CHAPTER 7 Triangles

Problem Set 7.1

1.

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$ Law of Sines

$$\frac{b}{\sin 60^\circ} = \frac{12}{\sin 40^\circ}$$ Substitute known values

$$b = \frac{12 \sin 60^\circ}{\sin 40^\circ}$$ Multiply both sides by sin 60°

$$= 16 \text{ cm}$$ Round to 2 significant digits

3.

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$ Law of Sines

$$\frac{b}{\sin 120^\circ} = \frac{28}{\sin 20^\circ}$$ Substitute known values

$$b = \frac{28 \sin 120^\circ}{\sin 20^\circ}$$ Multiply both sides by sin 120°

$$= 71 \text{ inches}$$ Round to 2 significant digits

5.

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$ Law of Sines

$$\frac{c}{\sin 100^\circ} = \frac{24}{\sin 10^\circ}$$ Substitute known values

$$c = \frac{24 \sin 100^\circ}{\sin 10^\circ}$$ Multiply both sides by sin 100°

$$= 140 \text{ yards}$$ Round to 2 significant digits

7.

$$C = 180^\circ - (A + B)$$
$$= 180^\circ - (50^\circ + 60^\circ)$$
$$= 180^\circ - 110^\circ = 70^\circ$$

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$ Law of Sines

$$\frac{c}{\sin 70^\circ} = \frac{36}{\sin 50^\circ}$$ Substitute known values

$$c = \frac{36 \sin 70^\circ}{\sin 50^\circ}$$ Multiply both sides by sin 70°

$$= 44 \text{ km}$$ Round to 2 significant digits

9. $C = 180^\circ - (A + B)$
$= 180^\circ - (52^\circ + 48^\circ)$
$= 180^\circ - 100^\circ = 80^\circ$

$\dfrac{a}{\sin A} = \dfrac{c}{\sin C}$ Law of Sines

$\dfrac{a}{\sin 52^\circ} = \dfrac{14}{\sin 80^\circ}$ Substitute known values

$a = \dfrac{14 \sin 52^\circ}{\sin 80^\circ}$ Multiply both sides by sin 52°

$= 11 \text{ cm}$ Round to 2 significant digits

11. $C = 180^\circ - (A + B)$
$= 180^\circ - (42.5^\circ + 71.4^\circ)$
$= 180^\circ - 113.9^\circ = 66.1^\circ$

$\dfrac{c}{\sin C} = \dfrac{a}{\sin A}$

$\dfrac{c}{\sin 66.1^\circ} = \dfrac{215}{\sin 42.5^\circ}$

$c = \dfrac{215 \sin 66.1^\circ}{\sin 42.5^\circ}$

$= 291 \text{ inches}$

$\dfrac{b}{\sin B} = \dfrac{a}{\sin A}$

$\dfrac{b}{\sin 71.4^\circ} = \dfrac{215}{\sin 42.5^\circ}$

$b = \dfrac{215 \sin 71.4^\circ}{\sin 42.5^\circ}$

$= 302 \text{ inches}$

13. $C = 180^\circ - (A + B)$
$= 180^\circ - (46^\circ + 95^\circ)$
$= 180^\circ - 141^\circ = 39^\circ$

$\dfrac{a}{\sin A} = \dfrac{c}{\sin C}$

$\dfrac{a}{\sin 46^\circ} = \dfrac{6.8}{\sin 39^\circ}$

$a = \dfrac{6.8 \sin 46^\circ}{\sin 39^\circ}$

$= 7.8 \text{ meters}$

$\dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$\dfrac{b}{\sin 95^\circ} = \dfrac{6.8}{\sin 39^\circ}$

$b = \dfrac{6.8 \sin 95^\circ}{\sin 39^\circ}$

$= 11 \text{ meters}$

15. $B = 180^\circ - (A + C)$
$= 180^\circ - (43.5^\circ + 120.5^\circ)$
$= 180^\circ - 164^\circ = 16^\circ$

