adjacent side $= \sqrt{3^2 - x^2}$

$= \sqrt{9 - x^2}$

Then, $\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{x}{3}\right)\left(\frac{\sqrt{9-x^2}}{3}\right)$

$= \frac{2}{9}x\sqrt{9-x^2}$

Last, we evaluate: $\frac{\theta}{2} - \frac{\sin 2\theta}{4} = \frac{1}{2}\sin^{-1}\frac{x}{3} - \frac{1}{4}\left(\frac{2}{9}x\sqrt{9-x^2}\right)$

$= \frac{1}{2}\sin^{-1}\frac{x}{3} - \frac{1}{18}x\sqrt{9-x^2}$

$= \frac{1}{2}\left(\sin^{-1}\frac{x}{3} - \frac{x\sqrt{9-x^2}}{9}\right)$

65. Let $y_1 = 2$ (a horizontal line) and $y_2 = -2\cos x$ (a cosine curve reflected across the x-axis with amplitude of 2). Graph y_1, y_2 and $y = y_1 + y_2$ on the same coordinate system.

67. Let $y_1 = 3$ (a horizontal line) and $y_2 = -3\cos x$ (a cosine curve reflected across the x-axis with amplitude of 3). Graph y_1, y_2 and $y = y_1 + y_2$ on the same coordinate system.

69. Let $y_1 = \cos x$ and $y_2 = \frac{1}{2}\sin 2x$ (a sine curve with amplitude of $\frac{1}{2}$ and a period of $\frac{2\pi}{2}$ or π). Graph y_1, y_2 and $y = y_1 + y_2$ on the same coordinate system.

71. Let $y_1 = \frac{1}{2}x$ (a line) and $y_2 = \sin \pi x$ (a sine curve with amplitude of 1 and period $= \frac{2\pi}{\pi} = 2$). Graph y_1, y_2 and $y = y_1 + y_2$ on the same coordinate system.

Problem Set 5.4

1. If A is in QIV, then $270^\circ < A < 360^\circ$ and $135^\circ < \frac{A}{2} < 180^\circ$. Therefore $\frac{A}{2}$ is in QII.

$\sin\frac{A}{2} = \sqrt{\frac{1-\cos A}{2}}$

$= \sqrt{\frac{1-\frac{1}{2}}{2}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

3. $\csc\frac{A}{2} = \frac{1}{\sin\frac{A}{2}} = \frac{1}{\frac{1}{2}} = 2$

5. If A is in QIII, then $180^\circ < A < 270^\circ$ and $90^\circ < \frac{A}{2} < 135^\circ$. Therefore, $\frac{A}{2}$ is in QII.

If $\sin A = -\frac{3}{5}$, then $\cos A = -\sqrt{1-\sin^2 A}$

$$= -\sqrt{1-\frac{9}{25}}$$

$$= -\sqrt{\frac{16}{25}}$$

$$= -\frac{4}{5}$$

Therefore, $\cos\frac{A}{2} = -\sqrt{\frac{1+\cos A}{2}}$

$$= -\sqrt{\frac{1-\frac{4}{5}}{2}}$$

$$= -\sqrt{\frac{\frac{1}{5}}{2}}$$

$$= -\sqrt{\frac{1}{10}} = -\frac{1}{\sqrt{10}}$$

7. $\sec\frac{A}{2} = \frac{1}{\cos\frac{A}{2}}$

$$= \frac{1}{-\frac{1}{\sqrt{10}}}$$

$$= -\sqrt{10}$$

9. If B is in QIII, then $\frac{B}{2}$ is in QII. (See Problem 5)

If $\sin B = -\frac{1}{3}$, then $\cos B = -\sqrt{1-\sin^2 B}$

$$= -\sqrt{1-\frac{1}{9}}$$

$$= -\sqrt{\frac{8}{9}}$$

$$= -\frac{2\sqrt{2}}{3}$$

Therefore $\sin\frac{B}{2} = -\sqrt{\frac{1-\cos B}{2}}$

$$= \sqrt{\frac{1+\frac{2\sqrt{2}}{3}}{2}}$$

$$= \sqrt{\frac{3+2\sqrt{2}}{6}}$$ (Multiply numerator and denominator by 3)

