

CHAPTER 6 Equations

Problem Set 6.1

1. $2\sin\theta = 1$

$$\sin\theta = \frac{1}{2}$$

$$\hat{\theta} = 30^\circ \text{ and } \theta \text{ is in QI or QII}$$

$$\theta = 30^\circ \text{ or } 150^\circ$$

3. $2\cos\theta - \sqrt{3} = 0$

$$2\cos\theta = \sqrt{3}$$

$$\cos\theta = \frac{\sqrt{3}}{2}$$

$$\hat{\theta} = 30^\circ \text{ and } \theta \text{ is in QI or QIV}$$

$$\theta = 30^\circ \text{ or } 330^\circ$$

5. $2\tan\theta + 2 = 0$

$$2\tan\theta = -2$$

$$\tan\theta = -1$$

$$\hat{\theta} = 45^\circ \text{ and } \theta \text{ in QII or QIV}$$

$$\theta = 135^\circ \text{ or } 315^\circ$$

7. $4\sin t - \sqrt{3} = 2\sin t$

$$2\sin t = \sqrt{3}$$

$$\sin t = \frac{\sqrt{3}}{2}$$

$$\hat{t} = \frac{\pi}{3} \text{ and } t \text{ is in QI or QII}$$

$$t = \frac{\pi}{3} \text{ or } \frac{2\pi}{3}$$

9. $2\cos t = 6\cos t - \sqrt{12}$

$$-4\cos t = -2\sqrt{3}$$

$$\cos t = \frac{\sqrt{3}}{2}$$

$$\hat{t} = \frac{\pi}{6} \text{ and } t \text{ is in QI or QIV}$$

$$t = \frac{\pi}{6} \text{ or } \frac{11\pi}{6}$$

11. $3\sin t + 5 = -2\sin t$

$$5\sin t = -5$$

$$\sin t = -1$$

$$t = \frac{3\pi}{2}$$

13. $4\sin\theta - 3 = 0$

$$4\sin\theta = 3$$

$$\sin\theta = 0.75$$

$$\hat{\theta} = 48.6^\circ \text{ and } \theta \text{ is in QI or QII}$$

$$\theta = 48.6^\circ \text{ or } 131.4^\circ$$

15. $2\cos\theta - 5 = 3\cos\theta - 2$

$$-\cos\theta = 3$$

$$\cos\theta = -3$$

There is no solution because $\cos\theta$ must be between -1 and 1 .

$$\begin{aligned}
 17. \quad \sin \theta - 3 &= 5 \sin \theta \\
 -3 &= 4 \sin \theta \\
 \sin \theta &= -0.75 \\
 \hat{\theta} &= 48.6^\circ \text{ and } \theta \text{ is in QIII or QIV} \\
 \theta &= 228.6^\circ \text{ or } 311.4^\circ
 \end{aligned}$$

$$\begin{aligned}
 19. \quad (\sin x - 1)(2 \sin x - 1) &= 0 \\
 \sin x - 1 &= 0 \text{ or } 2 \sin x - 1 = 0 \\
 \sin x &= 1 & 2 \sin x &= 1 \\
 x &= \frac{\pi}{2} & \sin x &= \frac{1}{2} \\
 & & \hat{x} &= \frac{\pi}{6} \\
 & & x &= \frac{\pi}{6} \text{ or } \frac{5\pi}{6}
 \end{aligned}$$

$$\begin{aligned}
 21. \quad \tan x (\tan x - 1) &= 0 \\
 \tan x &= 0 \text{ or } \tan x - 1 = 0 \\
 x &= 0, \pi & \tan x &= 1 \\
 & & x &= \frac{\pi}{4} \text{ or } \frac{5\pi}{4}
 \end{aligned}$$

$$\begin{aligned}
 23. \quad \sin x + 2 \sin x \cos x &= 0 \\
 \sin x (1 + 2 \cos x) &= 0 \\
 \sin x &= 0 \text{ or } 1 + 2 \cos x = 0 \\
 x &= 0, \pi & 2 \cos x &= -1 \\
 & & \cos x &= -\frac{1}{2} \\
 & & \hat{x} &= \frac{\pi}{3} \\
 & & x &= \frac{2\pi}{3} \text{ or } \frac{4\pi}{3}
 \end{aligned}$$

$$\begin{aligned}
 25. \quad 2 \sin^2 x - \sin x - 1 &= 0 \\
 (2 \sin x + 1)(\sin x - 1) &= 0 \\
 2 \sin x + 1 = 0 \text{ or } \sin x - 1 &= 0 \\
 2 \sin x = -1 & \quad \sin x = 1 \\
 \sin x = -\frac{1}{2} & \quad x = \frac{\pi}{2} \\
 x = \frac{7\pi}{6} \text{ or } \frac{11\pi}{6}
 \end{aligned}$$

