$ will look like this:

$$\boxed{\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}}$$

EXAMPLE 8 If $\sin A = \frac{3}{5}$ with A in QI and $\cos B = -\frac{5}{13}$ with B in QIII, find tan $(A + B)$ by using the formula

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

SOLUTION The angles A and B as given here are the same ones used previously in Example 7. Looking over Example 7 again, we find that

$$\tan A = \frac{3}{4} \qquad \text{and} \qquad \tan B = \frac{12}{5}$$

Therefore,

$$\begin{aligned}\tan(A+B) &= \frac{\tan A + \tan B}{1 - \tan A \tan B} \\ &= \frac{\frac{3}{4} + \frac{12}{5}}{1 - \frac{3}{4} \cdot \frac{12}{5}} \\ &= \frac{\frac{15}{20} + \frac{48}{20}}{1 - \frac{9}{5}} \\ &= \frac{\frac{63}{20}}{-\frac{4}{5}} \\ &= -\frac{63}{16}\end{aligned}$$

which is the same result we obtained previously.

GETTING READY FOR CLASS

After reading through the preceding section, respond in your own words and in complete sentences.

a. Why is it necessary to have sum and difference formulas for sine, cosine, and tangent?

b. Write both the sum and the difference formulas for cosine.

c. Write both the sum and the difference formulas for sine.

d. Write both the sum and the difference formulas for tangent.

PROBLEM SET 5.2

Find exact values for each of the following:

1. $\sin 15°$

2. $\sin 75°$

3. $\tan 15°$

4. $\tan 75°$

5. $\sin \frac{7\pi}{12}$

6. $\cos \frac{7\pi}{12}$

7. $\cos 105°$

8. $\sin 105°$

Show that each of the following is true:

9. $\sin(x + 2\pi) = \sin x$

10. $\cos(x - 2\pi) = \cos x$

11. $\cos\left(x - \frac{\pi}{2}\right) = \sin x$

12. $\sin\left(x - \frac{\pi}{2}\right) = -\cos x$

13. $\cos(180° - \theta) = -\cos\theta$

14. $\sin(180° - \theta) = \sin\theta$

15. $\sin(90° + \theta) = \cos\theta$

16. $\cos(90° + \theta) = -\sin\theta$

17. $\tan\left(x + \frac{\pi}{4}\right) = \frac{1 + \tan x}{1 - \tan x}$

18. $\tan\left(x - \frac{\pi}{4}\right) = \frac{\tan x - 1}{\tan x + 1}$

19. $\sin\left(\frac{3\pi}{2} - x\right) = -\cos x$

20. $\cos\left(x - \frac{3\pi}{2}\right) = -\sin x$

Write each expression as a single trigonometric function.

21. $\sin 3x \cos 2x + \cos 3x \sin 2x$

22. $\cos 3x \cos 2x + \sin 3x \sin 2x$

23. $\cos 5x \cos x - \sin 5x \sin x$

24. $\sin 8x \cos x - \cos 8x \sin x$

25. $\cos 15° \cos 75° - \sin 15° \sin 75°$

26. $\cos 15° \cos 75° + \sin 15° \sin 75°$

Graph each of the following from $x = 0$ to $x = 2\pi$.

27. $y = \sin 5x \cos 3x - \cos 5x \sin 3x$

28. $y = \sin x \cos 2x + \cos x \sin 2x$

29. $y = 3 \cos 7x \cos 5x + 3 \sin 7x \sin 5x$

30. $y = 2 \cos 4x \cos x + 2 \sin 4x \sin x$

31. Graph one complete cycle of $y = \sin x \cos \frac{\pi}{4} + \cos x \sin \frac{\pi}{4}$ by first rewriting the right side in the form $\sin(A + B)$.

32. Graph one complete cycle of $y = \sin x \cos \frac{\pi}{6} - \cos x \sin \frac{\pi}{6}$ by first rewriting the right side in the form $\sin(A - B)$.

