

## PROBLEM SET 5.2

Find exact values for each of the following:

1.  $\sin 15^\circ$
2.  $\sin 75^\circ$
3.  $\tan 15^\circ$
4.  $\tan 75^\circ$
5.  $\sin \frac{7\pi}{12}$
6.  $\cos \frac{7\pi}{12}$
7.  $\cos 105^\circ$
8.  $\sin 105^\circ$

Show that each of the following is true:

9.  $\sin(x + 2\pi) = \sin x$
10.  $\cos(x - 2\pi) = \cos x$
11.  $\cos\left(x - \frac{\pi}{2}\right) = \sin x$
12.  $\sin\left(x - \frac{\pi}{2}\right) = -\cos x$
13.  $\cos(180^\circ - \theta) = -\cos \theta$
14.  $\sin(180^\circ - \theta) = \sin \theta$
15.  $\sin(90^\circ + \theta) = \cos \theta$
16.  $\cos(90^\circ + \theta) = -\sin \theta$
17.  $\tan\left(x + \frac{\pi}{4}\right) = \frac{1 + \tan x}{1 - \tan x}$
18.  $\tan\left(x - \frac{\pi}{4}\right) = \frac{\tan x - 1}{\tan x + 1}$
19.  $\sin\left(\frac{3\pi}{2} - x\right) = -\cos x$
20.  $\cos\left(x - \frac{3\pi}{2}\right) = -\sin x$

Write each expression as a single trigonometric function.

21.  $\sin 3x \cos 2x + \cos 3x \sin 2x$
22.  $\cos 3x \cos 2x + \sin 3x \sin 2x$
23.  $\cos 5x \cos x - \sin 5x \sin x$
24.  $\sin 8x \cos x - \cos 8x \sin x$
25.  $\cos 15^\circ \cos 75^\circ - \sin 15^\circ \sin 75^\circ$
26.  $\cos 15^\circ \cos 75^\circ + \sin 15^\circ \sin 75^\circ$
35. Let  $\sin A = \frac{3}{5}$  with  $A$  in QII and  $\sin B = -\frac{5}{13}$  with  $B$  in QIII. Find  $\sin(A + B)$ ,  $\cos(A + B)$ , and  $\tan(A + B)$ . In what quadrant does  $A + B$  terminate?
36. Let  $\cos A = -\frac{5}{13}$  with  $A$  in QII and  $\sin B = \frac{3}{5}$  with  $B$  in QI. Find  $\sin(A - B)$ ,  $\cos(A - B)$ , and  $\tan(A - B)$ . In what quadrant does  $A - B$  terminate?
37. If  $\sin A = 1/\sqrt{5}$  with  $A$  in QI and  $\tan B = \frac{3}{4}$  with  $B$  in QI, find  $\tan(A + B)$  and  $\cot(A + B)$ . In what quadrant does  $A + B$  terminate?
38. If  $\sec A = \sqrt{5}$  with  $A$  in QI and  $\sec B = \sqrt{10}$  with  $B$  in QI, find  $\sec(A + B)$ . [First find  $\cos(A + B)$ .]
39. If  $\tan(A + B) = 3$  and  $\tan B = \frac{1}{2}$ , find  $\tan A$ .
40. If  $\tan(A + B) = 2$  and  $\tan B = \frac{1}{3}$ , find  $\tan A$ .
41. Write a formula for  $\sin 2x$  by writing  $\sin 2x$  as  $\sin(x + x)$  and using the formula for the sine of a sum.
42. Write a formula for  $\cos 2x$  by writing  $\cos 2x$  as  $\cos(x + x)$  and using the formula for the cosine of a sum.

Prove each identity.

43.  $\sin(90^\circ + x) + \sin(90^\circ - x) = 2 \cos x$
44.  $\sin(90^\circ + x) - \sin(90^\circ - x) = 0$
45.  $\cos(x - 90^\circ) - \cos(x + 90^\circ) = 2 \sin x$
46.  $\cos(x + 90^\circ) + \cos(x - 90^\circ) = 0$

## Sum and Difference Formulas [5.2]

$$\begin{aligned} \sin(A + B) &= \sin A \cos B + \cos A \sin B \\ \sin(A - B) &= \sin A \cos B - \cos A \sin B \\ \cos(A + B) &= \cos A \cos B - \sin A \sin B \\ \cos(A - B) &= \cos A \cos B + \sin A \sin B \end{aligned}$$

$$\begin{aligned} \tan(A + B) &= \frac{\tan A + \tan B}{1 - \tan A \tan B} \\ \tan(A - B) &= \frac{\tan A - \tan B}{1 + \tan A \tan B} \end{aligned}$$

47.  $\sin\left(\frac{\pi}{6} + x\right) + \sin\left(\frac{\pi}{6} - x\right) = \cos x$
48.  $\cos\left(\frac{\pi}{3} + x\right) + \cos\left(\frac{\pi}{3} - x\right) = \cos x$
49.  $\cos\left(x + \frac{\pi}{4}\right) + \cos\left(x - \frac{\pi}{4}\right) = \sqrt{2} \cos x$
50.  $\sin\left(\frac{\pi}{4} + x\right) + \sin\left(\frac{\pi}{4} - x\right) = \sqrt{2} \cos x$
51.  $\sin\left(\frac{3\pi}{2} + x\right) + \sin\left(\frac{3\pi}{2} - x\right) = -2 \cos x$
52.  $\cos\left(x + \frac{3\pi}{2}\right) + \cos\left(x - \frac{3\pi}{2}\right) = 0$

53.  $\sin(A + B) + \sin(A - B) = 2 \sin A \cos B$
54.  $\cos(A + B) + \cos(A - B) = 2 \cos A \cos B$
55.  $\frac{\sin(A - B)}{\cos A \cos B} = \tan A - \tan B$
56.  $\frac{\cos(A + B)}{\sin A \cos B} = \cot A - \tan B$

$$57. \sec(A + B) = \frac{\cos(A - B)}{\cos^2 A - \sin^2 B} \qquad 58. \sec(A - B) = \frac{\cos(A + B)}{\cos^2 A - \sin^2 B}$$

Section 6.3

Trigonometric Equations Involving Multiple Angles

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## PROBLEM SET 6.3

Find all solutions if  $0 \leq x < 2\pi$ . Use exact values only.

25.  $\sin 2x \cos x + \cos 2x \sin x = 1/2$
26.  $\sin 2x \cos x + \cos 2x \sin x = -1/2$
27.  $\cos 2x \cos x - \sin 2x \sin x = -\sqrt{3}/2$
28.  $\cos 2x \cos x - \sin 2x \sin x = 1/\sqrt{2}$

Find all solutions in radians using exact values only.

29.  $\sin 3x \cos 2x + \cos 3x \sin 2x = 1$
30.  $\sin 2x \cos 3x + \cos 2x \sin 3x = -1$