$$\begin{aligned}
 27. \quad (2 \cos \theta + \sqrt{3})(2 \cos \theta + 1) &= 0 \\
 2 \cos \theta + \sqrt{3} = 0 \text{ or } 2 \cos \theta + 1 &= 0 \\
 2 \cos \theta = -\sqrt{3} & \quad 2 \cos \theta = -1 \\
 \cos \theta = -\frac{\sqrt{3}}{2} & \quad \cos \theta = -\frac{1}{2} \\
 \hat{\theta} = 30^\circ & \quad \hat{\theta} = 60^\circ \\
 \theta = 150^\circ \text{ or } 210^\circ & \quad \theta = 120^\circ \text{ or } 240^\circ
 \end{aligned}$$

$$\begin{aligned}
 29. \quad \sqrt{3} \tan \theta - 2 \sin \theta \tan \theta &= 0 \\
 \tan \theta (\sqrt{3} - 2 \sin \theta) &= 0
 \end{aligned}$$

$$\begin{aligned}
 \tan \theta &= 0 \text{ or } \sqrt{3} - 2 \sin \theta = 0 \\
 \theta &= 0^\circ \text{ or } 180^\circ & -2 \sin \theta &= -\sqrt{3} \\
 & & \sin \theta &= \frac{\sqrt{3}}{2} \\
 & & \theta &= 60^\circ \text{ or } 120^\circ
 \end{aligned}$$

31. $2\cos^2\theta + 11\cos\theta + 5 = 0$
 $(2\cos\theta + 1)(\cos\theta + 5) = 0$
 $2\cos\theta + 1 = 0$ or $\cos\theta + 5 = 0$
 $2\cos\theta = -1$ $\cos\theta = -5$
 $\cos\theta = -\frac{1}{2}$ No solution
 $\hat{\theta} = 60^\circ$
 $\theta = 120^\circ$ or 240°

33. $2\sin^2\theta - 2\sin\theta - 1 = 0$ where $a = 2, b = -2, c = -1$
 $\sin\theta = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(-1)}}{2(2)}$
 $= \frac{2 \pm \sqrt{12}}{4}$
 $\sin\theta = 1.3666$ or $\sin\theta = -0.3660$
No solution $\hat{\theta} = 21.5^\circ$ and θ is in QIII or QIV
 $\theta = 201.5^\circ$ or 338.5°

35. $\cos^2\theta + \cos\theta - 1 = 0$ where $a = 1, b = 1, c = -1$
 $\cos\theta = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)}$
 $= \frac{-1 \pm \sqrt{5}}{2}$
 $\cos\theta = -1.6180$ or $\cos\theta = 0.6180$
No solution and $\hat{\theta} = 51.8^\circ$ and is in QI or QII
 $\theta = 51.8^\circ$ or 308.2°

37. $2\sin^2\theta + 1 = 4\sin\theta$
 $2\sin^2\theta - 4\sin\theta + 1 = 0$ where $a = 2, b = -4, c = 1$
 $\sin\theta = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(1)}}{2(2)}$
 $= \frac{4 \pm \sqrt{8}}{4}$
 $\sin\theta = 1.7071$ or $\sin\theta = 0.2929$
No solution $\hat{\theta} = 17.0^\circ$ and θ is in QI or QII
 $\theta = 17.0^\circ$ or 163.0°

39. From problem 1, $\hat{\theta} = 30^\circ$ and θ is in QI or QII
 $\theta = 30^\circ + 360^\circ k$ or $\theta = 150^\circ + 360^\circ k$

41. From problem 7, $\hat{t} = \frac{\pi}{3}$ and t is in QI or QII
 $t = \frac{\pi}{3} + 2k\pi$ or $t = \frac{2\pi}{3} + 2k\pi$

43. From problem 11, we found that $t = \frac{3\pi}{2}$.
 Therefore, $t = \frac{3\pi}{2} + 2k\pi$.

45. In problem 13 we found that $\theta = 48.6^\circ$ or 131.4° .
 Therefore, $\theta = 48.6^\circ + 360^\circ k$ or $\theta = 131.4^\circ + 360^\circ k$.

47. $\cos(2A - 50^\circ) = \frac{\sqrt{3}}{2}$
 $2A - 50^\circ = 30^\circ + 360^\circ k$ or $2A - 50^\circ = 330^\circ + 360^\circ k$
 $2A = 80^\circ + 360^\circ k$ or $2A = 380^\circ + 360^\circ k$
 $A = 40^\circ + 180^\circ k$ or $A = 190^\circ + 180^\circ k$

49. $\sin(3A + 30^\circ) = \frac{1}{2}$
 $3A + 30^\circ = 30^\circ + 360^\circ k$ or $3A + 30^\circ = 150^\circ + 360^\circ k$
 $3A = 360^\circ k$ or $3A = 120^\circ + 360^\circ k$
 $A = 120^\circ k$ or $A = 40^\circ + 120^\circ k$

