7.4: Function Operations and Composition of Functions

[Algebra 2(Y)]

HCPS III

- ***Standard 9***: Patterns, Functions, and Algebra: PATTERNS AND FUNCTIONAL RELATIONSHIPS: Understand various types of patterns and functional relationships.
- ***Benchmark MA.AII.9.4:*** Use the appropriate terminology and notation to define functions and their properties (e.g., domain, range, function composition, inverse, zeros).

Goal: Perform operations with functions, including composition of functions.

Operations on Functions

Operations on Functions

Let $f(x)$ and $g(x)$ be any two functions. You can add, subtract, multiply, or divide $f(x)$ and $g(x)$ to form a new function $h(x)$.

Operation	Definition	Example Let $f(x) = 2x$ and $g(x) = x + 1$.
Addition	$h(x) = f(x) + g(x)$	$h(x) = 2x + (x + 1) = 3x + 1$
Subtraction	$h(x) = f(x) - g(x)$	$h(x) = 2x - (x + 1) = x - 1$
Multiplication	$h(x) = f(x) \cdot g(x)$	$h(x) = (2x)(x + 1) = 2x^2 + 2x$
Division	$h(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$	$h(x) = \frac{2x}{x + 1}, x \neq -1$

The domain of h consists of the x-values that are in the domains of both f and g. When h involves division, the domain does not include x-values for which the denominator is equal to zero.

Example 1: Add and Subtract Functions

Let $f(x) = 3x^2$ and $g(x) = x - 1$. Find $h(x)$ and state its domain.

a.) $h(x) = f(x) + g(x)$

$= 3x^2 + (x - 1)$

$= \boxed{3x^2 + x - 1}$

b.) $h(x) = f(x) - g(x)$

$= 3x^2 - (x - 1)$

$= \boxed{3x^2 - x + 1}$

The domains of f+g and f−g are all real #'s.

Extra Example 1: Add and Subtract Functions

Let $f(x) = 4x^2$ and $g(x) = x + 1$. Find $h(x)$ and state its domain.

a.) $h(x) = f(x) + g(x)$

$= 4x^2 + (x+1)$

$= \boxed{4x^2 + x + 1}$

b.) $h(x) = f(x) - g(x)$

$= 4x^2 - (x + 1)$

$= \boxed{4x^2 - x - 1}$

The domains of f and g are all real #'s.
The domain of h is all real #'s

Example 2: Multiply and Divide Functions

Let $f(x) = x^4$ and $g(x) = 3x$. Find $h(x)$ and state its domain.

a.) $h(x) = f(x) \bullet g(x)$

$= (x^4)(3x)$

$= 3x^5$

The domains of f·g is all real #'s.

b.) $h(x) = \frac{f(x)}{g(x)}$

$= \frac{x^4}{3x}$

$= \frac{x^3}{3}$ or $\frac{1}{3}x^3$

The domain of $\frac{f}{g}$ is all real #'s except 0.

Extra Example 2: Multiply and Divide Functions

Let $f(x) = x^3$ and $g(x) = 2x$. Find $h(x)$ and state its domain.

a.) $h(x) = f(x) \bullet g(x)$

$= (x^3)(2x)$

$= 2x^4$

The domains of both f and g are all real #'s. So, the domain of h(x) is all real #'s.

b.) $h(x) = \frac{f(x)}{g(x)}$

$= \frac{x^3}{2x}$

$= \frac{x^2}{2}$ or $\frac{1}{2}x^2$

The domain is all real #'s except 0.

Composition of Functions

Composition of Functions: replacing the variable of a function with an entirely different function.

e.g., let $f(x) = 2x$ and $g(x) = 5x + 3$

The composition of f with g is:

$$\begin{aligned} f(g(x)) &= f(5x+3) \\ &= 2(5x+3) \\ &= 10x + 6 \end{aligned}$$

The composition of g with f is:

$$\begin{aligned} g(f(x)) &= g(2x) \\ &= 5(2x) + 3 \\ &= 10x + 3 \end{aligned}$$

Example 3: Write a Composition of Functions

Let $f(x) = x^2$ and $g(x) = 3x - 1$. Find the following.

a.) $f(g(x))$

$= f(3x-1)$

$= (3x-1)^2$

$= \boxed{9x^2 - 6x + 1}$

b.) $g(f(x))$

$= g(x^2)$

$= 3(x^2) - 1$

$= \boxed{3x^2 - 1}$

c.) the domain of each composition.

The domain of $f(g(x))$ is all real #'s.
The domain of $g(f(x))$ is all real #'s

Extra Example 3: Write a Composition of Functions

Let $f(x) = x^2$ and $g(x) = 2x + 3$. Find the following.

a.) $f(g(x))$

$= f(2x+3)$

$= (2x+3)^2$

$= \boxed{4x^2 + 12x + 9}$

b.) $g(f(x))$

$= g(x^2)$

$= 2(x^2) + 3$

$= \boxed{2x^2 + 3}$

c.) the domain of each composition.

The domain of the composition is all real #'s.

Example 4: Evaluate a Composition of Functions

a.) Let $f(x) = x^2 - 4$ and $g(x) = 4x$. Evaluate $f(g(2))$.

① Find $g(2)$

$g(2) = 4x$
$= 4(2)$
$= 8$

② Substitute $g(2)$ into $f(g(2))$.

$f(g(2)) = x^2 - 4$
$= (8)^2 - 4$
$= 64 - 4$
$= \boxed{60}$

b.) Let $f(x) = x^2 + 3$ and $g(x) = 5x$. Evaluate $f(g(2))$.

① Find $g(2)$

$g(2) = 5x$
$= 5(2)$
$= 10$

② Substitute $g(2)$ into $f(g(2))$.

$f(g(2)) = x^2 + 3$
$= (10)^2 + 3$
$= 100 + 3$
$= \boxed{103}$

Example 5: Model a Real-World Situation

You have a coupon for \$15 off all Spring clothes over \$50. For this weekend only, the department store is offering an additional 20% off all purchases. Let x be the cost of your purchases. Then $f(x) = x - 15$ and $g(x) = 0.80x$. Find $g(f(x))$. Tell what it represents.

$g(f(x)) = g(x - 15)$
$= 0.80(x - 15)$
$= \boxed{0.80x - 12}$

It represents the cost of your purchase when the \$15 coupon is applied before the 20% discount.