5.4 Half-Angle Formulas

$$\boxed{\sin \frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}}}$$

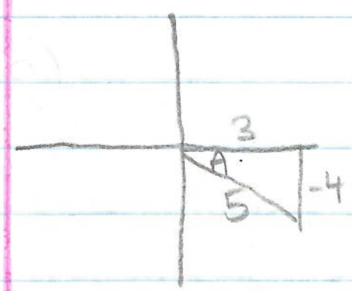
$$\boxed{\cos \frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}}}$$

+ or − depends on what quadrant $\frac{A}{2}$ is in

$$\boxed{\tan \frac{A}{2} = \frac{1-\cos A}{\sin A} \text{ or } \frac{\sin A}{1+\cos A}}$$

or do $\frac{\sin A/2}{\cos A/2}$

Ex 1 if $\cos A = \frac{3}{5}$ in QIV, find a) $\sin \frac{A}{2}$, b) $\cos \frac{A}{2}$ c) $\tan \frac{A}{2}$



if A is in QIV,
$270^\circ < A < 360^\circ$
$135^\circ < \frac{A}{2} < 180^\circ$,
which is in QII ! → sin + cos −

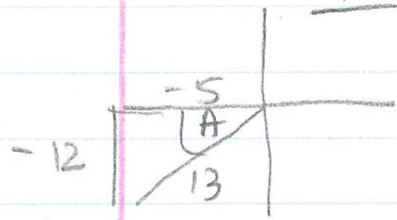
times every term by 5

a) $\sin \frac{A}{2} = \sqrt{\frac{1-\cos A}{2}} = \sqrt{\frac{1-\frac{3}{5}}{2}} = \sqrt{\frac{5-3}{10}} = \sqrt{\frac{2}{10}} = \sqrt{\frac{1}{5}} = \frac{1}{\sqrt{5}} = \boxed{\frac{\sqrt{5}}{5}}$

b) $\cos \frac{A}{2} = -\sqrt{\frac{1+\cos A}{2}} = -\sqrt{\frac{1+\frac{3}{5}}{2}} = -\sqrt{\frac{5+3}{10}} = -\sqrt{\frac{4}{5}} = -\frac{2}{\sqrt{5}} = \boxed{\frac{-2\sqrt{5}}{5}}$

c) $\tan \frac{A}{2} = \frac{\sin \frac{A}{2}}{\cos \frac{A}{2}} = \frac{\frac{1}{\sqrt{5}}}{-\frac{2}{\sqrt{5}}} = \boxed{-\frac{1}{2}}$

ex 2) $\sin A = \frac{-12}{13}$ in QIII, find all <u>six</u> trig ratios of $\frac{A}{2}$.



$180 < A < 270$

so $90 < \frac{A}{2} < 135$, in QII → sin +, cos −

a) $\sin\frac{A}{2} = \sqrt{\frac{1-\frac{-5}{13}}{2}} = \sqrt{\frac{1+\frac{5}{13}}{2}} = \sqrt{\frac{13+5}{26}} = \sqrt{\frac{18}{26}} = \sqrt{\frac{9}{13}} = \frac{3}{\sqrt{13}} = \boxed{\frac{3\sqrt{13}}{13}}$

b) $\csc\frac{A}{2} = \boxed{\frac{\sqrt{13}}{3}}$

c) $\cos\frac{A}{2} = -\sqrt{\frac{1+-\frac{5}{13}}{2}} = \sqrt{\frac{1-\frac{5}{13}}{2}} = -\sqrt{\frac{13-5}{26}} = -\sqrt{\frac{8}{26}} = -\sqrt{\frac{4}{13}} = \frac{-2}{\sqrt{13}}$

$= \boxed{\frac{-2\sqrt{13}}{13}}$

$\sec\frac{A}{2} = \boxed{\frac{-\sqrt{13}}{2}}$

d) $\tan\frac{A}{2} = \frac{\frac{3}{\sqrt{13}}}{-2/\sqrt{13}} = \boxed{\frac{-3}{2}}$ e) $\cot\frac{A}{2} = \boxed{\frac{-2}{3}}$

ex 3 <u>Find tan 15°</u> $\tan\frac{A}{2} = \frac{\sin A}{1+\cos A}$

if $\frac{A}{2} = 15$, $A = 30$!

$\frac{\sin 30}{1+\cos 30} = \frac{\frac{1}{2}}{1+\frac{\sqrt{3}}{2}} = \frac{1}{2+\sqrt{3}}$ ← we rationalize the denominator w/ the conjugate

$\frac{1}{2+\sqrt{3}} \cdot \frac{(2-\sqrt{3})}{(2-\sqrt{3})}$ $\frac{2-\sqrt{3}}{4-3} = \frac{2-\sqrt{3}}{1} = \boxed{2-\sqrt{3}}$

ex 4) Condense into a graphable equation.

$$y = 4\cos^2\left(\frac{x}{2}\right)$$

$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos x}{2}}$ so $4\cos^2\left(\frac{x}{2}\right)$

$$= 4\cos\left(\frac{x}{2}\right)\cdot\cos\left(\frac{x}{2}\right)$$

$$= 4\left(\pm\sqrt{\frac{1+\cos x}{2}}\right)^2$$

$$= 4\left(\frac{1+\cos x}{2}\right) = \frac{4+4\cos x}{2}$$

$$\boxed{= 2 + 2\cos x}$$

ex 5) Proove that $\frac{\tan x - \sin x}{2\tan x} = \sin^2\left(\frac{x}{2}\right)$

$$\frac{\cos\cdot\frac{\sin x}{\cos x} - \frac{\sin x}{1}\cdot\cos}{2\frac{\sin x}{\cos x}\cos} \Rightarrow \frac{\cancel{\sin x} - \cancel{\sin x}\cos x}{2\cancel{\sin x}}$$

$$\frac{1-\cos x}{2} \Rightarrow \left(\sqrt{\frac{1-\cos x}{2}}\right)^2 = \left(\sin\frac{x}{2}\right)^2$$

$$= \boxed{\sin^2\frac{x}{2}}$$

HW: 5.4 # 1-47 odds
6.2 # 29, 31