Name: Key Per: ________ Date: ____________

Serafino ▪ Algebra 2E

5B Rational Equations & Applications

1. You need to paint a house. If you work alone, the job will take 60 hours. The rational function that represents how long the job will take (h) as a function of the number of people (p) painting is:

$$h(p) = \frac{60}{p}$$

Using this formula, we can now solve for any input or output.

Let's start with evaluation.

a. Fill in the table of values for how long it would take, based on how many people help....

p	1	3	5	15	30	60	120	300	600	1,000
h(p)	60	20	12	4	2	1	.5	.2	.1	.06

(30 min) (12 min) (6 min) 3.6

b. What's the time saved, comparatively, between hiring...

2 people or 3 people?

10 hours

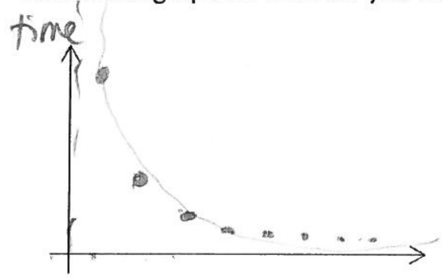
300 people or 600 people?

.1 hours (6 min)

1,000 or 2,000 people?

.03 hours (1.8 min)

c. Sketch the graph ... what do you notice about the real life meanings of asymptotes?



Vertical Asymptote – it will take ∞ hours! (it will never get done)

Horizontal Asymptote – no matter how big the group is, it will never take "0 hours" - just less and less time.

Now, let's solve for input!

Let's say you need the job to take exactly 10 hours (you need to bill a certain amount, for example).

d. How many people do you need for the job to take exactly 10 hours?

$h(p) = \frac{60}{p}$ $\frac{10}{1} = \frac{60}{p}$ $\frac{10p}{10} = \frac{60}{10}$ $p = 6$

$p \cdot \frac{10}{1} = \frac{60}{p} \cdot p$ $10p = 60$

6 people

e. It's 9:00 am and you hear there's a huge snow storm coming at 5:00 pm. If you can get people there to start painting at 10:00 am, how many people need to show up to get the job done before the storm?

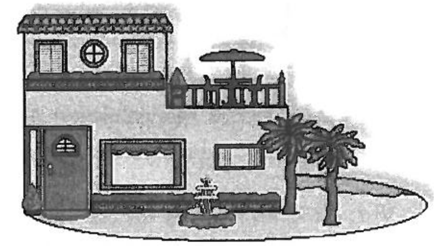
you have 7 hours

$7 = \frac{60}{p}$ $\frac{60}{7} = 8.57$

you need 9 people

Don't bring this guy →

2. You want to rent a beach house with some friends. The house is $10,500 for the week.



a. Write a function the represents the cost of the house, per person, based on how many people split it.

$C = \frac{10500}{p}$

b. How much would it cost each person if 12 people go? **$875 each**

c. How much MORE would it cost each person if one person dropped out at the last minute before the deposit was due? $\frac{10500}{11} = 954.\overline{54}$ **$79.55 more**

d. Your friends can't afford to spend more than $450 each. How many people have to go to make it affordable?

$450 = \frac{10500}{p}$ $450p = 10500$ $23.\overline{3}$ **24 people**

3. The monthly rent of an apartment in the city is related to the distance from the city's center.

$$C(d) = \frac{4500d}{d^2 + 32}$$

C = cost d = distance in miles

a. How much is rent if you live 2.5 miles from the center? 1 mile?

$\frac{4500(2.5)}{(2.5)^2 + 32} =$ **$294.12** **$136.36**

b. You can only afford a place that has a rent of $375. How far from the city center should you live?

$375 = \frac{4500d}{d^2+32}$

$375(d^2+32) = 4500d$

$375d^2 + 12000 = 4500d$

$375d^2 - 4500d + 12000 = 0$

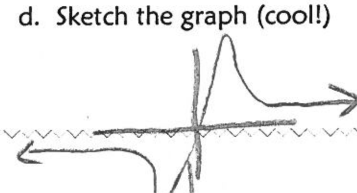
$375(d^2 - 12d + 32) = 0$

$(d-4)(d-8)$

d = 4, d = 8

c. Use your graphing calculator to find the highest rent in the city. **$397.75** (5.65 miles away)

d. Sketch the graph (cool!)



The worksheet on solving equations is sloppy looking but still good. It's about finding the intersection of two rational functions. Pay attention to extraneous solutions and WHY they occur!

LESSON 9.6

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Practice A

For use with pages 568–574

Determine whether the given x-value is a solution of the equation.

1. $\frac{2}{x-3} = \frac{3}{x+1}, x = -1$
2. $\frac{7}{x+3} = \frac{x}{4}, x = 4$
3. $\frac{x}{x-5} + 4 = \frac{1}{x+3}, x = 4$
4. $\frac{3x-1}{x-2} + 3 = \frac{x}{x-2}, x = -2$

Solve the equation by using the LCD. Check each solution.

