81. $\sin(30^\circ + 60^\circ) = \sin 90^\circ = 1$ and $\sin 30^\circ + \sin 60^\circ = \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$

Since $1 \neq \frac{1+\sqrt{3}}{2}$, this statement is false.

83.
$$\cos A = \sqrt{1-\sin^2 A} = \sqrt{1-\left(\frac{3}{5}\right)^2} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

$$\tan A = \frac{\sin A}{\cos A} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$$

85. $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

87. $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

89.
$$\frac{\pi}{12} = \frac{\pi}{12} \cdot \frac{180}{\pi} = 15^\circ$$

91.
$$\frac{7\pi}{12} = \frac{7\pi}{12} \cdot \frac{180}{\pi} = 105^\circ$$

Problem Set 5.2

1.
$$\sin 15^\circ = \sin(45^\circ - 30^\circ) = \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ = \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

3.
$$\tan 15^\circ = \tan(45^\circ - 30^\circ) = \frac{\tan 45^\circ - \tan 30^\circ}{1+\tan 45^\circ \tan 30^\circ} = \frac{1-\frac{1}{\sqrt{3}}}{1+1\left(\frac{1}{\sqrt{3}}\right)} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

Multiply numerator and denominator by $\sqrt{3}$

5. $\sin\frac{7\pi}{12}=\sin\left(\frac{3\pi}{12}+\frac{4\pi}{12}\right)$

$=\sin\left(\frac{\pi}{4}+\frac{\pi}{3}\right)$

$=\sin\frac{\pi}{4}\cos\frac{\pi}{3}+\cos\frac{\pi}{4}\sin\frac{\pi}{3}$

$=\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)+\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$=\frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{4}$

$=\frac{\sqrt{2}+\sqrt{6}}{4}$

7. $\cos 105^\circ=\cos\left(60^\circ+45^\circ\right)$

$=\cos 60^\circ\cos 45^\circ-\sin 60^\circ\sin 45^\circ$

$=\frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)-\frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right)$

$=\frac{\sqrt{2}}{4}-\frac{\sqrt{6}}{4}$

$=\frac{\sqrt{2}-\sqrt{6}}{4}$

9. $\sin(x+2\pi)=\sin x\cos 2\pi+\cos x\sin 2\pi$

$=(\sin x)(1)+(\cos x)(0)$

$=\sin x$

11. $\cos\left(x-\frac{\pi}{2}\right)=\cos x\cos\frac{\pi}{2}+\sin x\sin\frac{\pi}{2}$

$=\cos x(0)+\sin x(1)$

$=\sin x$

13. $\cos\left(180^\circ-\theta\right)=\cos 180^\circ\cos\theta+\sin 180^\circ\sin\theta$

$=-1(\cos\theta)+0(\sin\theta)$

$=-\cos\theta$

15. $\sin\left(90^\circ+\theta\right)=\sin 90^\circ\cos\theta+\cos 90^\circ\sin\theta$

$=1(\cos\theta)+0(\sin\theta)$

$=\cos\theta$

17. $\tan\left(x+\frac{\pi}{4}\right)=\frac{\tan x+\tan\frac{\pi}{4}}{1-\tan x\tan\frac{\pi}{4}}$

$=\frac{\tan x+1}{1-(\tan x)(1)}$

$=\frac{1+\tan x}{1-\tan x}$

19. $\sin\left(\frac{3\pi}{2}-x\right)=\sin\frac{3\pi}{2}\cos x-\cos\frac{3\pi}{2}\sin x$

$=-1(\cos x)-0(\sin x)$

$=-\cos x$

21. $\sin 3x\cos 2x+\cos 3x\sin 2x=\sin(3x+2x)=\sin 5x$

23. $\cos 5x\cos x-\sin 5x\sin x=\cos(5x+x)=\cos 6x$

25. $\cos 15^\circ \cos 75^\circ - \sin 15^\circ \sin 75^\circ = \cos(15^\circ + 75^\circ) = \cos 90^\circ = 0$

27. $y = \sin 5x \cos 3x - \cos 5x \sin 3x$

$y = \sin(5x - 3x) = \sin 2x$

The graph is a sine curve with amplitude of 1 and period of $\frac{2\pi}{2} = \pi$.