33. Graph one complete cycle of $y = 2(\sin x \cos \frac{\pi}{3} + \cos x \sin \frac{\pi}{3})$ by first rewriting the right side in the form $2\sin(A + B)$.

34. Graph one complete cycle of $y = 2(\sin x \cos \frac{\pi}{3} - \cos x \sin \frac{\pi}{3})$ by first rewriting the right side in the form $2\sin(A - B)$.

35. Let $\sin A = \frac{3}{5}$ with A in QII and $\sin B = -\frac{5}{13}$ with B in QIII. Find $\sin(A + B)$, $\cos(A + B)$, and $\tan(A + B)$. In what quadrant does $A + B$ terminate?

36. Let $\cos A = -\frac{5}{13}$ with A in QII and $\sin B = \frac{3}{5}$ with B in QI. Find $\sin(A - B)$, $\cos(A - B)$, and $\tan(A - B)$. In what quadrant does $A - B$ terminate?

37. If $\sin A = 1/\sqrt{5}$ with A in QI and $\tan B = \frac{3}{4}$ with B in QI, find $\tan(A + B)$ and $\cot(A + B)$. In what quadrant does $A + B$ terminate?

38. If $\sec A = \sqrt{5}$ with A in QI and $\sec B = \sqrt{10}$ with B in QI, find $\sec(A + B)$. [First find $\cos(A + B)$.]

39. If $\tan(A + B) = 3$ and $\tan B = \frac{1}{2}$, find $\tan A$.

40. If $\tan(A + B) = 2$ and $\tan B = \frac{1}{3}$, find $\tan A$.

41. Write a formula for $\sin 2x$ by writing $\sin 2x$ as $\sin(x + x)$ and using the formula for the sine of a sum.

42. Write a formula for $\cos 2x$ by writing $\cos 2x$ as $\cos(x + x)$ and using the formula for the cosine of a sum.

Prove each identity.

43. $\sin(90° + x) + \sin(90° - x) = 2\cos x$

44. $\sin(90° + x) - \sin(90° - x) = 0$

45. $\cos(x - 90°) - \cos(x + 90°) = 2\sin x$

46. $\cos(x + 90°) + \cos(x - 90°) = 0$

47. $\sin\left(\frac{\pi}{6}+x\right)+\sin\left(\frac{\pi}{6}-x\right)=\cos x$

48. $\cos\left(\frac{\pi}{3}+x\right)+\cos\left(\frac{\pi}{3}-x\right)=\cos x$

49. $\cos\left(x+\frac{\pi}{4}\right)+\cos\left(x-\frac{\pi}{4}\right)=\sqrt{2}\cos x$

50. $\sin\left(\frac{\pi}{4}+x\right)+\sin\left(\frac{\pi}{4}-x\right)=\sqrt{2}\cos x$

51. $\sin\left(\frac{3\pi}{2}+x\right)+\sin\left(\frac{3\pi}{2}-x\right)=-2\cos x$

52. $\cos\left(x+\frac{3\pi}{2}\right)+\cos\left(x-\frac{3\pi}{2}\right)=0$

53. $\sin(A+B)+\sin(A-B)=2\sin A\cos B$

54. $\cos(A+B)+\cos(A-B)=2\cos A\cos B$

55. $\frac{\sin(A-B)}{\cos A\cos B}=\tan A-\tan B$

56. $\frac{\cos(A+B)}{\sin A\cos B}=\cot A-\tan B$

57. $\sec(A+B)=\frac{\cos(A-B)}{\cos^2 A-\sin^2 B}$

58. $\sec(A-B)=\frac{\cos(A+B)}{\cos^2 A-\sin^2 B}$

Use your graphing calculator to determine if each equation appears to be an identity by graphing the left expression and right expression together. If so, then prove the identity.

