Problem Set 6.3

1. $\sin 2\theta = \frac{\sqrt{3}}{2}$

$2\theta = 60^\circ + 360^\circ k$ or $2\theta = 120^\circ + 360^\circ k$

$\theta = 30^\circ + 180^\circ k$ $\quad\theta = 60^\circ + 180^\circ k$

If we let $k = 0$ and 1, we get

$\theta = 30^\circ$ $\quad\theta = 60^\circ$

$\theta = 210^\circ$ $\quad\theta = 240^\circ$

3. $\tan 2\theta = -1$

$2\theta = 135^\circ + 360^\circ k$ or $2\theta = 315^\circ + 360^\circ k$

$\theta = 67.5^\circ + 180^\circ k$ $\quad\theta = 157.5^\circ + 180^\circ k$

If we let $k = 0$ and 1, we get

$\theta = 67.5^\circ$ $\quad\theta = 157.5^\circ$

$\theta = 247.5^\circ$ $\quad\theta = 337.5^\circ$

5. $\cos 3\theta = -1$

$3\theta = 180^\circ + 360^\circ k$

$\theta = 60^\circ + 120^\circ k$

If we let $k = 0$, 1, and 2, we get

$\theta = 60^\circ$ $\quad\theta = 300^\circ$

$\theta = 180^\circ$

7. $\sin 2x = \frac{1}{\sqrt{2}}$

$2x = \frac{\pi}{4} + 2k\pi$ or $2x = \frac{3\pi}{4} + 2k\pi$

$x = \frac{\pi}{8} + k\pi$ or $x = \frac{3\pi}{8} + k\pi$

If we let $k = 0$ and 1, we get

$x = \frac{\pi}{8}$ $\quad x = \frac{3\pi}{8}$

$x = \frac{9\pi}{8}$ $\quad x = \frac{11\pi}{8}$

9. $\sec 3x = -1$

$\cos 3x = -1$ (Reciprocal identity)

$3x = \pi + 2k\pi$

$x = \frac{\pi}{3} + \frac{2k\pi}{3}$

If we let $k = 0$, 1, and 2, we get

$\theta = \frac{\pi}{3}$ $\quad\theta = \frac{\pi}{3} + \frac{4\pi}{3} = \frac{5\pi}{3}$

$\theta = \frac{\pi}{3} + \frac{2\pi}{3} = \pi$

11. $\tan 2x = \sqrt{3}$

$2x = \frac{\pi}{3} + k\pi$

$x = \frac{\pi}{6} + \frac{k\pi}{2}$

If we let $k = 0$, 1, 2 and 3, we get

$x = \frac{\pi}{6}$ $\quad x = \frac{7\pi}{6}$

$x = \frac{4\pi}{6} = \frac{2\pi}{3}$ $\quad x = \frac{10\pi}{6} = \frac{5\pi}{3}$

13. $\sin 2\theta = \frac{1}{2}$

$2\theta = 30^\circ + 360^\circ k$ or $2\theta = 150^\circ + 360^\circ k$

$\theta = 15^\circ + 180^\circ k$ $\quad\theta = 75^\circ + 180^\circ k$

15. $\cos 3\theta = 0$

$3\theta = 90^\circ + 360^\circ k$ or $3\theta = 270^\circ + 360^\circ k$

$\theta = 30^\circ + 120^\circ k$ $\quad\theta = 90^\circ + 120^\circ k$

17. $\sin 10\theta = \frac{\sqrt{3}}{2}$

$$10\theta = 60^\circ + 360^\circ k \quad \text{or} \quad 10\theta = 120^\circ + 360^\circ k$$
$$\theta = 6^\circ + 36^\circ k \qquad\qquad \theta = 12^\circ + 36^\circ k$$

25. $2\sin 2x\cos x + \cos 2x \sin x = \frac{1}{2}$

$$\sin(2x + x) = \frac{1}{2} \quad \text{(Sum formula)}$$
$$3x = \frac{\pi}{6} + 2k\pi \quad \text{or} \quad 3x = \frac{5\pi}{6} + 2k\pi$$
$$x = \frac{\pi}{18} + \frac{2k\pi}{3} \qquad x = \frac{5\pi}{18} + \frac{2k\pi}{3}$$
$$x = \frac{\pi + 12k\pi}{18} \qquad x = \frac{5\pi + 12k\pi}{3}$$

If we let $k = 0$, 1, and 2, we get

$$x = \frac{\pi}{18} \qquad x = \frac{5\pi}{18}$$
$$x = \frac{13\pi}{18} \qquad x = \frac{17\pi}{18}$$
$$x = \frac{25\pi}{18} \qquad x = \frac{29\pi}{18}$$