29. $y = 3\cos 7x \cos 5x + 3\sin 7x \sin 5x$

$= 3[\cos 7x \cos 5x + \sin 7x \sin 5x]$

$y = 3\cos(7x - 5x) = 3\cos 2x$

The graph is a cosine curve with an amplitude of 3 and a period of $\frac{2\pi}{2} = \pi$.

31. $y = \sin x \cos\frac{\pi}{4} + \cos x \sin\frac{\pi}{4}$

$y = \sin\left(x + \frac{\pi}{4}\right)$

The graph is a sine curve with: Amplitude $= 1$ Period $= 2\pi$ Phase shift $= -\frac{\pi}{4}$

Spacing $= \frac{1}{4}(2\pi) = \frac{\pi}{2}$

$\text{a} = -\frac{\pi}{4}$

$\text{b} = -\frac{\pi}{4} + \frac{\pi}{2} = \frac{\pi}{4}$

$\text{c} = \frac{\pi}{4} + \frac{\pi}{2} = \frac{3\pi}{4}$

$\text{d} = \frac{3\pi}{4} + \frac{\pi}{2} = \frac{5\pi}{4}$

$\text{e} = \frac{5\pi}{4} + \frac{\pi}{2} = \frac{7\pi}{4}$

The 5 points on the x-axis we use are: $-\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$.

The 2 points on the y-axis we use are: -1 and 1.

33. $y = 2\left(\sin x \cos\frac{\pi}{3} + \cos x \sin\frac{\pi}{3}\right)$

$y = 2\sin\left(x + \frac{\pi}{3}\right)$

The graph is a sine curve with: Amplitude $= 2$ Period $= 2\pi$ Phase shift $= -\frac{\pi}{3}$

Spacing $= \frac{1}{4}(2\pi) = \frac{\pi}{2}$

$\text{a} = -\frac{\pi}{3}$

$\text{b} = -\frac{\pi}{3} + \frac{\pi}{2} = \frac{\pi}{6}$

$\text{c} = \frac{\pi}{6} + \frac{\pi}{2} = \frac{2\pi}{3}$

$\text{d} = \frac{2\pi}{3} + \frac{\pi}{2} = \frac{7\pi}{6}$

$\text{e} = \frac{7\pi}{6} + \frac{\pi}{2} = \frac{5\pi}{3}$

The 5 points on the x-axis we use are: $-\frac{\pi}{3}, \frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3}$.

The 2 points on the y-axis we use are: 2 and -2.

35. If $\sin A = \frac{3}{5}$ with A in QII, then

$$\cos A = -\sqrt{1-\sin^2 A} = -\sqrt{1-\frac{9}{25}} = -\sqrt{\frac{16}{25}} = -\frac{4}{5}$$

Also, if $\sin B = -\frac{5}{13}$ B in QIII, then

$$\cos B = -\sqrt{1-\frac{25}{169}} = -\sqrt{\frac{144}{169}} = -\frac{12}{13}$$

$$\sin(A+B) = \sin A\cos B + \cos A\sin B = \frac{3}{5}\left(-\frac{12}{13}\right)+\left(-\frac{4}{5}\right)\left(-\frac{5}{13}\right) = -\frac{36}{65}+\frac{20}{65} = -\frac{16}{65}$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B = -\frac{4}{5}\left(-\frac{12}{13}\right)-\frac{3}{5}\left(-\frac{5}{13}\right) = \frac{48}{65}+\frac{15}{65} = \frac{63}{65}$$