59. $\sin x=\cos\left(\frac{\pi}{2}-x\right)$

60. $\cos x=\sin\left(\frac{\pi}{2}-x\right)$

61. $-\cos x=\sin\left(\frac{\pi}{2}+x\right)$

62. $\sin x=\cos\left(\frac{\pi}{2}+x\right)$

63. $-\sin x=\cos\left(\frac{\pi}{2}+x\right)$

64. $-\cos x=\cos(\pi-x)$

REVIEW PROBLEMS

The problems that follow review material we covered in Section 4.2. Graph one complete cycle of each of the following:

65. $y=4\sin 2x$

66. $y=2\sin 4x$

67. $y=3\sin\frac{1}{2}x$

68. $y=5\sin\frac{1}{3}x$

69. $y=2\cos\pi x$

70. $y=\cos 2\pi x$

71. $y=\csc 3x$

72. $y=\sec 3x$

73. $y=\frac{1}{2}\cos 3x$

74. $y=\frac{1}{2}\sin 3x$

75. $y=\frac{1}{2}\sin\frac{\pi}{2}x$

76. $y=2\sin\frac{\pi}{2}x$

PROBLEM SET 5.2

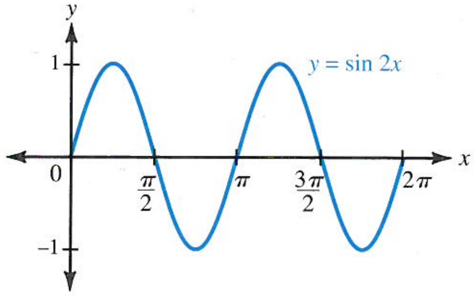
1. $\dfrac{\sqrt{6}-\sqrt{2}}{4}$ **3.** $\dfrac{\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}$ or $\dfrac{\sqrt{3}-1}{\sqrt{3}+1}$ **5.** $\dfrac{\sqrt{6}+\sqrt{2}}{4}$ **7.** $\dfrac{\sqrt{2}-\sqrt{6}}{4}$

9. $\sin(x+2\pi) = \sin x \cos 2\pi + \cos x \sin 2\pi$
$= \sin x(1) + \cos x(0)$
$= \sin x$

For problems 11–19, proceed as in Problem 9. Expand the left side and simplify. Solutions are given in the Solutions Manual.

21. $\sin 5x$ **23.** $\cos 6x$ **25.** $\cos 90° = 0$

27.



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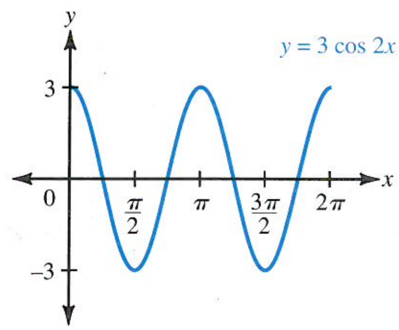

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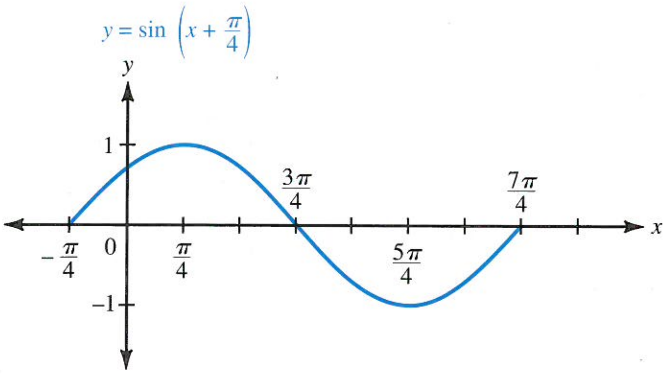

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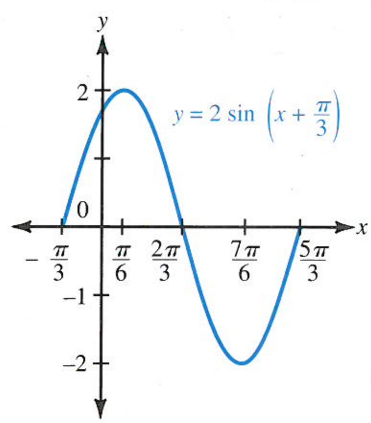


