5.4 Half-Angle Expressions & Equations

PROBLEM SET 5.4

NOTE For the following problems, assume that all the given angles are in simplest form, so that if A is in QIV you may assume that $270° < A < 360°$.

If $\cos A = \frac{1}{2}$ with A in QIV, find

1. $\sin \frac{A}{2}$
2. $\cos \frac{A}{2}$
3. $\csc \frac{A}{2}$
4. $\sec \frac{A}{2}$

If $\sin A = -\frac{3}{5}$ with A in QIII, find

5. $\cos \frac{A}{2}$
6. $\sin \frac{A}{2}$
7. $\sec \frac{A}{2}$
8. $\csc \frac{A}{2}$

If $\sin B = -\frac{1}{3}$ with B in QIII, find

9. $\sin \frac{B}{2}$
10. $\csc \frac{B}{2}$
11. $\cos \frac{B}{2}$
12. $\sec \frac{B}{2}$
13. $\tan \frac{B}{2}$
14. $\cot \frac{B}{2}$

If $\sin A = \frac{4}{5}$ with A in QII, and $\sin B = \frac{3}{5}$ with B in QI, find

15. $\sin \frac{A}{2}$
16. $\cos \frac{A}{2}$
17. $\cos 2A$
18. $\sin 2A$
19. $\sec 2A$
20. $\csc 2A$
21. $\cos \frac{B}{2}$
22. $\sin \frac{B}{2}$
23. $\sin (A + B)$
24. $\cos (A + B)$
25. $\cos (A - B)$
26. $\sin (A - B)$

condense into a graphable equation

Graph each of the following from $x = 0$ to $x = 4\pi$.

27. $y = 4 \sin^2 \frac{x}{2}$
28. $y = 6 \cos^2 \frac{x}{2}$
29. $y = 2 \cos^2 \frac{x}{2}$
30. $y = 2 \sin^2 \frac{x}{2}$

Use half-angle formulas to find exact values for each of the following:

31. $\cos 15°$
32. $\tan 15°$
33. $\sin 75°$
34. $\cos 75°$
35. $\cos 105°$
36. $\sin 105°$

Prove the following identities. (in the direction of arrow)

37. $\sin^2 \frac{\theta}{2} = \frac{\csc \theta - \cot \theta}{2 \csc \theta}$
38. $2 \cos^2 \frac{\theta}{2} = \frac{\sin^2 \theta}{1 - \cos \theta}$
39. $\sec^2 \frac{A}{2} = \frac{2 \sec A}{\sec A + 1}$
40. $\csc^2 \frac{A}{2} = \frac{2 \sec A}{\sec A - 1}$
41. $\tan \frac{B}{2} = \csc B - \cot B$
42. $\tan \frac{B}{2} = \frac{\sec B}{\sec B \csc B + \csc B}$
43. $\tan \frac{x}{2} + \cot \frac{x}{2} = 2 \csc x$
44. $\tan \frac{x}{2} - \cot \frac{x}{2} = -2 \cot x$
45. $\cos^2 \frac{\theta}{2} = \frac{\tan \theta + \sin \theta}{2 \tan \theta}$
46. $2 \sin^2 \frac{\theta}{2} = \frac{\sin^2 \theta}{1 + \cos \theta}$
47. $\cos^4 \theta = \frac{1}{4} + \frac{\cos 2\theta}{2} + \frac{\cos^2 2\theta}{4}$
48. $4 \sin^4 \theta = 1 - 2 \cos 2\theta + \cos^2 2\theta$

PROBLEM SET 6.2

Solve for θ if $0° \le \theta < 360°$.

25. $\sqrt{3} \sin \theta + \cos \theta = \sqrt{3}$
26. $\sin \theta - \sqrt{3} \cos \theta = \sqrt{3}$
27. $\sqrt{3} \sin \theta - \cos \theta = 1$
28. $\sin \theta - \sqrt{3} \cos \theta = 1$
29. $\sin \frac{\theta}{2} - \cos \theta = 0$
30. $\sin \frac{\theta}{2} + \cos \theta = 1$
31. $\cos \frac{\theta}{2} - \cos \theta = 1$
32. $\cos \frac{\theta}{2} - \cos \theta = 0$

Answers

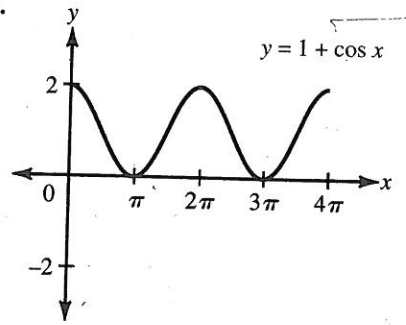
PROBLEM SET 5.4

1. $\frac{1}{2}$ **3.** 2 **5.** $-\frac{1}{\sqrt{10}}$ **7.** $-\sqrt{10}$ **9.** $\sqrt{\frac{3+2\sqrt{2}}{6}}$ **11.** $-\sqrt{\frac{3-2\sqrt{2}}{6}}$ **13.** $-3-2\sqrt{2}$

15. $\frac{2}{\sqrt{5}}$ **17.** $-\frac{7}{25}$ **19.** $-\frac{25}{7}$ **21.** $\frac{3}{\sqrt{10}}$ **23.** $\frac{7}{25}$ **25.** 0

27. See the solution to Problem 65 in Problem Set 5.3.

29.



31. $\frac{\sqrt{2+\sqrt{3}}}{2}$ **33.** $\frac{\sqrt{2+\sqrt{3}}}{2}$ **35.** $-\frac{\sqrt{2-\sqrt{3}}}{2}$

For Problems 37–47, see the Solutions Manual.

PROBLEM SET 6.2

1. 30°, 330° **3.** 225°, 315° **5.** 45°, 135°, 225°, 315° **7.** 30°, 150° **9.** 30°, 90°, 150°, 270°

11. 60°, 180°, 300° **13.** $\frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$ **15.** $0, \frac{2\pi}{3}, \frac{4\pi}{3}$ **17.** $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$ **19.** $\frac{\pi}{3}, \frac{5\pi}{3}$

21. $\frac{2\pi}{3}, \frac{4\pi}{3}$ **23.** $\frac{\pi}{4}$ **25.** 30°, 90° **27.** 60°, 180° **29.** 60°, 300° **31.** 120°, 180° **33.** 210°, 330°

35. 36.9°, 48.2°, 311.8°, 323.1° **37.** 36.9°, 143.1°, 216.9°, 323.1° **39.** $225° + 360°k$, $315° + 360°k$

41. $\frac{\pi}{4} + 2k\pi$ **43.** $120° + 360°k$, $180° + 360°k$ **45.** See the Solutions Manual. **47.** 68.5°, 291.5°

49. 218.2°, 321.8° **51.** 73.0°, 287.0° **53.** 0.3630, 2.1351 **55.** 3.4492, 5.9756 **57.** 0.3166, 1.9917

59. $\sqrt{\frac{3-\sqrt{5}}{6}}$ **61.** $\sqrt{\frac{6}{3-\sqrt{5}}}$ **63.** $\sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}}$ or $\frac{3-\sqrt{5}}{2}$

65. See the solution to Problem 65 in Problem Set 5.3. **67.** $\frac{\sqrt{2-\sqrt{2}}}{2}$