51. $\cos(4A - 20^\circ) = -\frac{1}{2}$
 $4A - 20^\circ = 120^\circ + 360^\circ k$ or $4A - 20^\circ = 240^\circ + 360^\circ k$
 $4A = 140^\circ + 360^\circ k$ or $4A = 260^\circ + 360^\circ k$
 $A = 35^\circ + 90^\circ k$ or $A = 65^\circ + 90^\circ k$

$$53. \quad \sin(5A+15^\circ) = -\frac{1}{\sqrt{2}}$$

$$5A+15^\circ = 225^\circ + 360^\circ k \quad \text{or} \quad 5A+15^\circ = 315^\circ + 360^\circ k$$

$$5A = 210^\circ + 360^\circ k \qquad \qquad \qquad 5A = 300^\circ + 360^\circ k$$

$$A = 42^\circ + 72^\circ k \qquad \qquad \qquad A = 60^\circ + 72^\circ k$$

$$75. \quad h = -16t^2 + vt \sin \theta \quad \text{where } v = 1,500 \text{ ft/sec and } \theta = 30^\circ$$

$$= -16t^2 + 1500t \sin 30^\circ$$

$$= -16t^2 + 1500t \left(\frac{1}{2} \right)$$

$$= -16t^2 + 750t$$

$$77. \quad h = -16t^2 + 750t \quad \text{where } t = 2 \text{ sec}$$

$$= -16(2^2) + 750(2)$$

$$= -64 + 1500$$

$$= 1,436 \text{ ft}$$

$$79. \quad h = -16t^2 + vt \sin \theta \quad \text{where } v = 1,500 \text{ ft/sec, } t = 2 \text{ sec, and } h = 750 \text{ ft}$$

$$750 = -16(2^2) + 1,500(2) \sin \theta$$

$$750 = -64 + 3,000 \sin \theta$$

$$814 = 3,000 \sin \theta$$

$$\sin \theta = \frac{814}{3,000}$$

$$\theta = 15.7^\circ$$

$$85. \quad \sin(\theta + 45^\circ) = \sin \theta \cos 45^\circ + \cos \theta \sin 45^\circ$$

$$= \sin \theta \left(\frac{1}{\sqrt{2}} \right) + \cos \theta \left(\frac{1}{\sqrt{2}} \right)$$

$$= \frac{1}{\sqrt{2}} \sin \theta + \frac{1}{\sqrt{2}} \cos \theta$$

$$87. \quad \sin 75^\circ = \sin(45^\circ + 30^\circ)$$

$$= \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$$

$$= \frac{\sqrt{2}}{2} \left(\frac{\sqrt{3}}{2} \right) + \frac{\sqrt{2}}{2} \left(\frac{1}{2} \right)$$

$$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

$$\begin{aligned}
 89. \quad \frac{1 - \tan^2 x}{1 + \tan^2 x} &= \frac{1 - \frac{\sin^2 x}{\cos^2 x}}{1 + \frac{\sin^2 x}{\cos^2 x}} \\
 &= \frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x} \\
 &= \frac{\cos 2x}{1} \\
 &= \cos 2x
 \end{aligned}$$

Ratio identity

Multiply numerator and denominator by $\cos^2 x$

Double-angle identity and Pythagorean identity

Simplify

Problem Set 6.2

$$1. \quad \sqrt{3} \sec \theta = 2$$

$$\sec \theta = \frac{2}{\sqrt{3}}$$

$$\cos \theta = \frac{\sqrt{3}}{2}$$

$$\hat{\theta} = 30^\circ \text{ and } \theta \text{ is in QI or QIV}$$

$$\theta = 30^\circ \text{ or } 330^\circ$$

$$3. \quad \sqrt{2} \csc \theta + 5 = 3$$

$$\sqrt{2} \csc \theta = -2$$

$$\csc \theta = -\frac{2}{\sqrt{2}}$$

$$\sin \theta = -\frac{\sqrt{2}}{2}$$

$$\hat{\theta} = 45^\circ \text{ and } \theta \text{ is in QIII or QIV}$$

$$\theta = 225^\circ \text{ or } 315^\circ$$

$$5. \quad 4 \sin \theta - 2 \csc \theta = 0$$

$$4 \sin \theta - \frac{2}{\sin \theta} = 0$$

$$4 \sin^2 \theta - 2 = 0$$

$$4 \sin^2 \theta = 2$$

$$\sin^2 \theta = \frac{1}{2}$$

$$\sin \theta = \pm \frac{1}{\sqrt{2}}$$

$$\hat{\theta} = 45^\circ \text{ and } \theta \text{ is in QI, QII, QIII, or QIV}$$

$$\theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ$$

$$\csc \theta = \frac{1}{\sin \theta}$$

Multiply both sides by $\sin \theta$

Add 2 to both sides

Divide both sides by 4

Take square root of both sides