27. $\cos 2x \cos x - \sin 2x \sin x = -\frac{\sqrt{3}}{2}$

$$\cos(2x + x) = -\frac{\sqrt{3}}{2} \quad \text{(Sum formula)}$$
$$\cos 3x = -\frac{\sqrt{3}}{2}$$
$$3x = \frac{5\pi}{6} + 2k\pi \quad \text{or} \quad 3x = \frac{7\pi}{6} + 2k\pi$$
$$x = \frac{5\pi}{18} + \frac{2k\pi}{3} \qquad x = \frac{7\pi}{18} + \frac{2k\pi}{3}$$
$$x = \frac{5\pi + 12k\pi}{18} \qquad x = \frac{7\pi + 12k\pi}{18}$$

If we let $k = 0$, 1, and 2, we get

$$x = \frac{5\pi}{18} \qquad x = \frac{7\pi}{18}$$
$$x = \frac{17\pi}{18} \qquad x = \frac{19\pi}{18}$$
$$x = \frac{29\pi}{18} \qquad x = \frac{31\pi}{18}$$

29. $\sin 3x \cos 2x + \cos 3x \sin 2x = 1$

$$\sin(3x + 2x) = 1$$
$$5x = \frac{\pi}{2} + 2k\pi$$
$$x = \frac{\pi}{10} + \frac{2k\pi}{5}$$

31. $\sin^2 4x = 1$

$\sin 4x = \pm 1$

$$4x = \frac{\pi}{2} + 2k\pi \quad \text{or} \quad 4x = \frac{3\pi}{2} + 2k\pi$$

$$x = \frac{\pi}{8} + \frac{k\pi}{2} \qquad x = \frac{3\pi}{8} + \frac{k\pi}{2}$$

We could also write this as $4x = \frac{\pi}{2} + k\pi$ or $x = \frac{\pi}{8} + \frac{k\pi}{4}$

33. $\cos^3 5x = -1$

$\cos x = -1$

$5x = \pi + 2k\pi$

$$x = \frac{\pi}{5} + \frac{2k\pi}{5}$$

35. $2\sin^2 3\theta + \sin 3\theta - 1 = 0$

$(\sin 3\theta - 1)(\sin 3\theta + 1) = 0$

$2\sin 3\theta - 1 = 0$ or $\sin 3\theta + 1 = 0$

$2\sin 3\theta = 1$ $\qquad$ $\sin 3\theta = -1$

$\sin 3\theta = \frac{1}{2}$ $\qquad$ $3\theta = 270^\circ + 360^\circ k$

$3\theta = 30^\circ + 360^\circ k$ or $3\theta = 150^\circ + 360^\circ k$ $\qquad$ $\theta = 90^\circ + 120^\circ k$

$\theta = 10^\circ + 120^\circ k$ $\qquad$ $\theta = 50^\circ + 120^\circ k$

37. $2\cos^2 2\theta + 3\cos 2\theta + 1 = 0$

$(2\cos 2\theta + 1)(\cos 2\theta + 1) = 0$

$2\cos 2\theta + 1 = 0$ or $\cos 2\theta + 1 = 0$

$2\cos 2\theta = -1$ $\qquad$ $2\theta = 180^\circ + 360^\circ k$

$\cos 2\theta = -\frac{1}{2}$ $\qquad$ $\theta = 90^\circ + 180^\circ k$

$2\theta = 120^\circ + 360^\circ k$ or $2\theta = 240^\circ + 360^\circ k$

$\theta = 60^\circ + 180^\circ k$ $\qquad$ $\theta = 120^\circ + 180^\circ k$

39. $\tan^2 3\theta = 3$

$\tan 3\theta = \pm\sqrt{3}$

$3\theta = 60^\circ + 180^\circ k$ or $3\theta = 120^\circ + 180^\circ k$

$\theta = 20^\circ + 60^\circ k$ $\qquad$ $\theta = 40^\circ + 60^\circ k$

41. $\cos\theta - \sin\theta = 1$

$\cos^2\theta - 2\sin\theta\cos\theta + \sin^2\theta = 1$ Square both sides

$-2\sin\theta\cos\theta + 1 = 1$ Pythagorean identity

$\sin 2\theta = 0$ Double-angle identity

$2\theta = 0^\circ + 360^\circ k$ or $2\theta = 180^\circ + 360^\circ k$

$\theta = 180^\circ k$ $\theta = 90^\circ + 180^\circ k$

If we let $k = 0$ and 1, we get

$\theta = 0^\circ$ $\theta = 90^\circ$

$\theta = 180^\circ$ $\theta = 270^\circ$

Since we squared both sides, we must check. Only 0° and 270° check.