35. $-\dfrac{16}{65}, \dfrac{63}{65}, -\dfrac{16}{63}$, QIV **37.** $2, \dfrac{1}{2}$, QI **39.** 1

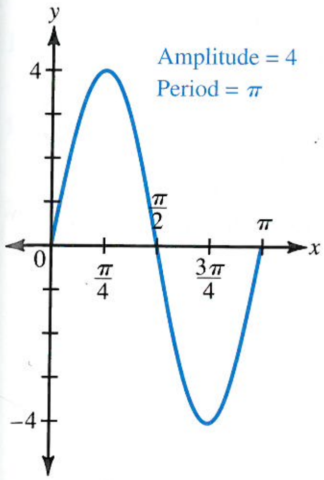
41. $\sin 2x = 2 \sin x \cos x$

For Problems 43–57, solutions are given in the Solutions Manual.

NOTE For Problems 59–63, when the equation is an identity, the proof is given in the Solutions Manual.

59. Is an identity **61.** Not an identity **63.** Is an identity

65.



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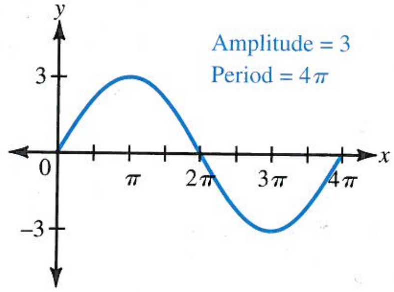

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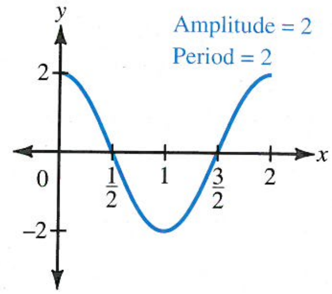
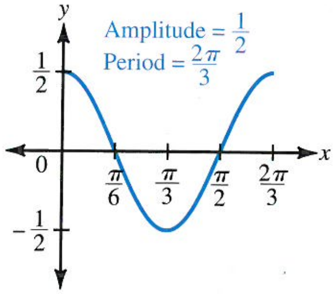
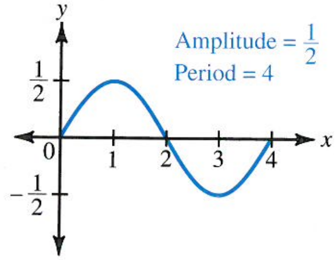


71. See the solution to Problem 9 in Problem Set 4.2.

73.



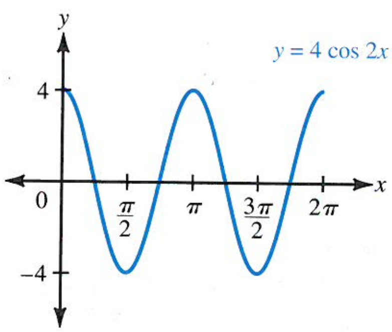
75.



PROBLEM SET 5.3

1. $\frac{24}{25}$ **3.** $\frac{24}{7}$ **5.** $-\frac{4}{5}$ **7.** $\frac{4}{3}$ **9.** $\frac{120}{169}$ **11.** $\frac{169}{120}$ **13.** $\frac{3}{5}$ **15.** $\frac{5}{3}$

17. $y = 4 - 8\sin^2 x$
$= 4(1 - 2\sin^2 x)$
$= 4\cos 2x$



PROBLEM SET 6.3

Find all solutions if $0° \le \theta < 360°$. Verify your answer graphically.

1. $\sin 2\theta = \sqrt{3}/2$
2. $\sin 2\theta = -\sqrt{3}/2$
3. $\tan 2\theta = -1$
4. $\cot 2\theta = 1$
5. $\cos 3\theta = -1$
6. $\sin 3\theta = -1$

Find all solutions if $0 \le x < 2\pi$. Use exact values only. Verify your answer graphically.

