Name: Answer Key Per: _____ Date: _____

Serafino • Precalculus S2

7AB Matrices (Chapters 14 & 12)

Mid–unit review & Applications

Matrix Operations: Use the matrices to answer the questions below. Do all work on a separate sheet of paper.

$$A=\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix} \quad B=\begin{bmatrix}-2 & 3 & 1\\ 4 & -5 & 0\end{bmatrix} \quad C=\begin{bmatrix}-4 & 1/2\\ 3 & -1\end{bmatrix} \quad D=\begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix}$$

$$E=\begin{bmatrix}2 & 0\\ 4 & -1\\ 6 & 3\end{bmatrix} \quad F=\begin{bmatrix}4 & 5 & -2\\ 1 & 3 & -4\\ 0 & 6 & 8\end{bmatrix} \quad G=\begin{bmatrix}1 & -5\\ 3 & 2\end{bmatrix} \quad H=\begin{bmatrix}5 & 1/4\\ 2 & -3\end{bmatrix}$$

1. C+D $\begin{bmatrix}-3 & 4.5\\ 5 & 2\end{bmatrix}$
2. B^t $\begin{bmatrix}-2 & 4\\ 3 & -5\\ 1 & 0\end{bmatrix}$
3. $-2F$ $\begin{bmatrix}-8 & -10 & 4\\ -2 & -6 & 8\\ 0 & 12 & -16\end{bmatrix}$
4. A^t+E $\begin{bmatrix}3 & 4\\ 6 & 4\\ 9 & 9\end{bmatrix}$
5. $(C+D)^T$ $\begin{bmatrix}-3 & 5\\ 4.5 & 2\end{bmatrix}$
6. $A-B$ $\begin{bmatrix}3 & -1 & 2\\ 0 & 10 & 6\end{bmatrix}$
7. D^t+G^t $\begin{bmatrix}2 & 5\\ -1 & 5\end{bmatrix}$
8. E^t+A $\begin{bmatrix}3 & 6 & 9\\ 4 & 4 & 9\end{bmatrix}$
9. 10D · 2G $\begin{bmatrix}12 & 30\\ 26 & 34\end{bmatrix}$
10. G·D + D·G $\begin{bmatrix}4 & -8\\ 18 & 14\end{bmatrix}$
11. $2G^2$ $\begin{bmatrix}-28 & -30\\ 18 & -22\end{bmatrix}$
12. Solve for X: $2G+X=H$ $\begin{bmatrix}3 & 10.25\\ -4 & -7\end{bmatrix}$

13. All of Serafino's Precalculus classes want to go on a fieldtrip to Six Flags, so decide to sell T–shirts. We sell both short sleeve and long sleeve. The tables show the quantities sold by each Period, and the price of each shirt.

	Short Sleeve	Long Sleeve
Period 1	20	15
Period 3	8	6
Period 5	32	14

	Price ($)
Short Sleeve	10
Long Sleeve	15

Create two matrices S and P that represent Sales & Price, and multiply them to get matrix R, the revenue generated by each class.

$S=\begin{bmatrix}20 & 15\\ 8 & 6\\ 32 & 14\end{bmatrix}$ $P=\begin{bmatrix}10\\ 15\end{bmatrix}$

$SP=\begin{bmatrix}425\\ 170\\ 530\end{bmatrix}$

	Total Revenue ($)
Period 1	425
Period 3	170
Period 5	530

Period 5 wins!

