Name: ______________________________________ Per: _______ Date: ________________
Serafino ▪ Precalculus

6A Using Basic Identities to Solve Equations

We have done proofs using basic identities. We have already solved equations. Now we put them together.
ALL methods of solving equations still apply – factoring, square roots, etc…

1) Using Reciprocal/Ratio Identities: $\mathbf{2\cos x - 1 = \sec x}$

We cannot solve an equation with two variables at the same time (unless they are parts of individual factors given to us that we merely have to set equal to zero… but that's not the case here). So we look to get them into the same.

$2\cos x - 1 = \frac{1}{\cos x}$ We turn secant into 1/cosine…
Well.. now we have to get that cosine out of the denominator if we want to solve.

$2\cos^2 x - \cos x = 1$ Multiply each term on both sides by a cos x.

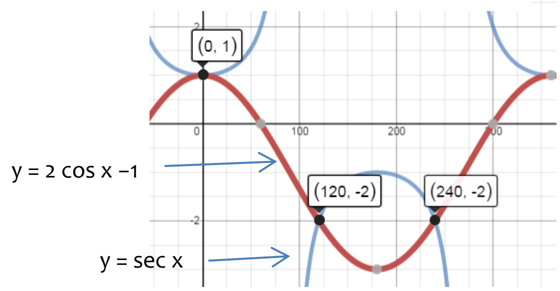
$2\,u^2 - u = 1$ Let: u = cos x
$2\,u^2 - u - 1 = 0$
$(2u + 1)(u - 1) = 0 \quad u = -\frac{1}{2}\ or\ u = 1$
So …. cos x = –1/2 or 1

YOU MUST CHECK FOR EXTRANEOUS!! We did not work with the original equation. We changed a function that is undefined in some places to a function that is defined across the reals… AND we turns a linear variable into a quadratic one, potentially getting us more solutions than the original had. We have to check.

Potential Solutions x ∈ [0, 360): x = 0°, 120° or 240°.

Check: 2cos(0) – 1 = sec (0)? 2(1) – 1 = 1? 1 = 1 ✓
2cos(120) – 1 = sec (120)? $2(-\frac{1}{2}) - 1 =$ –2? –2 = –2 ✓
2cos(240) – 1 = sec (240)? $2(-\frac{1}{2}) - 1 =$ –2? –2 = –2 ✓

They all work! And we can confirm by graphing →

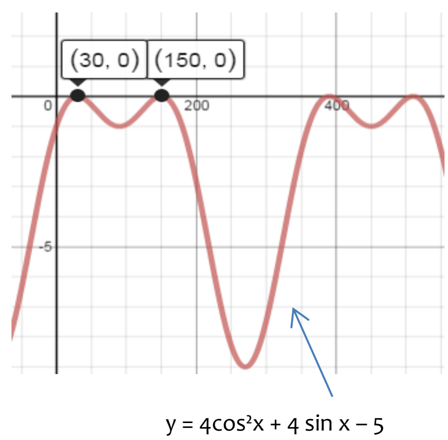


Solutions: x = 0°, 120° 240°

2) Using Pythagorean Identities $\mathbf{4\cos^2 x + 4\sin x - 5 = 0}$

Again, we can't solve for two variables.

$4(1 - \sin^2 x) + 4\sin x - 5 = 0.$ Turn cos² into 1–sin²
$4 - 4\sin^2 x + 4\sin x - 5 = 0$ Distribute and simplify the left side.
$4\sin^2 x - 4\sin x + 1 = 0$
$4u^2 - 4u + 1 = 0$ Let: u = sin x
$(4u - 2)(4u - 2) = 0$
$(2u - 1)(2u - 1) = 0,$ u= 1/2



Potential Solutions: x = 30°, 150°
We didn't really change anything about the original equation so we don't really have to check for extraneous, but let's do it anyway.