43. $\sin\theta + \cos\theta = -1$

$\sin^2 + 2\sin\theta\cos\theta + \cos^2\theta = 1$ Square both sides

$1 + 2\sin\theta\cos\theta = 1$ Pythagorean identity

$\sin 2\theta = 0$ Double-angle identity

$2\theta = 0^\circ + 360^\circ k$ or $2\theta = 180^\circ + 360^\circ k$

$\theta = 180^\circ k$ $\theta = 90^\circ + 180^\circ k$

If we let $k = 0$ and 1, we get

$\theta = 0^\circ$ $\theta = 90^\circ$

$\theta = 180^\circ$ $\theta = 270^\circ$

Since we squared both sides, we must check. Only 180° and 270° check.

45. $\sin^2 2\theta - 4\sin 2\theta - 1 = 0$

We use the quadratic formula with $a = 1, b = -4, c = -1$,

$$\sin 2\theta = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{20}}{2}$$

$\sin 2\theta = \dfrac{4 + \sqrt{20}}{2}$ or $\sin 2\theta = \dfrac{4 - \sqrt{20}}{2}$

$= 4.236$

No solution

$= -0.236$

$2\theta = 193.7^\circ + 360^\circ k$ or $2\theta = 346.3^\circ + 360^\circ k$

$\theta = 96.8^\circ + 180^\circ k$ $\theta = 173.2^\circ + 180^\circ k$

If we let $k = 0$ and 1, we get:

$\theta = 96.8^\circ$ $\theta = 173.2^\circ$

$\theta = 276.8^\circ$ $\theta = 353.2^\circ$

47. $4\cos^2 3\theta - 8\cos 3\theta + 1 = 0$ $\qquad a = 4, b = -8, c = 1$

$$\cos 3\theta = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(4)(1)}}{2(4)}$$

$$= \frac{8 \pm \sqrt{48}}{8}$$

$$\cos 3\theta = \frac{8 \pm 6.9282}{8}$$

$\cos 3\theta = 1.8660$ or $\cos 3\theta = 0.1340$

No solution

$3\theta = 82.3^\circ + 360^\circ k \qquad 3\theta = 277.7^\circ + 360^\circ k$

$\theta = 27.4^\circ + 120^\circ k \qquad \theta = 92.6^\circ + 120^\circ k$

If we let $k = 0$, 1 and 2, we get

$\theta = 27.4^\circ \qquad \theta = 92.6^\circ$

$\theta = 147.4^\circ \qquad \theta = 212.6^\circ$

$\theta = 267.4^\circ \qquad \theta = 332.6^\circ$

49.

$$2\cos^2 4\theta + 2\sin 4\theta = 1$$

$$2\left(1 - \sin^2 4\theta\right) + 2\sin 4\theta - 1 = 0 \qquad \text{Pythagorean identity}$$

$$2 - 2\sin^2 4\theta + 2\sin 4\theta - 1 = 0$$

$$-2\sin^2 4\theta + 2\sin 4\theta + 1 = 0$$

$$2\sin^2 4\theta - 2\sin 4\theta - 1 = 0$$

We apply the quadratic formula with $a = 2$, $b = -2$, and $c = -1$:

$$\sin 4\theta = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(-1)}}{2(2)}$$

$$= \frac{2 \pm \sqrt{12}}{4}$$

$\sin 4\theta = \frac{2 + \sqrt{12}}{4} = 1.366$ or $\sin 4\theta = \frac{2 - \sqrt{12}}{4} = -0.366$

No solution

$4\theta = 201.5^\circ + 360^\circ k$ or $4\theta = 338.5^\circ + 360^\circ k$

$\theta = 50.4^\circ + 90^\circ k \qquad \theta = 84.6^\circ + 90^\circ k$

If we let $k = 0$, 1, 2, and 3, we get

$\theta = 50.4^\circ \qquad \theta = 84.6^\circ$

$\theta = 140.4^\circ \qquad \theta = 174.6^\circ$

$\theta = 230.4^\circ \qquad \theta = 264.6^\circ$

$\theta = 320.4^\circ \qquad \theta = 354.6^\circ$

51. We want to find t when $h=100$.

$$100=139-125\cos\frac{\pi}{10}t$$

$$125\cos\frac{\pi}{10}t=39$$

$$\cos\frac{\pi}{10}t=0.312$$

$$\frac{\pi}{10}t=1.253+2\pi k \quad \text{or} \quad \frac{\pi}{10}t=(2\pi-1.253)+2\pi k$$

$$t=4.0+20k \qquad \frac{\pi}{10}t=5.030+2\pi k$$

$$t=16.0+20k \text{ (where } k=0, 1, 2, 3, \ldots, 9)$$

It will be at 100 ft after 4.0 min, 16.0 min, 24.0 min. 28.0 min, and so on.

