Name: Answer key   Per: ________ Date: ______________

Serafino ▪ Algebra 2E

7B Compound & Continuous Growth

If there's one thing you want to grow exponentially in your life, it's your money (and happiness). When you invest or store money somewhere, you typically earn a percent of it in interest. You will end up with more than when you started.

1. You have \$10,000 you'd like to put away in savings. You find a bank that offers 0.9% annual interest.

a) How much will you have in that account in 10 years?

$10{,}000(1+.009)^{10} = \boxed{\$10{,}937.34}$

b) 50 years?

$10{,}000(1+.009)^{10} = \boxed{\$15{,}651.58}$

It also works the other way. When you borrow it money, you end up paying interest. You end up giving back more than you borrowed. (This is why you pay your debts on time!)

2. You borrow \$500 from a friend. Your friend charges you 5% interest per year. How much will you owe your friend if you pay her back in:

a) 6 months?

$500(1+.05)^{0.5} = \boxed{\$512.38}$

b) 1 year?

$500(1+.05)^{1} = \boxed{\$525}$

b) At your 10–year high school reunion?

$500(1+.05)^{12} = \boxed{\$897.93}$

d) At your kids' wedding in 50 years?

$500(1+.05)^{50} = \boxed{\$5{,}733.70}$

COMPOUND INTEREST

If you Google "Compound Interest Formula":

$$A = P(1 + \tfrac{r}{n})^{nt}$$

No memorizing! Let's analyze and compare it to $y = a \cdot b^{x}$

A = Amount (\$) (y)   P = Principle Starting value (a)   r = % interest / growth rate as decimal   t = time (x)

So the only thing that's new is the n. It represents how many times you compound your annual interest per year. Every banking interest rate you see is annual. That means they will, by the end of the year, have given you that extra percentage of your money to you. But interest isn't only calculated at the end of the year. It's calculated (compounded) **n**–number of times per year.

<u>It's going to chop up the annual interest rate per compounding period.</u>

If 4% interest in compounded annually, n = 1, you get 4% at the end of every.
If 4% interest is compounded semiannually, n = 2, you get 2% at the end of every 6 months.
If 4% interest is compounded quarterly, n = 4, you get 1% at the end of every 3 months. Etc...

I invite you to think of this formula in this way

$$y = a\left(1 + \frac{r}{n}\right)^{\#\ COMPOUNDING\ PERIODS}$$

How many times you're compounding

3. Let's revisit my original $10,000 investment at 0.9% interest. Compare my 50–year balance if interest is...

a) Compounded annually

$\boxed{\$15,651.58}$

(from prev. page)

b) Compounded semiannually

$10{,}000\left(1+\frac{.009}{2}\right)^{100 \text{ or } (2\cdot 50)}$

$= \boxed{\$15,667.29}$

c) Compounded monthly

$10000\left(1+\frac{.009}{12}\right)^{600 \text{ or } (12\cdot 50)}$

$= \boxed{\$15,680.48}$

4. Let's compare how different compounding periods grow your money over time.
You have $25,000 to put away in savings.

	1 year	5 years	10 years	25 years	50 years
BANK A 3.5% Compounded Annually	$25875	$29,692.16	$35,264.97	$59,081.12	$139,623.17
BANK B 3.5% Compounded Semiannually	$25882.66	$29,736.41	$35,369.45	$59,519.72	$141,703.90
BANK C 3.5% Compounded Quarterly	$25,886.55	$29,758.49	$35,422.72	$59744.07	$142,774.14
BANK D 3.5% Compounded Monthly	$25,889.17	$29,773.57	$35,458.62	$59,895.55	$143,499.09
BANK E 3.5% Compounded Daily	$25,890.45	$29,780.91	$35,476.09	$59,969.37	$143,852.99

Understanding compound interest gives you the tools necessary to make smart choices about your money.

5. Let's say you have $7,500 to put in savings, but you can't touch it for 10 years. Pick the best offer!

BANK A offers 3% interest, compounded annually. $\boxed{A: 7500(1+.03)^{10} = \$10079.37}$
BANK B offers 2.9% interest compounded monthly. $B: 7500(1+.029/12)^{120} = 10019.70$
BANK C offers 2.8% interest, compounded daily. $C: 7500(1+.028/365)^{3650} = 9923.34$

6. Again, lets revisit my $10,000 investment at 0.9% interest. Compare my 50–year balance if interest is...

d) Compounded daily

$10{,}000\left(1+\frac{.009}{365}\right)^{50\cdot 365}$

$\boxed{\$15,683.03}$

e) Compounded hourly

$10{,}000(1+.009/8760)^{(8760\cdot 50)}$

$\boxed{\$15,683.12}$

f) Compounded "minutely"

$10{,}000(1+.009/525600)^{(525600\cdot 50)}$

$\boxed{\$15,683.13}$

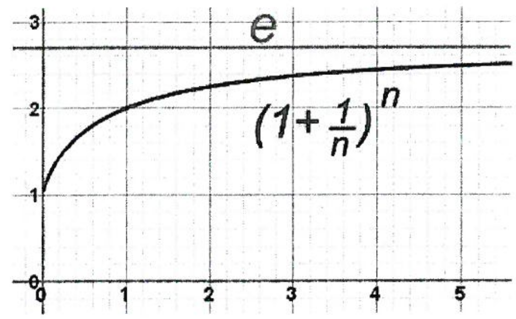
9¢

1¢

CONTINUOUS INTERST

What if your interest is compounded every second of every minute of every single day? That would get you the maximum amount of interest possible. Cool. But how you do represent that infinite "n" as a number? You're compounding a bazillion times every millisecond.