7. $\sin 2x = 1/\sqrt{2}$
8. $\cos 2x = 1/\sqrt{2}$
9. $\sec 3x = -1$
10. $\csc 3x = 1$
11. $\tan 2x = \sqrt{3}$
12. $\tan 2x = -\sqrt{3}$

Find all degree solutions for each of the following:

13. $\sin 2\theta = 1/2$
14. $\sin 2\theta = -\sqrt{3}/2$
15. $\cos 3\theta = 0$
16. $\cos 3\theta = -1$
17. $\sin 10\theta = \sqrt{3}/2$
18. $\cos 8\theta = 1/2$

Use your graphing calculator to find all degree solutions in the interval $0° \le x < 360°$ for each of the following equations.

19. $\sin 2x = -\dfrac{1}{\sqrt{2}}$
20. $\cos 2x = -\dfrac{1}{2}$
21. $\cos 3x = \dfrac{1}{2}$
22. $\sin 3x = \dfrac{\sqrt{3}}{2}$
23. $\tan 2x = \dfrac{1}{\sqrt{3}}$
24. $\tan 2x = 1$

Find all solutions if $0 \le x < 2\pi$. Use exact values only.

25. $\sin 2x \cos x + \cos 2x \sin x = 1/2$
26. $\sin 2x \cos x + \cos 2x \sin x = -1/2$
27. $\cos 2x \cos x - \sin 2x \sin x = -\sqrt{3}/2$
28. $\cos 2x \cos x - \sin 2x \sin x = 1/\sqrt{2}$

Find all solutions in radians using exact values only.

29. $\sin 3x \cos 2x + \cos 3x \sin 2x = 1$
30. $\sin 2x \cos 3x + \cos 2x \sin 3x = -1$
31. $\sin^2 4x = 1$
32. $\cos^2 4x = 1$
33. $\cos^3 5x = -1$
34. $\sin^3 5x = -1$

Find all degree solutions.

35. $2\sin^2 3\theta + \sin 3\theta - 1 = 0$
36. $2\sin^2 3\theta + 3\sin 3\theta + 1 = 0$
37. $2\cos^2 2\theta + 3\cos 2\theta + 1 = 0$
38. $2\cos^2 2\theta - \cos 2\theta - 1 = 0$
39. $\tan^2 3\theta = 3$
40. $\cot^2 3\theta = 1$

Find all solutions if $0° \le \theta < 360°$.

41. $\cos\theta - \sin\theta = 1$
42. $\sin\theta - \cos\theta = 1$
43. $\sin\theta + \cos\theta = -1$
44. $\cos\theta - \sin\theta = -1$
45. $\sin^2 2\theta - 4\sin 2\theta - 1 = 0$
46. $\cos^2 3\theta - 6\cos 3\theta + 4 = 0$
47. $4\cos^2 3\theta - 8\cos 3\theta + 1 = 0$
48. $2\sin^2 2\theta - 6\sin 2\theta + 3 = 0$
49. $2\cos^2 4\theta + 2\sin 4\theta = 1$
50. $2\sin^2 4\theta - 2\cos 4\theta = 1$

37. 17.0°, 163.0° **39.** 30° + 360°k, 150° + 360°k **41.** $\frac{\pi}{3} + 2k\pi$, $\frac{2\pi}{3} + 2k\pi$ **43.** $\frac{3\pi}{2} + 2k\pi$

45. 48.6° + 360°k, 131.4° + 360°k **47.** 40° + 180°k, 190° + 180°k **49.** 120°k, 40° + 120°k

51. 35° + 90°k, 65° + 90°k **53.** 42° + 72°k, 60° + 72°k

For Problems 55–73, see the answer for the corresponding problem.

75. $h = -16t^2 + 750t$ **77.** 1,436 ft **79.** 15.7° **81.** $\sin 2A = 2 \sin A \cos A$ **83.** $\cos 2A = 2\cos^2 A - 1$

85. $\frac{1}{\sqrt{2}} \sin\theta + \frac{1}{\sqrt{2}} \cos\theta$ **87.** $\frac{\sqrt{6}+\sqrt{2}}{4}$ **89.** See the Solutions Manual.

