Problem Set 5.3

1. If $\sin A = -\frac{3}{5}$ with A in QIII, then

$$\cos A = -\sqrt{1-\sin^2 A} = -\sqrt{1-\frac{9}{25}} = -\sqrt{\frac{16}{25}} = -\frac{4}{5}$$

Therefore,

$$\sin 2A = 2\sin A\cos A = 2\left(-\frac{3}{5}\right)\left(\frac{4}{5}\right) = \frac{24}{25}$$

3. $$\cos 2A = \cos^2 A - \sin^2 A = \left(-\frac{4}{5}\right)^2 - \left(-\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

$$\tan 2A = \frac{\sin 2A}{\cos 2A} = \frac{\frac{24}{25}}{\frac{7}{25}} = \frac{24}{7}$$

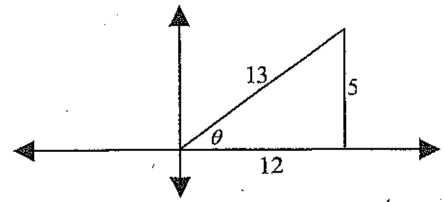
5. If $\cos x = \frac{1}{\sqrt{10}}$ with x in QIV, then

$$\cos 2x = 2\cos^2 x - 1 = 2\left(\frac{1}{\sqrt{10}}\right)^2 - 1 = 2\left(\frac{1}{10}\right) - 1 = \frac{1}{5} - 1 = -\frac{4}{5}$$

7. $$\sin 2x = -\sqrt{1-\cos^2(2x)} = -\sqrt{1-\left(\frac{16}{25}\right)} = -\sqrt{\frac{9}{25}} = -\frac{3}{5}$$

$$\cot 2x = \frac{\cos 2x}{\sin 2x} = \frac{-\frac{4}{5}}{-\frac{3}{5}} = \frac{4}{3}$$

9. If $\tan\theta = \frac{5}{12}$ with θ in QI, we can draw the triangle at the right and find the hypotenuse using the Pythagorean Theorem.



$$\begin{aligned}\text{hypotenuse} &= \sqrt{5^2+12^2}\\ &= \sqrt{25+144}\\ &= \sqrt{169} = 13\end{aligned}$$

Then, $\sin\theta = \frac{5}{13}$ and $\cos\theta = \frac{12}{13}$.

Therefore, $\sin 2\theta = 2\sin\theta\cos\theta$

$$= 2\left(\frac{5}{13}\right)\left(\frac{12}{13}\right) = \frac{120}{169}$$

11. $\csc 2\theta = \frac{1}{\sin 2\theta} = \frac{1}{\frac{120}{169}} = \frac{169}{120}$

13. If $\csc t = \sqrt{5}$ with t in QII, then $\sin t = \frac{1}{\sqrt{5}}$.

Therefore, $\cos 2t = 1 - 2\sin^2 t$

$$= 1 - 2\left(\frac{1}{\sqrt{5}}\right)^2$$

$$= 1 - \frac{2}{5} = \frac{3}{5}$$

15. $\sec 2t = \frac{1}{\cos 2t}$

$$= \frac{1}{3/5} = \frac{5}{3}$$

17. $y = 4 - 8\sin^2 x$

$= 4\left(1 - 2\sin^2 x\right)$

$= 4\cos 2x$

The graph is a cosine curve with amplitude $= 4$ and period $= \frac{2\pi}{2} = \pi$.

19. $y = 6\cos^2 x - 3$

$= 3\left(2\cos^2 x - 1\right)$

$= 3\cos 2x$

The graph is a cosine curve with amplitude $= 3$ and period $= \frac{2\pi}{2} = \pi$.

21. $y = 1 - 2\sin^2 2x$

$= \cos 2(2x)$

$= \cos 4x$

The graph is a cosine curve with amplitude $= 1$ and period $= \frac{2\pi}{4} = \frac{\pi}{2}$.