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$$\frac{b}{\sin B}=\frac{a}{\sin A}$$
$$\frac{b}{\sin 16^\circ}=\frac{3.48}{\sin 43.5^\circ}$$
$$b=\frac{3.48\sin 16^\circ}{\sin 43.5^\circ}=1.39 \text{ ft}$$

$$\frac{c}{\sin C}=\frac{a}{\sin A}$$
$$\frac{c}{\sin 120.5^\circ}=\frac{3.48}{\sin 43.5^\circ}$$
$$c=\frac{3.48\sin 120.5^\circ}{\sin 43.5^\circ}=4.36 \text{ ft}$$

17. $A=180^\circ-(B+C)$
$=180^\circ-(13.4^\circ+24.8^\circ)$
$=180^\circ-38.2^\circ = 141.8^\circ$

$$\frac{b}{\sin B}=\frac{a}{\sin A}$$
$$\frac{b}{\sin 13.4^\circ}=\frac{315}{\sin 141.8^\circ}$$
$$b=\frac{315\sin 13.4^\circ}{\sin 141.8^\circ}$$
$$=118 \text{ cm}$$

$$\frac{c}{\sin C}=\frac{b}{\sin B}$$
$$\frac{c}{\sin 24.8^\circ}=\frac{315}{\sin 141.8^\circ}$$
$$c=\frac{315\sin 24.8^\circ}{\sin 141.8^\circ}$$
$$=214 \text{ cm}$$

19. $$\frac{\sin B}{b}=\frac{\sin A}{a}$$
$$\frac{\sin B}{20}=\frac{\sin 30^\circ}{2}$$
$$\sin B=\frac{20\sin 30^\circ}{2}=5$$
This is impossible because the sine function must be between – 1 and 1.

21.

$s=r\theta \quad (\theta \text{ is } \angle C)$	Arc length formula
$11=12\cdot\theta$	Substitute known values
$\theta=\frac{11}{12}$	Divide both sides by 12

$\angle C=\frac{11}{12}$ radians. Converting this to degrees, we get:

$$\angle C=\left(\frac{11}{12}\cdot\frac{180}{\pi}\right)^\circ=53^\circ$$

Also: $D=180^\circ-(C+A)$
$=180^\circ-(53^\circ+31^\circ)$
$=180^\circ-84^\circ=96^\circ$

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Using the Law of Sines, we find x:

$$\frac{x+r}{\sin D} = \frac{r}{\sin A}$$

$$\frac{x+12}{\sin 96^\circ} = \frac{12}{\sin 31^\circ}$$

$$x+12 = \frac{12\sin 96^\circ}{\sin 31^\circ}$$

$$x+12 = 23$$

$$x = 11$$

23. $s = r\theta$ (θ is $\angle C$) Arc length formula

$18 = 15\cdot\theta$ Substitute known values

$\theta = \frac{18}{15} = \frac{6}{5}$ Divide both sides by 12

$\angle C = \frac{6}{5}$ radians. Converting this to degrees, we get:

$$\angle C = \left(\frac{6}{5}\cdot\frac{180}{\pi}\right)^\circ = 69^\circ$$

Also:
$$D = 180^\circ - (C + A)$$
$$= 180^\circ - (69^\circ + 45^\circ)$$
$$= 180^\circ - 114^\circ = 66^\circ$$

Using the Law of Sines, we find y:

$$\frac{y}{\sin C} = \frac{r}{\sin A}$$

$$\frac{y}{\sin 69^\circ} = \frac{15}{\sin 45^\circ}$$

$$y = \frac{15\sin 69^\circ}{\sin 45^\circ}$$

$$y = 20$$

25. We find the missing angles first:

$\angle ABD = 180^\circ - 64^\circ = 116^\circ$

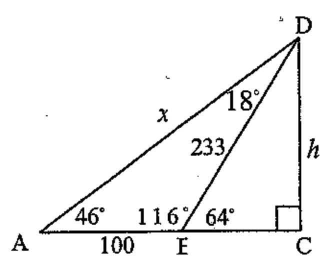
$\angle ADB = 180^\circ - (46^\circ + 116^\circ) = 18^\circ$