$$\tan A = \frac{\sin A}{\cos A} = \frac{\frac{3}{5}}{-\frac{4}{5}} = -\frac{3}{4}$$

$$\tan B = \frac{\sin B}{\cos B} = \frac{-\frac{5}{13}}{-\frac{12}{13}} = \frac{5}{12}$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A\tan B} = \frac{-\frac{3}{4}+\frac{5}{12}}{1-\left(-\frac{3}{4}\right)\left(\frac{5}{12}\right)} = \frac{-\frac{1}{3}}{1+\frac{5}{16}} = \frac{-\frac{1}{3}}{\frac{21}{16}} = -\frac{16}{63}$$

The angle $(A + B)$ must terminate in QIV because the cosine is positive and the sine and tangent are negative.

37. If $\sin A = \frac{1}{\sqrt{5}}$ with A in QI, then

$$\cos A = \sqrt{1-\sin^2 A} = \sqrt{1-\frac{1}{5}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$$

Also, $\tan A = \frac{\sin A}{\cos A} = \frac{1/\sqrt{5}}{2\sqrt{5}} = \frac{1}{2}$

We have $\tan A = \frac{1}{2}$ and $\tan B = \frac{3}{4}$

This problem is continued on the next page.

Therefore, $\tan(A+B)=\dfrac{\frac{1}{2}+\frac{3}{4}}{1-\left(\frac{1}{2}\right)\left(\frac{3}{4}\right)}$

$$=\frac{5/4}{5/8}$$
$$=2$$

$\cot(A+B)=\dfrac{1}{\tan(A+B)}$

$$=\frac{1}{2}$$

The angle $(A + B)$ terminates in QI because its tangent is positive. (If its tangent were negative, it would terminate in QII.)

39.

$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$$
$$3=\frac{\tan A+\frac{1}{2}}{1-(\tan A)\left(\frac{1}{2}\right)}$$
$$\frac{3}{1}=\frac{2\tan A+1}{2-\tan A}$$
$$6-3\tan A=2\tan A+1$$
$$-5\tan A=-5$$
$$\tan A=1$$

41.

$$\sin 2x=\sin(x+x)$$
$$=\sin x\cos x+\cos x\sin x$$
$$=2\sin x\cos x$$

43.

$\sin(90^\circ+x)+\sin(90^\circ-x)$	
$=(\sin 90^\circ\cos x+\cos 90^\circ\sin x)+(\sin 90^\circ\cos x-\cos 90^\circ\sin x)$	Sum and difference formulas
$=2\sin 90^\circ\cos x$	Combine
$=2(1)\cos x$	Substitute exact value
$=2\cos x$	Multiply

45.

$\cos(x-90^\circ)-\cos(x+90^\circ)$	
$=[\cos x\cos 90^\circ+\sin x\sin 90^\circ]-[\cos x\cos 90^\circ-\sin x\sin 90^\circ]$	Sum and difference formulas
$=[(\cos x)(0)+(\sin x)(1)]-[(\cos x)(0)-(\sin x)(1)]$	Substitute exact values
$=\sin x-(-\sin x)$	Multiply
$=2\sin x$	Subtract

47. $\sin\left(\frac{\pi}{6}+x\right)+\sin\left(\frac{\pi}{6}-x\right)$

$=\left(\sin\frac{\pi}{6}\cos x+\cos\frac{\pi}{6}\sin x\right)+\left(\sin\frac{\pi}{6}\cos x-\cos\frac{\pi}{6}\sin x\right)$ Sum and difference formulas

$=2\sin\frac{\pi}{6}\cos x$ Combine

$=2\left(\frac{1}{2}\right)\cos x$ Substitute exact values

$=\cos x$ Multiply

49. $\cos\left(x+\frac{\pi}{4}\right)+\cos\left(x-\frac{\pi}{4}\right)$

$=\left(\cos x\cos\frac{\pi}{4}-\sin x\sin\frac{\pi}{4}\right)+\left(\cos x\cos\frac{\pi}{4}+\sin x\sin\frac{\pi}{4}\right)$ Sum and difference formulas