23. $\sin 60^\circ = \frac{\sqrt{3}}{2}$

$$2\sin 30^\circ \cos 30^\circ = 2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{2}$$

Therefore, they are equal.

25. $\cos 120^\circ = -\cos 60^\circ = -\frac{1}{2}$

$$\cos^2 60^\circ - \sin^2 60^\circ = \left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} - \frac{3}{4} = -\frac{1}{2}$$

Therefore, they are equal.

27.
$$\tan 2A = \frac{2\tan A}{1-\tan^2 A} = \frac{2\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)^2} = \frac{\frac{3}{2}}{1-\frac{9}{16}} = \frac{\frac{3}{2}}{\frac{7}{16}} = \frac{24}{7}$$

29. $2\sin 15^\circ \cos 15^\circ = \sin 2(15^\circ) = \sin 30^\circ = \frac{1}{2}$

31.
$$1-2\sin^2 75^\circ = \cos 2(75^\circ) = \cos 150^\circ = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$$

33.
$$\sin\frac{\pi}{12}\cos\frac{\pi}{12} = \frac{1}{2}\left(2\sin\frac{\pi}{12}\cos\frac{\pi}{12}\right) = \frac{1}{2}\sin 2\left(\frac{\pi}{12}\right) = \frac{1}{2}\sin\frac{\pi}{6} = \frac{1}{2}\left(\frac{1}{2}\right) = \frac{1}{4}$$

35. $$\frac{\tan 22.5^\circ}{1-\tan^2 22.5^\circ} = \frac{1}{2}\tan 2\left(22.5^\circ\right)$$
$$= \frac{1}{2}\tan 45^\circ$$
$$= \frac{1}{2}(1)$$
$$= \frac{1}{2}$$

37.

$(\sin x - \cos x)^2 = \sin^2 x - 2\sin x \cos x + \cos^2 x$	Expand
$= \left(\sin^2 x + \cos^2 x\right) - 2\sin x \cos x$	Commutative property
$= 1 - 2\sin x \cos x$	Pythagorean identity
$= 1 - \sin 2x$	Double-angle identity

39.

$\frac{1+\cos 2\theta}{2} = \frac{1+2\cos^2\theta - 1}{2}$	Double-angle identity
$= \frac{2\cos^2\theta}{2}$	Combine
$= \cos^2\theta$	Reduce

41.

$\frac{\sin 2\theta}{1-\cos 2\theta} = \frac{2\sin\theta\cos\theta}{1-\left(1-2\sin^2\theta\right)}$	Double-angle identities
$= \frac{2\sin\theta\cos\theta}{2\sin^2\theta}$	Subtract
$= \frac{\cos\theta}{\sin\theta}$	Reduce
$= \cot\theta$	Ratio identity

43.

$\tan x + \cot x = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$	Ratio identities
$= \frac{\sin x}{\cos x}\cdot\frac{\sin x}{\sin x} + \frac{\cos x}{\sin x}\cdot\frac{\cos x}{\cos x}$	LCD is $\sin x \cos x$
$= \frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$	Multiply and add fractions
$= \frac{1}{\sin x \cos x}$	Pythagorean identity
$= \frac{1}{\frac{1}{2}\sin 2x}$	Double-angle identity
$= 2\csc 2x$	Reciprocal identity

45. $\sin 3\theta = \sin(2\theta + \theta)$ Addition
$= \sin 2\theta \cos\theta + \cos 2\theta \sin\theta$ Sum formula
$= (2\sin\theta\cos\theta)\cos\theta + (1 - 2\sin^2\theta)\sin\theta$ Double-angle identities
$= 2\sin\theta\cos^2\theta + \sin\theta - 2\sin^3\theta$ Multiply
$= 2\sin\theta(1 - \sin^2\theta) + \sin\theta - 2\sin^3\theta$ Pythagorean identity
$= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$ Multiply
$= 3\sin\theta - 4\sin^3\theta$ Combine