Now we find BD using the Law of Sines:

$$\frac{BD}{\sin A} = \frac{AB}{\sin ADB}$$

$$\frac{BD}{\sin 46^\circ} = \frac{100}{\sin 18^\circ}$$

$$BD = \frac{100\sin 46^\circ}{\sin 18^\circ} = 233$$



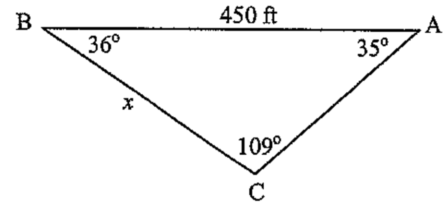
Then we find h, using the sine ratio:

$$\sin 64^\circ = \frac{h}{233}$$

$$h = 233\sin 64^\circ = 209 \text{ feet}$$

27. First, we'll find C:

$$C = 180^\circ - (36^\circ + 35^\circ)$$
$$= 180^\circ - 71^\circ = 109^\circ$$



Next, we'll find the distance that the man travels from A to B using the distance formula:

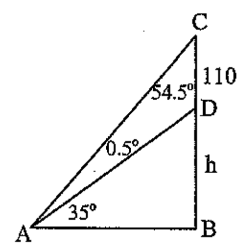
$$d = r \cdot t$$
$$= 5\frac{\text{ft}}{\text{sec}} \cdot 90\text{ sec} = 450\text{ ft}$$

Now we use the Law of Sines to find x:

$$\frac{x}{\sin 35^\circ} = \frac{450}{\sin 109^\circ}$$
$$x = \frac{450 \sin 35^\circ}{\sin 109^\circ} = 273\text{ ft}$$

29. We find the missing angles first:

$$\angle ADB = 90^\circ - 35^\circ = 55^\circ$$
$$\angle ADC = 180^\circ - 55^\circ = 125^\circ$$
$$\angle C = 180^\circ - \left(125^\circ - 0.5^\circ\right) = 54.5^\circ$$



Now we find AD using the Law of Sines:

$$\frac{AD}{\sin 54.5^\circ} = \frac{110}{\sin 0.5^\circ}$$
$$AD = \frac{110 \sin 54.5}{\sin 0.5} = 10{,}262$$

Then we find h, using the sine ratio:

$$\sin 35^\circ = \frac{h}{10{,}262}$$
$$h = 10{,}262 \sin 35^\circ = 5{,}900\text{ ft}$$

31. Using triangle ABC on the ground, $C = 180^\circ - (105^\circ + 44^\circ) = 31^\circ$.

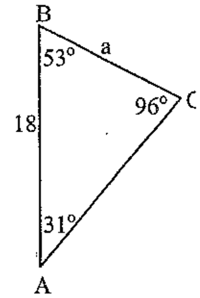
Now we can use the Law of Sines to find b:

$$\frac{b}{\sin 44^\circ} = \frac{25}{\sin 31^\circ}$$
$$b = \frac{25 \sin 44^\circ}{\sin 31^\circ} = 34$$

Then we can find h using the tangent ratio:

$$\tan 51^\circ = \frac{h}{34}$$
$$h = 34 \tan 51^\circ = 42\text{ ft}$$

33. We find the missing angle first: $\angle C = 180^\circ - (53^\circ + 31^\circ) = 96^\circ$

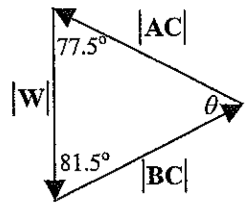


Then we find the missing sides using the Law of Sines:

$$\frac{18}{\sin 96^\circ} = \frac{a}{\sin 31^\circ} \qquad \frac{18}{\sin 96^\circ} = \frac{b}{\sin 53^\circ}$$

$$a = \frac{18 \sin 31^\circ}{\sin 96^\circ} = 9.3 \text{ miles} \qquad b = \frac{18 \sin 53^\circ}{\sin 96^\circ} = 14 \text{ miles}$$

35. We can redraw the two tension vectors **AC** and **BC** and the vector **W** due to gravity. We know that the magnitude of **W** is 125 pounds.