$=(\cos x)\frac{\sqrt{2}}{2}-(\sin x)\frac{\sqrt{2}}{2}+(\cos x)\frac{\sqrt{2}}{2}+(\sin x)\frac{\sqrt{2}}{2}$ Substitute exact values

$=\sqrt{2}\cos x$ Combine

51. $\sin\left(\frac{3\pi}{2}+x\right)+\sin\left(\frac{3\pi}{2}-x\right)$

$=\left(\sin\frac{3\pi}{2}\cos x+\cos\frac{3\pi}{2}\sin x\right)+\left(\sin\frac{3\pi}{2}\cos x-\cos\frac{3\pi}{2}\sin x\right)$ Sum and difference formulas

$=2\sin\frac{3\pi}{2}\cos x$ Combine

$=2(-1)\cos x$ Substitute exact value

$=-2\cos x$ Multiply

53. $\sin(A+B)+\sin(A-B)$

$=(\sin A\cos B+\cos A\sin B)+(\sin A\cos B-\cos A\sin B)$ Sum and difference formulas

$=2\sin A\cos B$ Combine

55. $\frac{\sin(A-B)}{\cos A\cos B}=\frac{\sin A\cos B-\cos A\sin B}{\cos A\cos B}$ Difference formula

$=\frac{\sin A\cos B}{\cos A\cos B}-\frac{\cos A\sin B}{\cos A\cos B}$ Separate into 2 fractions

$=\frac{\sin A}{\cos A}-\frac{\sin B}{\cos B}$ Reduce

$=\tan A-\tan B$ Ratio identity

57. $\sec(A+B)$

$=\dfrac{1}{\cos(A+B)}$ Reciprocal identity

$=\dfrac{1}{\cos(A+B)}\cdot\dfrac{\cos(A-B)}{\cos(A-B)}$ Multiply numerator and denominator by $\cos(A-B)$

$=\dfrac{\cos(A-B)}{(\cos A\cos B-\sin A\sin B)(\cos A\cos B+\sin A\sin B)}$ Sum and differences formulas

$=\dfrac{\cos(A-B)}{\cos^2 A\cos^2 B-\sin^2 A\sin^2 B}$ Multiply

$=\dfrac{\cos(A-B)}{\cos^2 A\cos^2 B+\sin^2 A\cos^2 B-\sin^2 A\sin^2 B-\sin^2 A\cos^2 B}$ Add & subtract $\sin^2 A\cos^2 B$ in denominator

$=\dfrac{\cos(A-B)}{\cos^2 B(\cos^2 A+\sin^2 A)-\sin^2 A(\sin^2 B+\cos^2 B)}$ Factor by grouping

$=\dfrac{\cos(A-B)}{\cos^2 B(1)-\sin^2 A(1)}$ Pythagorean identity

$=\dfrac{\cos(A-B)}{\cos^2 B-\sin^2 A}$ Multiply

65. Amplitude = 4

Period $=\dfrac{2\pi}{2}=\pi$

67. Amplitude = 3

Period $=\dfrac{2\pi}{1/2}=4\pi$

69. Amplitude = 2

Period $=\dfrac{2\pi}{\pi}=2$

71. Period $=\dfrac{2\pi}{3}$

Asymptotes occur where $y=\sin 3x$ is zero, or at 0, $\dfrac{\pi}{3}$, and $\dfrac{2\pi}{3}$.

We use the graph of $y=\sin 3x$ and the asymptotes to sketch the graph of $y=\csc 3x$.

73. Amplitude $=\dfrac{1}{2}$

Period $=\dfrac{2\pi}{3}$

75. Amplitude $=\dfrac{1}{2}$

Period $=\dfrac{2\pi}{\pi/2}=4$