53. $l=2r\sin\frac{180^\circ}{n}$ where $l=r$

$$r=2r\sin\frac{180^\circ}{n}$$

$$\sin\frac{180^\circ}{n}=\frac{r}{2r}$$

$$\sin\frac{180^\circ}{n}=\frac{1}{2}$$

$$\frac{180^\circ}{n}=30^\circ \quad \text{or} \quad \frac{180^\circ}{n}=150^\circ$$

$$180^\circ=30^\circ n \qquad 180^\circ=150^\circ n$$

$$n=6 \qquad n=1.2$$

Not possible

The polygon has 6 sides.

55. $d=10\tan\pi t$ where $d=10$

$$10=10\tan\pi t$$

$$\tan\pi t=1$$

$$\pi t=\frac{\pi}{4}+k\pi$$

$$t=\frac{1}{4}+k$$

t is $\frac{1}{4}$ second and every second after that.

57. $\sin 2\pi t = \frac{1}{2}$

$$2\pi t = \frac{\pi}{6} + 2k\pi$$

$$t = \frac{1}{12} + k \qquad \text{Let } k = 0\text{, then } t = \frac{1}{12}.$$

59. $\frac{\sin x}{1+\cos x} = \frac{\sin x}{1+\cos x} \cdot \frac{1-\cos x}{1-\cos x}$ Multiply by a fraction equal to one

$= \frac{\sin x(1-\cos x)}{1-\cos^2 x}$ Multiply

$= \frac{\sin x(1-\cos x)}{\sin^2 x}$ Pythagorean identity

$= \frac{1-\cos x}{\sin x}$ Reduce

61.

$$\frac{1}{1+\cos t} + \frac{1}{1-\cos t} = \frac{1}{1+\cos t} \cdot \frac{1-\cos t}{1-\cos t} + \frac{1}{1-\cos t} \cdot \frac{1+\cos t}{1+\cos t}$$

$$= \frac{1-\cos t}{1-\cos^2 t} + \frac{1+\cos t}{1-\cos^2 t}$$

$$= \frac{1-\cos t + 1 + \cos t}{1-\cos^2 t}$$

$$= \frac{2}{\sin^2 t}$$

$$= 2\csc^2 t$$

63. $\tan\frac{A}{2} = \frac{\sin\frac{A}{2}}{\cos\frac{A}{2}}$ Ratio identity

$= \frac{\sqrt{\frac{1-\cos A}{2}}}{\sqrt{\frac{1+\cos A}{2}}}$ Half-angle identities

$= \frac{\sqrt{1-\cos A}}{\sqrt{1+\cos A}}$ Divide

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$$=\frac{\sqrt{1-\cos A}}{\sqrt{1+\cos A}}\cdot\frac{\sqrt{1+\cos A}}{\sqrt{1+\cos A}}$$ Multiply by a fraction equal to one

$$=\frac{\sqrt{1-\cos^2 A}}{1+\cos A}$$ Multiply

$$=\frac{\sqrt{\sin^2 A}}{1+\cos A}$$ Pythagorean identity

$$=\frac{\sin A}{1+\cos A}$$ Simplify

65. If $\sin A=\frac{1}{3}$ and A is in QII, then

$$\cos A=-\sqrt{1-\sin^2 A}$$
$$=\sqrt{1-\frac{1}{9}}=-\sqrt{\frac{8}{9}}=-\frac{2\sqrt{2}}{3}$$
$$\sin 2A=2\sin A\cos A$$
$$=2\left(\frac{1}{3}\right)\left(-\frac{2\sqrt{2}}{3}\right)$$
$$=-\frac{4\sqrt{2}}{9}$$

67. If $90^\circ\le A\le 180^\circ$, then $45^\circ\le\frac{A}{2}\le 90^\circ$

Also, $\sin A=\frac{1}{3}$ and $\cos A=-\frac{2\sqrt{2}}{3}$ (from problem 65)

Therefore, $\cos\frac{A}{2}=\sqrt{\frac{1+\cos A}{2}}$

$$=\sqrt{\frac{1+\left(\frac{-2\sqrt{2}}{3}\right)}{2}}$$
$$=\sqrt{\frac{3-2\sqrt{2}}{6}}$$

69. If $\sin B=\frac{3}{5}$ with B in QI, then

$$\cos B=\sqrt{1-\sin^2 B}$$
$$=\sqrt{1-\frac{9}{25}}=\sqrt{\frac{16}{25}}=\frac{4}{5}$$

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