First, we find the missing angle:

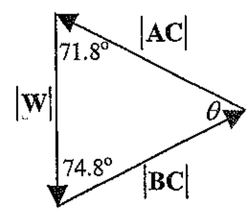
$\theta = 180^\circ - (77.5^\circ + 81.5^\circ) = 21^\circ$

Then, we find $|\mathbf{AC}|$ and $|\mathbf{BC}|$ using the Law of Sines:

$$\frac{|\mathbf{AC}|}{\sin 81.5^\circ} = \frac{125}{\sin 21^\circ} \qquad \frac{|\mathbf{BC}|}{\sin 77.5^\circ} = \frac{125}{\sin 21^\circ}$$

$$|\mathbf{AC}| = \frac{125 \sin 81.5^\circ}{\sin 21^\circ} = 345 \text{ lbs} \qquad |\mathbf{BC}| = \frac{125 \sin 77.5^\circ}{\sin 21^\circ} = 341 \text{ lbs}$$

37. We can redraw the two tension vectors **AC** and **BC** and the vector **W** due to gravity. We know that the magnitude of **W** is 1850 pounds.



First, we find the missing angle:

$\theta = 180^\circ - (71.8^\circ + 74.8^\circ) = 33.4^\circ$

Then, we find $|\mathbf{AC}|$ and $|\mathbf{BC}|$ using the Law of Sines:

$$\frac{|\mathbf{AC}|}{\sin 71.8^\circ} = \frac{1850}{\sin 33.4^\circ} \qquad \frac{|\mathbf{BC}|}{\sin 74.8^\circ} = \frac{1850}{\sin 33.4^\circ}$$

$$|\mathbf{AC}| = \frac{1850 \sin 71.8^\circ}{\sin 33.4^\circ} = 3190 \text{ lbs} \qquad |\mathbf{BC}| = \frac{1850 \sin 74.8^\circ}{\sin 33.4^\circ} = 3240 \text{ lbs}$$

39.

$$2\sin\theta - \sqrt{2} = 0$$
$$2\sin\theta = \sqrt{2}$$
$$\sin\theta = \frac{\sqrt{2}}{2}$$
$$\hat{\theta} = 45^\circ$$
$$\theta = 45^\circ \text{ or } 135^\circ$$

41.

$$\sin\theta\cos\theta - 2\cos\theta = 0$$
$$\cos\theta(\sin\theta - 2) = 0$$
$$\sin\theta = 2 \text{ or } \cos\theta = 0$$

No solution $\qquad \theta = 90^\circ \text{ or } 270^\circ$

43. $2\sin^2\theta - 3\sin\theta + 1 = 0$

$(2\sin\theta - 1)(\sin\theta - 1) = 0$

$2\sin\theta - 1 = 0$ or $\sin\theta - 1 = 0$

$\sin\theta = \dfrac{1}{2}$ $\qquad \sin\theta = 1$

$\theta = 30^\circ$ or 150° $\qquad \theta = 90^\circ$

45. $\cos^2\theta - 4\cos\theta + 2 = 0$ where a = 1, b = – 4, and c = 2

$$\cos\theta = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{8}}{2}$$

$\cos\theta = 3.4142$ or $\cos\theta = 0.5858$

No solution $\qquad \theta = 54.1^\circ$ or 305.9°

47. $(\sin x + 1)(2\sin x - 1) = 0$

$\sin x + 1 = 0$ or $\qquad 2\sin x - 1 = 0$

$\sin x = -1$ $\qquad \sin x = \dfrac{1}{2}$

$x = \dfrac{3\pi}{2} + 2k\pi$ $\qquad x = \dfrac{\pi}{6} + 2k\pi$ or $x = \dfrac{5\pi}{6} + 2k\pi$

49. $\sin\theta = 0.7380$

$\hat{\theta} = 47.6^\circ$ and θ is in QI or QII

$\theta = 47.6^\circ$ or 132.4°

51. $\sin\theta = 0.9668$

$\hat{\theta} = \sin^{-1}(0.9668) = 75.2^\circ$

$\theta = 75.2^\circ$ or 104.8°