47. $\cos^4 x - \sin^4 x = (\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x)$ Factor
$= 1(\cos^2 x - \sin^2 x)$ Pythagorean identity
$= \cos 2x$ Double-angle identity

49. $\dfrac{\cos 2\theta}{\sin\theta\cos\theta} = \dfrac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta}$ Double-angle identity
$= \dfrac{\cos^2\theta}{\sin\theta\cos\theta} - \dfrac{\sin^2\theta}{\sin\theta\cos\theta}$ Separate into 2 fractions
$= \dfrac{\cos\theta}{\sin\theta} - \dfrac{\sin\theta}{\cos\theta}$ Reduce
$= \cot\theta - \tan\theta$ Ratio identities

51. $\sin 4A = 2\sin 2A \cos 2A$ Double-angle identity
$= 2(2\sin A\cos A)(\cos^2 A - \sin^2 A)$ Double-angle identities
$= 4\sin A\cos^3 A - 4\sin^3 A\cos A$ Distributive property

53. $\dfrac{1 - \tan x}{1 + \tan x} = \dfrac{1 - \dfrac{\sin x}{\cos x}}{1 + \dfrac{\sin x}{\cos x}}$ Ratio identity
$= \dfrac{\cos x\left(1 - \dfrac{\sin x}{\cos x}\right)}{\cos x\left(1 + \dfrac{\sin x}{\cos x}\right)}$ Multiply numerator and denominator by LCD
$= \dfrac{\cos x - \sin x}{\cos x + \sin x}$ Distributive property
$= \dfrac{\cos x - \sin x}{\cos x + \sin x} \cdot \dfrac{\cos x - \sin x}{\cos x - \sin x}$ Multiply by a fraction equal to 1

This problem is continued on the next page

$$= \frac{\cos^2 x - 2\sin x\cos x + \sin^2 x}{\cos^2 x - \sin^2 x}$$ Multiply fractions

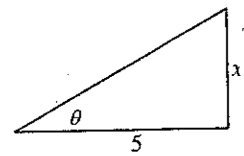
$$= \frac{1 - 2\sin x\cos x}{\cos^2 x - \sin^2 x}$$ Pythagorean identity

$$= \frac{1 - \sin 2x}{\cos 2x}$$ Double-angle identities

61. First, we find θ: $x = 5\tan\theta$

$$\tan\theta = \frac{x}{5}$$

$$\theta = \tan^{-1}\frac{x}{5}$$



Next, we find $\sin 2\theta$ by drawing the triangle at the right and using the Pythagorean Theorem:

$$\text{hypotenuse } = \sqrt{x^2 + 5^2} = \sqrt{x^2 + 25}$$

Then $\sin 2\theta = 2\sin\theta\cos\theta$

$$= 2\left(\frac{x}{\sqrt{x^2+25}}\right)\left(\frac{5}{\sqrt{x^2+25}}\right)$$

$$= \frac{10x}{x^2+25}$$

Last, we evaluate: $\frac{\theta}{2} - \frac{\sin 2\theta}{4} = \frac{1}{2}\tan^{-1}\frac{x}{5} - \frac{1}{4}\left(\frac{10x}{x^2+25}\right)$

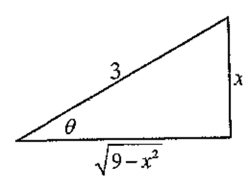
$$= \frac{1}{2}\tan^{-1}\frac{x}{5} - \frac{1}{2}\left(\frac{5x}{x^2+25}\right)$$

$$= \frac{1}{2}\left(\tan^{-1}\frac{x}{5} - \frac{5x}{x^2+25}\right)$$

63. First we find θ: $x = 3\sin\theta$

$$\sin\theta = \frac{x}{3}$$

$$\theta = \sin^{-1}\frac{x}{3}$$



Next, we find $\sin 2\theta$ by drawing the triangle at the right and using the Pythagorean Theorem:

This problem is continued on the next page