PROBLEM SET 6.2

1. 30°, 330° **3.** 225°, 315° **5.** 45°, 135°, 225°, 315° **7.** 30°, 150° **9.** 30°, 90°, 150°, 270°

11. 60°, 180°, 300° **13.** $\frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$ **15.** $0, \frac{2\pi}{3}, \frac{4\pi}{3}$ **17.** $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$ **19.** $\frac{\pi}{3}, \frac{5\pi}{3}$

21. $\frac{2\pi}{3}, \frac{4\pi}{3}$ **23.** $\frac{\pi}{4}$ **25.** 30°, 90° **27.** 60°, 180° **29.** 60°, 300° **31.** 120°, 180° **33.** 210°, 330°

35. 36.9°, 48.2°, 311.8°, 323.1° **37.** 36.9°, 143.1°, 216.9°, 323.1° **39.** 225° + 360°k, 315° + 360°k

41. $\frac{\pi}{4} + 2k\pi$ **43.** 120° + 360°k, 180° + 360°k **45.** See the Solutions Manual. **47.** 68.5°, 291.5°

49. 218.2°, 321.8° **51.** 73.0°, 287.0° **53.** 0.3630, 2.1351 **55.** 3.4492, 5.9756 **57.** 0.3166, 1.9917

59. $\sqrt{\frac{3-\sqrt{5}}{6}}$ **61.** $\sqrt{\frac{6}{3-\sqrt{5}}}$ **63.** $\sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}}$ or $\frac{3-\sqrt{5}}{2}$

65. See the solution to Problem 65 in Problem Set 5.3. **67.** $\frac{\sqrt{2-\sqrt{2}}}{2}$

PROBLEM SET 6.3

1. 30°, 60°, 210°, 240° **3.** 67.5°, 157.5°, 247.5°, 337.5° **5.** 60°, 180°, 300° **7.** $\frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}$

9. $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$ **11.** $\frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3}$ **13.** 15° + 180°k, 75° + 180°k **15.** 30° + 120°k, 90° + 120°k

17. 6° + 36°k, 12° + 36°k **19.** 112.5°, 157.5°, 292.5°, 337.5° **21.** 20°, 100°, 140°, 220°, 260°, 340°

23. 15°, 105°, 195°, 285° **25.** $\frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \frac{25\pi}{18}, \frac{29\pi}{18}$ **27.** $\frac{5\pi}{18}, \frac{7\pi}{18}, \frac{17\pi}{18}, \frac{19\pi}{18}, \frac{29\pi}{18}, \frac{31\pi}{18}$

29. $\frac{\pi}{10} + \frac{2k\pi}{5}$ **31.** $\frac{\pi}{8} + \frac{k\pi}{2}, \frac{3\pi}{8} + \frac{k\pi}{2}$ **33.** $\frac{\pi}{5} + \frac{2k\pi}{5}$ **35.** 10° + 120°k, 50° + 120°k, 90° + 120°k

37. 60° + 180°k, 90° + 180°k, 120° + 180°k **39.** 20° + 60°k, 40° + 60°k **41.** 0°, 270° **43.** 180°, 270°

45. 96.8°, 173.2°, 276.8°, 353.2° **47.** 27.4°, 92.6°, 147.4°, 212.6°, 267.4°, 332.6°

49. 50.4°, 84.6°, 140.4°, 174.6°, 230.4°, 264.6°, 320.4°, 354.6°

51. 4.0 min and 16.0 min **53.** 6 **55.** $\frac{1}{4}$ second (and every second after that) **57.** $\frac{1}{12}$

For Problems 59–63, see the Solutions Manual.

65. $-\frac{4\sqrt{2}}{9}$ **67.** $\sqrt{\frac{3-2\sqrt{2}}{6}}$ **69.** $\frac{4-6\sqrt{2}}{15}$ **71.** $\frac{15}{4-6\sqrt{2}}$