5. $\frac{3}{x} - \frac{2}{x+1} = \frac{4}{x}$
6. $\frac{x}{x-4} + 1 = \frac{4}{x-4}$
7. $\frac{15}{x} - 4 = \frac{6}{x} + 3$
8. $\frac{4}{x} - \frac{1}{x+2} = \frac{2}{x}$
9. $\frac{2x}{x+3} + 5 = \frac{3}{x+3}$
10. $\frac{1}{x+2} + \frac{1}{x+2} = \frac{4}{x^2-4}$

Solve the equation by cross multiplying. Check each solution.

11. $\frac{2x-3}{x+3} = \frac{3x}{x+4}$
12. $\frac{x}{2x+1} = \frac{5}{4-x}$
13. $\frac{x}{x-3} = \frac{6}{x-3}$
14. $\frac{2}{x-1} = \frac{x-8}{x+1}$
15. $\frac{7}{x+3} = \frac{x}{4}$
16. $\frac{x}{x^2-10} = \frac{3}{2x+1}$

Solve the equation using any method. Check each solution.

17. $\frac{3}{x-1} - 6 = \frac{5x}{x-1}$
18. $\frac{5x}{x-1} - 2 = \frac{14}{x^2-1}$
19. $\frac{5x-7}{x-2} = \frac{8}{x-2}$
20. $\frac{1}{x-5} + \frac{1}{x+5} = \frac{x+3}{x^2-25}$
21. $\frac{2x-4}{x-4} = \frac{4}{x-4}$
22. $\frac{1}{x-2} + \frac{1}{x+3} = \frac{5}{x^2+x-6}$

23. *Population Density* The population density in a large city is related to the distance from the center of the city. It can be modeled by

$$D = \frac{5000x}{x^2+36}$$

where D is the population density (in people per square mile) and x is the distance (in miles) from the center of the city. Find the areas where the population density is 400 people per square mile.

Practice A

1. no 2. yes 3. no 4. no 5. $-\frac{1}{3}$
6. no solution 7. $\frac{7}{6}$ 8. -4 9. $-\frac{12}{7}$
10. 4 11. no solution 12. $-5, -1$
13. 6 14. 2, 5 15. $-7, 4$ 16. $-5, 6$
17. $\frac{9}{11}$ 18. $-3, \frac{3}{4}$ 19. 3 20. 3 21. no solution 22. 2 23. 4.5 miles, 8 miles

LESSON 9.6

NAME ______________________________ DATE ____________

Practice B

For use with pages 568–574

Determine whether the given x-value is a solution of the equation.

1. $\frac{1}{x-3} + \frac{1}{x+3} = \frac{10}{x^2-9}$, $x = 5$

2. $\frac{x}{x-4} + 1 = \frac{4}{x-4}$, $x = 4$

Solve the equation by using the LCD. Check each solution.

3. $\frac{3x}{x-2} = 1 + \frac{6}{x-2}$

4. $\frac{3x}{x-2} + \frac{1}{x+2} = -\frac{4}{x^2-4}$

5. $\frac{2}{2x+5} + \frac{3}{2x-5} = \frac{5x+5}{4x^2-25}$

6. $\frac{5}{2x+3} + \frac{4}{2x-3} = \frac{14x+3}{4x^2-9}$

7. $-\frac{15}{x} - 4 = \frac{6}{x} + 3$

8. $\frac{3x-1}{x-2} + 3 = \frac{x}{x-2}$

Solve the equation by cross multiplying. Check each solution.

9. $\frac{x+1}{x+3} = 2$

10. $\frac{2}{x-3} = \frac{3}{x+1}$

11. $\frac{7}{x+3} = \frac{x}{4}$

12. $\frac{6+5x}{3x} = \frac{7}{x}$

13. $\frac{x}{x^2-8} = \frac{2}{x}$

14. $\frac{2x}{5} = \frac{x^2-5x}{5x}$

Solve the equation using any method. Check each solution.

15. $\frac{5x}{x-2} = 7 + \frac{10}{x-2}$

16. $\frac{2x}{4-x} = \frac{x^2}{x-4}$

17. $\frac{3x}{x+1} = \frac{12}{x^2-1} + 2$

18. $\frac{6}{x} - \frac{7x}{5} = \frac{x}{10}$

19. $\frac{3}{x} + 12 = 2 + \frac{4}{3x}$

20. $\frac{x^2+2x+2}{x-1} = \frac{2x+3}{x-1}$

21. *Average Cost* A greeting card manufacturer can produce a dozen cards for \$6.50. If the initial investment by the company was \$60,000, how many dozen cards must be produced before the average cost per dozen falls to \$11.50?

22. *Brakes* The braking distance of a car can be modeled by $d = s + \frac{s^2}{20}$ where d is the distance (in feet) that the car travels before coming to a stop, and s is the speed at which the car is traveling (in miles per hour). Find the speed that results in a braking distance of 75 feet.

Lesson 9.6

Practice B

1. yes 2. no 3. no solution 4. $-\frac{1}{3}$ 5. 0
6. no solution 7. -3 8. $\frac{7}{5}$ 9. -5 10. 11
11. $-7, 4$ 12. 3 13. $-4, 4$ 14. -5 15. no solution 16. $0, -2$ 17. $5, -2$ 18. $-2, 2$
19. $-\frac{1}{6}$ 20. -1 21. 12,000 dozen cards
22. 30 miles per hour