Lets evaluate $f(n) = \left(1 + \frac{1}{n}\right)^n$ so see what happens as n → ∞.



f(1) = 2   f(2) = 2.25   f(5) = 2.488

f(50) = 2.6916   f(5000) = 2.71801

e ≈ 2.71828

So, e is an actual number. You've never needed it before but now you're ready for it.

So when we compound continuously, we need a base that represents an infinitely big "n". That's e.

The Continuous Compound Interest Formula: $A = P \cdot e^{rt}$

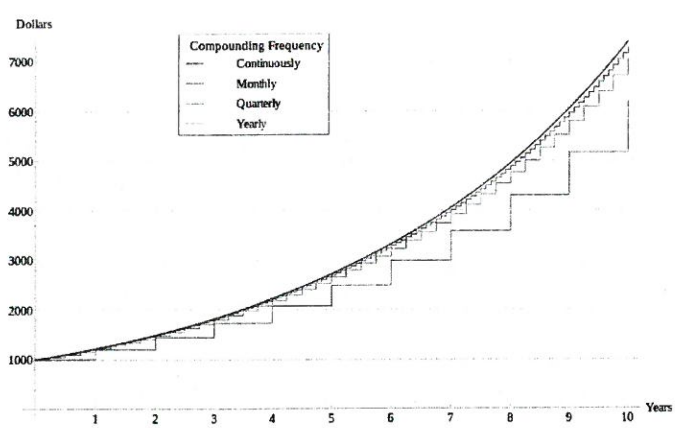
7. If I have $10,000 and invest it in an account that earns 0.9% interest, compounded continuously.

a) How much will I have in 10 years?   50 years?

$10{,}000(e)^{(.009 \cdot 10)} = \boxed{\$10{,}941.74}$ $10{,}000e^{.009 \cdot 50} = \boxed{\$15683.12}$

b) Compare that to my compound interest in #8

Pretty cool chart of Periodic vs. Continuous compound interest on an investment of $1,000.



So, some things DO grow and decay periodically. Cells, for example double every … whatever.
For your edification, here's a video on how credit card interest works: http://youtu.be/qAQnlwSeih8

Continuous Growth is actually more practical in the real world – think about a tree. If I tell you it grows 6% every year, that doesn't mean it stays the same height all year and then BAM! – at midnight – it shoots up 6% of it's height. It's growing the whole time. In fact, most things in the real world grow/decay continuously.

8. You have an 8–ft tree in your backyard that has been growing continuously at a rate of 2.5% since it's been planted there.

a) If the tree was planted 10 years ago, how tall was it when it was planted?

$y = Pe^{rt}$ $8 = Pe^{(.025)(10)}$

$8 = 1.2840P$ $P = \boxed{6.23 \text{ ft}}$

b) How tall will your tree be 10 years from now?

$8e^{.025(10)} = \boxed{10.27 \text{ ft}}$

9. Josie is 8 and weighs 76 lbs. Over the next 5 years, she is going to continuously gain 12% of her body weight. For the next 5 years after that, she is going to gain 1.2% of her body weight.

a) How much will she weigh when she is 18?

$76e^{.12 \cdot 5} = \boxed{138.48}$ $138.48e^{.012 \cdot 5} = \boxed{147.04 \text{ lbs}}$

b) What was her average rate of change per year during puberty vs. during adolescence?

Puberty $\frac{62.48}{5} = \boxed{12.496 \text{ lbs/year}}$ Adolosence $\frac{8.56}{5} = \boxed{1.712 \text{ lbs/yer}}$

c) BONUS: What kind of function uses different functions for different intervals of the domain? Write one if you dare!

(☺ see me)

Do the following on a separate piece of paper: HW on separate page

1. I have \$50 in savings, earning 0.5% annual interest, compounded monthly. How much will I have in 6 months? 10 years? 20 years? $\boxed{\$50.13}$ $\boxed{\$52.56}$ $\boxed{\$55.26}$
2. The polar ice cap is receding, and the polar bear population is decreasing by 1.8% per year. Today, there are approximately 25,000 polar bears. If the trend continues, how many polar bears will there be in 50 years? 200 years? $\boxed{10,081 \text{ bears}}$ $\boxed{661 \text{ bears}}$
3. When you were born, you had \$20,000 put away for college in an account that earns 0.85% annual interest, compounded continuously. How much will you have when you are 18? $\boxed{\$23,306.50}$
4. You lend your friend \$200, but since he's unreliable, you charge him a 15% interest rate, compounded quarterly. How much will your friend owe you in 6 months? 1 year? 20 years? $\boxed{\$215.28}$ $\boxed{\$231.73}$ $\boxed{\$3,802.58}$
5. A 10–ft tree is expected to grow 20% of its height every year for the next 5 years, then 5% continuously after that. How tall would that tree be in 6 months? 5 years? 8 years? $\boxed{10.95 \text{ ft}}$ $\boxed{24.88 \text{ ft}}$ $\boxed{37.12 \text{ ft}}$
6. You have \$10,000 to invest and you have two banks you could invest with. One can offer 0.9%, compounded monthly, and the other can offer 1%, compounded daily. Which should you invest in? How much more money could you have 50 years with the better offer? $\boxed{\$806.62 \text{ more}}$
7. At 7:30 am, you drink a Starbucks coffee that has 120 mg of caffeine. Caffeine decreases at a rate of 12% per hour. How much caffeine is left in your body at noon? $\boxed{67.5 \text{ mg}}$