Use the information from problem 9 to solve #11 and #13:

11. $$\cos\frac{B}{2} = -\sqrt{\frac{1+\cos B}{2}} = -\sqrt{\frac{1-\frac{2\sqrt{2}}{3}}{2}} = -\sqrt{\frac{3-2\sqrt{2}}{6}}$$

13. $$\tan\frac{B}{2} = \frac{1-\cos A}{\sin A} = \frac{1+\frac{2\sqrt{2}}{3}}{-\frac{1}{3}} = \frac{3+2\sqrt{2}}{-1} = -3-2\sqrt{2}$$

15. $$\cos A = -\sqrt{1-\sin^2 A} = -\sqrt{1-\frac{16}{25}} = -\sqrt{\frac{9}{25}} = -\frac{3}{5}$$

$$\sin\frac{A}{2} = \sqrt{\frac{1-\cos A}{2}} \quad \text{(in QI)}$$

$$= \sqrt{\frac{1-\left(-\frac{3}{5}\right)}{2}} = \sqrt{\frac{\frac{8}{5}}{2}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$$

17. $$\cos 2A = 1-2\sin^2 A = 1-2\left(\frac{4}{5}\right)^2 = 1-\frac{32}{25} = -\frac{7}{25}$$

19. $$\sec 2A = \frac{1}{\cos 2A} = \frac{1}{-\frac{7}{25}} = -\frac{25}{7}$$

21. If B is in QI, then $\frac{B}{2}$ must be in QI.

If $\sin B = \frac{3}{5}$, then $\cos B = \sqrt{1-\sin^2 B}$

$$= \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

Therefore, $\cos\frac{B}{2} = \sqrt{\frac{1+\cos B}{2}}$

$$= \sqrt{\frac{1+\frac{4}{5}}{2}} = \sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}}$$

23. $\sin(A+B) = \sin A\cos B + \cos A\sin B$

$$= \left(\frac{4}{5}\right)\left(\frac{4}{5}\right) + \left(-\frac{3}{5}\right)\left(\frac{3}{5}\right)$$

$$= \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

25. If $\sin A = \dfrac{4}{5}$, then $\cos A = -\sqrt{1-\sin^2 A}$

$$= -\sqrt{1-\frac{16}{25}} = -\sqrt{\frac{9}{25}} = -\frac{3}{5}$$

Therefore, $\cos(A-B) = \cos A\cos B + \sin A\sin B$

$$= -\frac{3}{5}\left(\frac{4}{5}\right) + \left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$$

$$= -\frac{12}{25} + \frac{12}{25}$$

$$= 0$$

27. $y = 4\sin^2\dfrac{x}{2}$

$$= 4\left(\pm\sqrt{\frac{1-\cos x}{2}}\right)^2$$

$$= 4\left(\frac{1-\cos x}{2}\right)$$

$$= 2 - 2\cos x$$

This graph is a cosine curve with amplitude of 2. It has been reflected across the x-axis and has a vertical translation of 2.

29. $y = 2\cos^2\dfrac{x}{2}$

$$= 2\left(\pm\sqrt{\frac{1+\cos x}{2}}\right)^2$$

$$= 2\left(\frac{1+\cos x}{2}\right)$$

$= 1 + \cos x$ This graph is the standard cosine curve with a vertical translation of 1.