14. Alfie, Bernardo and Cassie are taking a college class and want to know their final course grade. Their grades are based on five tests, all weighted as shown. What matrix represents their grades?

$$T = \begin{array}{c} \begin{matrix} A & B & C \end{matrix} \\ \begin{bmatrix} 82 & 92 & 74 \\ 85 & 88 & 68 \\ 78 & 95 & 73 \\ 75 & 85 & 82 \\ 84 & 94 & 81 \end{bmatrix} \end{array}$$

5 × 3

$$W = \begin{bmatrix} 15\% & 15\% & 25\% & 15\% & 30\% \end{bmatrix}$$

test 1 test 2 midterm test 4 final

1 × 5

Can't multiply T·W, must multiply W·T

$G = \begin{bmatrix} 8 & 91.7 & 76.15 \end{bmatrix}$

15. A zombie virus sweeps through a high school, infecting 30% of the 11th graders and 20% of the 12th graders, as represented by matrix P. There are 100 11th grade boys, 110 11th grade girls, 120 12th grade boys, and 130 12th grade girls, as represented by matrix S.

$$P = \begin{array}{c} \begin{matrix} 11^{th} & 12^{th} \end{matrix} \\ \begin{bmatrix} 0.3 & 0.2 \\ 0.7 & 0.8 \end{bmatrix} \begin{matrix} Ill \\ Well \end{matrix} \end{array}$$

$$S = \begin{array}{c} \begin{matrix} Boys & Girls \end{matrix} \\ \begin{bmatrix} 100 & 110 \\ 120 & 130 \end{bmatrix} \begin{matrix} 11^{th}\ grade \\ 12^{th}\ grade \end{matrix} \end{array}$$

$PS = \begin{bmatrix} 54 & 59 \\ 166 & 181 \end{bmatrix}$

$SP = \begin{bmatrix} 107 & 108 \\ 127 & 128 \end{bmatrix}$

a. Show that the product P S does not equal S P.
b. What does the matrix PS represent?
c. What does the matrix SP represent?

$PS = \begin{bmatrix} \text{sick boys} & \text{sick girls} \\ \text{well boys} & \text{well girls} \end{bmatrix}$

$SP = \begin{bmatrix} \text{doesn't make sense.} \end{bmatrix}$

The Determinant Calculate #1–4s' by hand and 5 with a calculator (maybe by hand too, in case your calc dies):

1. $\begin{vmatrix} 6 & 2 \\ -1 & 3 \end{vmatrix}$ 20

2. $\begin{vmatrix} -1 & 5 \\ 3 & 4 \end{vmatrix}$ −17

whoa! ↓ let's talk!

3. $\begin{vmatrix} -4 & 6 \\ -2 & 3 \end{vmatrix}$ 0

4. $\begin{vmatrix} 0 & -1 \\ 7 & 8 \end{vmatrix}$ 7

5. $\begin{vmatrix} 5 & 2 & 8 \\ 3 & 4 & 1 \\ 7 & -1 & 6 \end{vmatrix}$ −145

6. Solve for x. $\begin{vmatrix} 5 & 7 \\ x & -6 \end{vmatrix} = -2$ $x = -4$

Inverse Matrices: Calculate A^{-1} for #6–8 by hand and 10 with a calculator.

7. $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $\begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$

8. $A = \begin{bmatrix} -4 & -3 \\ 12 & 9 \end{bmatrix}$ Doesn't have an inverse (det can't be 0)

9. $A = \begin{bmatrix} -6 & 3 \\ 5 & -3 \end{bmatrix}$ $\begin{bmatrix} -1 & -1 \\ -\frac{5}{3} & -2 \end{bmatrix}$

10. $A = \begin{vmatrix} 5 & 2 & 8 \\ 3 & 4 & 1 \\ 7 & -1 & 6 \end{vmatrix}$

$\begin{bmatrix} -\frac{5}{29} & \frac{4}{29} & \frac{6}{29} \\ \frac{11}{145} & \frac{26}{145} & -\frac{19}{145} \\ \frac{31}{145} & -\frac{19}{145} & -\frac{14}{145} \end{bmatrix}$

Solve the following using the inverses (no calculator allowed)

1. $\begin{cases} 5x-2y=-9 \\ -7x+3y=14 \end{cases}$ $x=1$, $y=7$

2. $\begin{bmatrix} 3 & -2 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 10 \\ 7 \end{bmatrix}$ $\begin{bmatrix} -24/5 \\ -61/5 \end{bmatrix}$