Check: 4cos²(30) + 4sin(30)–5 = 0? 4(3/4) +4(1/2)–5 = 0? 3 + 2 –5 = 0 ✓
4cos²(150) +4sin(150)–5 = 0? 4(3/4) +4(1/2)–5 = 0? 3 + 2 –5 = 0 ✓

Solutions: x = 30°, 150°

3a) Isolate One Variable & Square Both Sides: $\sin x - \cos x = 1$

Isolate One Variable, then Square

$\sin x = 1 + \cos x$	Add cos x to both sides.
$(\sin x)^2 = (\cos x + 1)^2$	Square both sides
$1 - \cos^2 x = (\cos x + 1)^2$	Turn $\sin^2 x$ into $1 - \cos^2 x$.
$1 - \cos^2 x = \cos^2 x + cosx + 1$	Combine/move like terms

$2\cos^2 x + 2\cos x = 0$
$2\cos x\,(\cos x + 1) = 0$
$\cos x = 0, \quad \cos x = -1$

Potential Solutions: x = 90°, 270°, 180°

Square with Both Variables

$(\sin x - \cos x)^2 = (1)^2$	Square both sides.
$\sin^2 x - 2\sin x\cos x + \cos^2 x = 1$	Expand…..
$1 - 2\sin x\cos x = 1$	$\sin^2 + \cos^2 = 1$
$-2\sin x\cos x = 0$	Combine terms

* *Note:* Whew! The 1's canceled out and we were left with =0. *If we had −2sin x cos x = 1 we would be stuck unless we used a Double Angle identity… which we WILL learn.*

$2\sin x\cos x = 0$
$\sin x = 0 \qquad \cos x = 0$

Potential Solutions: x = 90°, 270°, 0°, 180°

CHECK FOR EXTRANEOUS! Quadratics have MORE solutions than linear equations so we may get some solutions that do not satisfy the original problem.

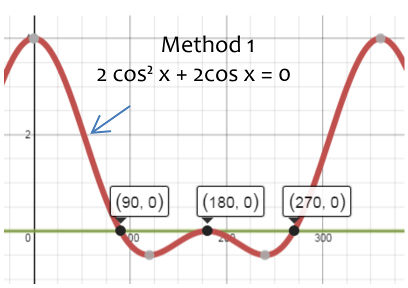
sin(90) – cos(90) = 1?	1 – 0= 1?	1 = 1 ✓
sin(270) – cos(270) = 1?	–1 – 0= 1?	–1 = 1 ×
sin(180) – cos(180) = 1?	0 – –1 = 1?	1 = 1 ✓

Solutions: x = 90 °, 180°
(Extraneous solutions: 270°)

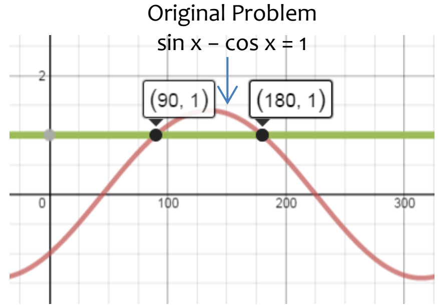
sin(90) – cos(90) = 1?	1 – 0= 1?	1 = 1 ✓
sin(270) – cos(270) = 1?	–1 – 0= 1?	–1 = 1 ×
sin(0) – cos(0) = 1?	0 –1 = 1?	–1 = 1 ×
sin(180) – cos(180) = 1?	0 – –1 = 1?	1 = 1 ✓

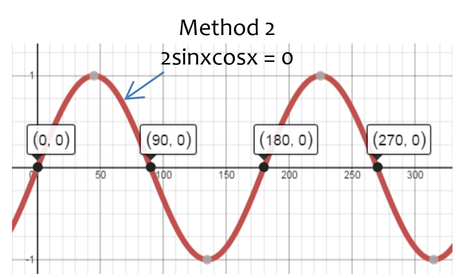
Solutions: x = 90°, 180°
(Extraneous solutions: 0 °, 270°)

Here are the graphs of each equation, so you can see what they look like and where the extraneous solutions come from:



Original Problem





6.2 Problem Set: Solve for 0° ≤ x < 360°. State any extraneous solutions that arose from your method.

1. $\sqrt{3}\sec x = 2$
3. $\sqrt{2}\csc x + 5 = 3$
5. $4\sin x - 2\csc x = 0$
7. $\sec x - 2\tan x = 0$
11. $2\cos x + 1 = \sec x$
17. $2\cos^2 x + \sin x - 1 = 0$
19. $4\sin^2 x + 4\cos x - 5 = 0$
21. $2\sin x + \cot x - \csc x = 0$
25. $\sqrt{3}\sin x + \cos x = \sqrt{3}$
27. $\sqrt{3}\sin x - \cos x = 1$
33. $6\cos x + 7\tan x = \sec x$

* Challenges *

23. $\sin x + \cos x = \sqrt{2}$
35. $23\csc^2 x - 22\cot x\csc x - 15 = 0$