31. $\cos 15^\circ = \cos\left(\dfrac{30^\circ}{2}\right)$

$$= \sqrt{\frac{1+\cos 30^\circ}{2}}$$

$$= \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}}$$

$$= \sqrt{\frac{2+\sqrt{3}}{4}}$$

$$= \frac{\sqrt{2+\sqrt{3}}}{2}$$

33.
$$\sin 75^\circ = \sin\left(\frac{150^\circ}{2}\right) = -\sqrt{\frac{1-\cos 150^\circ}{2}} = \sqrt{\frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{2+\sqrt{3}}}{2}$$

35.
$$\cos 105^\circ = \cos\left(\frac{210^\circ}{2}\right) = -\sqrt{\frac{1+\cos 210^\circ}{2}} = -\sqrt{\frac{1+\left(-\frac{\sqrt{3}}{2}\right)}{2}} = -\sqrt{\frac{2-\sqrt{3}}{4}} = -\frac{\sqrt{2-\sqrt{3}}}{2}$$

37.

$$\frac{\csc\theta-\cot\theta}{2\csc\theta} = \frac{\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}}{\frac{2}{\sin\theta}}$$ Reciprocal and ratio identities

$$= \frac{\frac{1-\cos\theta}{\sin\theta}}{\frac{2}{\sin\theta}}$$ Subtract

$$= \frac{1-\cos\theta}{2}$$ Divide

$$= \sin^2\frac{\theta}{2}$$ Half-angle formula

39.

$$\sec^2\frac{A}{2} = \frac{1}{\cos^2\frac{A}{2}}$$ Reciprocal identity

$$= \frac{1}{\left(\pm\sqrt{\frac{1+\cos A}{2}}\right)^2}$$ Half-angle identity

$$= \frac{1}{\frac{1+\cos A}{2}}$$ Simplify

$$= \frac{2}{1+\cos A}$$ Divide

$$= \frac{2}{1+\frac{1}{\sec A}}$$ Reciprocal identity

$$= \frac{2\sec A}{\sec A+1}$$ Multiply numerator and denominator by sec$A$

41. $\tan\frac{B}{2}=\frac{1-\cos B}{\sin B}$ Half-angle identity

$=\frac{1}{\sin B}-\frac{\cos B}{\sin B}$ Separate fractions

$=\csc B-\cot B$ Reciprocal and ratio identities

43. $\tan\frac{x}{2}+\cot\frac{x}{2}=\frac{1-\cos x}{\sin x}+\frac{1}{\frac{\sin x}{1+\cos x}}$ Half-angle identities and reciprocal identity

$=\frac{1-\cos x}{\sin x}+\frac{1+\cos x}{\sin x}$ Divide second fraction

$=\frac{1-\cos x+1+\cos x}{\sin x}$ Add

$=\frac{2}{\sin x}$ Combine

$=2\csc x$ Reciprocal identity

45. $\frac{\tan\theta+\sin\theta}{2\tan\theta}=\frac{\frac{\sin\theta}{\cos\theta}+\sin\theta}{\frac{2\sin\theta}{\cos\theta}}$ Ratio identity

$=\frac{\sin\theta+\sin\theta\cos\theta}{2\sin\theta}$ Multiply numerator and denominator by $\cos\theta$

$=\frac{\sin\theta(1+\cos\theta)}{2\sin\theta}$ Factor

$=\frac{1+\cos\theta}{2}$ Reduce

$=\cos^2\frac{\theta}{2}$ Half-angle identity

47. $\frac{1}{4}+\frac{\cos 2\theta}{2}+\frac{\cos^2 2\theta}{4}=\frac{1}{4}+\frac{2\cos^2\theta-1}{2}+\frac{(2\cos^2\theta-1)^2}{4}$ Double-angle identity

$=\frac{1}{4}+\frac{2(2\cos^2\theta-1)}{4}+\frac{4\cos^4\theta-4\cos^2\theta+1}{4}$ LCD is 4

$=\frac{1+4\cos^2\theta-2+4\cos^4\theta-4\cos^2\theta+1}{4}$ Combine

$=\frac{4\cos^4\theta}{4}$ Simplify

$=\cos^4\theta$ Reduce