3. $\begin{bmatrix} 0 & -3 \\ 2 & 4 \end{bmatrix}X=\begin{bmatrix} -18 \\ 32 \end{bmatrix}$ $\begin{bmatrix} 4 \\ 6 \end{bmatrix}$

4. $\begin{bmatrix} -1 & -2 \\ 2 & 9 \end{bmatrix}B=\begin{bmatrix} -3 & -5 & 13 \\ 21 & 0 & -36 \end{bmatrix}$ $\begin{bmatrix} -3 & 9 & -9 \\ 3 & -2 & -2 \end{bmatrix}$

5. $\begin{cases} 3x+5y=7 \\ 4x+9y=11 \end{cases}$ $x=8/7$, $y=5/7$

6. $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}X=\begin{bmatrix} 4 & -1 \\ 5 & 2 \end{bmatrix}$ $\begin{bmatrix} -2 & 7 \\ 3 & -4 \end{bmatrix}$

7. $\begin{bmatrix} 7 & -3 \\ 9 & -1 \end{bmatrix}=\begin{bmatrix} -4 & -3 \\ 2 & -1 \end{bmatrix}Z$ $\begin{bmatrix} 2 & 0 \\ -5 & 1 \end{bmatrix}$

8. $\begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}X-\begin{bmatrix} 4 & -6 \\ -2 & 8 \end{bmatrix}=\begin{bmatrix} 5 & 0 \\ -2 & 3 \end{bmatrix}$ $\begin{bmatrix} 24/11 & 9/11 \\ 17/11 & -28/11 \end{bmatrix}$

Solve WITH a calculator:

9. $\begin{bmatrix} 6 & 4 & 3 \\ 1 & -2 & -2 \\ 1 & 1 & 1 \end{bmatrix}\times\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 610 \\ 0 \\ 120 \end{bmatrix}$ $\begin{bmatrix} 80 \\ 10 \\ 30 \end{bmatrix}$

10. $\begin{bmatrix} 5 & -6 & -7 \\ 6 & -4 & 10 \\ 2 & 4 & -3 \end{bmatrix}\times\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 7 \\ -34 \\ 29 \end{bmatrix}$ $\begin{bmatrix} 2 \\ 4 \\ -3 \end{bmatrix}$

11. $\begin{bmatrix} 1 & -1 & 3 \\ 2 & 1 & 2 \\ -2 & -2 & 1 \end{bmatrix}\cdot\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$ $\begin{bmatrix} -3 \\ 14/5 \\ 13/5 \end{bmatrix}$

For the following you must set up a) a linear system of equations and b) a matrix equation. You may solve these with a calculator (using inverses), but you must be able to set them up to

12. N'Sync: N 'Sync is having a reunion performance at RHS and 1200 tickets are sold. Student tickets cost $1.50 each and teacher tickets cost $5.00 each. The total revenue collected for the concert was $3200. Who bought more, students or teachers? Also, name one N 'Sync song.

$s+t=1200$

$1.5s+5t=3200$

$\begin{bmatrix} 1 & 1 \\ 1.5 & 5 \end{bmatrix}\begin{bmatrix} s \\ t \end{bmatrix}=\begin{bmatrix} 1200 \\ 3200 \end{bmatrix}$

800 students
400 teachers
students bought more

13. Leggy Aquarium: There are starfish and octopi in a fish tank. You don't know how many of each you have, but somehow, you know you have 8 more starfish than octopi, and you have 261 total legs.

$p+8=s$

$5s+8p=261$

$-s+p=-8$

$\rightarrow \begin{bmatrix} 5 & 8 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} s \\ p \end{bmatrix}=\begin{bmatrix} 261 \\ -8 \end{bmatrix}$

25 starfish
17 octopi

14. Fifty Shades of Grey is coming out, and the Ridgewood movie theater sells adult tickets for $9.00 each. Senior citizens (I don't know why senior citizens are going to see this movie) receive a discount of $3.00. Last night, they sold 636 tickets and took in $4974 in revenue. How many of each ticket was sold?

$a + s = 636$

$9a + 6s = 4974$

$\begin{bmatrix} 1 & 1 \\ 9 & 6 \end{bmatrix}\begin{bmatrix} a \\ s \end{bmatrix} = \begin{bmatrix} 636 \\ 4974 \end{bmatrix}$

336 adults
300 seniors

15. Money Magic: You want to impress your friend doing a little "money magic". It's super lame, but your friend lets you do it anyway. You ask your friend how much money he has in his wallet: $85 ... composed only of $10 and $5 bills. You ask if you can count his bills (with your eyes closed) and count that he has 14 bills in his wallet. Can you figure out how many of each bill he has?

$f + t = 14$

$5f + 10t = 85$

$\begin{bmatrix} 1 & 1 \\ 5 & 10 \end{bmatrix}\begin{bmatrix} f \\ t \end{bmatrix} = \begin{bmatrix} 14 \\ 85 \end{bmatrix}$

11 $5 bills
3 $10 bills

16. Fistful of Coins: You're sitting in class and get thirsty. You want a Gatorade, which costs $2.25. In your locker, you have a jar of dimes and quarters. You grab a handful of 18 coins and run. Magically, you have exactly the right amount. How many of each coin do you have?

$d + q = 18$

$.1d + .25q = 2.25$

$\begin{bmatrix} 1 & 1 \\ .1 & .25 \end{bmatrix}\begin{bmatrix} d \\ q \end{bmatrix} = \begin{bmatrix} 18 \\ 2.25 \end{bmatrix}$

15 dimes
3 quarters

17. Pizzas party: You're having a party! Everyone wants pizza. You ask people to tell you how many slices each person wants, and you find out you need 56 slices of pizza. People put money in a hat... and you have a total of $44. A large pizza has 10 slices and costs $8. A medium pizza costs $6 and has 8 slices. How many of each pizza should you get?

$ $6m + 8l = 44$

Slices $8m + 10l = 56$

$\begin{bmatrix} 6 & 8 \\ 8 & 10 \end{bmatrix}\begin{bmatrix} m \\ L \end{bmatrix} = \begin{bmatrix} 44 \\ 56 \end{bmatrix}$

2 medium
4 large

18. You park like an idiot and get a parking violation of $156. You decide to pay for it by selling your clothes at a garage sale. You are willing to get rid of 12 shoes, 14 pants and 26 tops... and need to figure out how to price each item. Your shoes are twice as expensive as your tops, so you want to sell them for twice as much. To make things easy, you'd like to be able to sell an entire outfit (shoes, pants and shirt) for $10. How much should each item cost if you want to reach your goal?

$2t = s$

$s + p + t = 10$

$12s + 14p + 26t = 156$

$-s + 2t = 0$

$\begin{bmatrix} 1 & 1 & 1 \\ 12 & 14 & 26 \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} s \\ p \\ t \end{bmatrix} = \begin{bmatrix} 10 \\ 156 \\ 0 \end{bmatrix}$

$4 for shoes
$4 for pants
$2 for tops

19. In the last football game, the Maroons and their opponents scored a total of 72 points. The points came from a total of 20 scoring plays, which were a combination of touchdowns (6 pts), field goals (3 pts) and PATs (1 pt). The same number of extra points were scored as field goals were kicked. How many of each were scored?

$f = p$

$f + t + p = 20$

$3f + 6t + p = 72$

$f - p = 0$

$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 6 & 1 \\ 1 & 0 & -1 \end{bmatrix}\begin{bmatrix} f \\ t \\ p \end{bmatrix} = \begin{bmatrix} 20 \\ 72 \\ 0 \end{bmatrix}$

6 field goals
3 touchdowns
6 PATs