

Name: Answer key No. 0  
 Serafino · Algebra 2E

Per: 1 Date: 4/21/16  
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**7C**

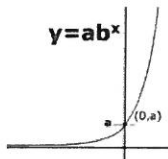
**Solve Exponential Equations → 1) via Common Base**

Notes & Practice

$$36^x = 216^{x-3}$$

$$(6^2)^x = (6^3)^{x-3}$$

$$6^{2x} = 6^{3x-9}$$



So far, we can do three (3) things with exponential functions:

- 1) Evaluate/solve for "y": Predict the amount of something down the road
- 2) Solve for "a": Figure out the initial or prior value of something
- 3) Solve for "b": Figure out the rate of growth/decay.

What can't we do yet? Solve for "x", the input, which is almost always time, which answers questions like, "How long will it take for the tree to be 20 feet tall?" or "When will you be able to afford that car?" "In how many weeks will the bacteria take over?"

There are TWO ways to solve exponential equations, depending on the kinds solutions that exist.

*This is kind of like quadratics; when solutions are rational, we can factor; when they are irrational, we solve with square roots or the quadratic formula.*

This packet is dedicated to the exponential equations we can solve by hand; when solutions are rational.

**Method 1: Convert base to like (or common) bases.**

**Ex 1) Drive to do Dishes:** You want a car. Your rich aunt wants to teach you some responsibility. She says she will give you \$5,000 as a show of good faith, and will double that amount of money every year, paying you one big payment as long as you did the dishes every day up until that point.



Start with  $y = a(b)^x$  model:  $y = 5,000(2)^x$

How long will it take you to afford...



a) A \$40,000 car?

$$\frac{5000(2)^x = 40000}{5000} \quad x = 3$$

$$2^x = 8 \rightarrow 2^x = 2^3$$

**3 years**

b) a \$320,000 car?

$$5000(2)^x = 320,000$$

$$2^x = 64$$

$$2^x = 2^6$$

$$x = 6$$

**6 years**

**Ex 2) Billions of Bacteria:** There are 2 million bacteria sitting on a subway seat. It doesn't get cleaned, so the number of bacteria nonuples every week (X9). In how many weeks will there be...

a) 54 million bacteria?

$$\frac{2(9)^x = 54}{2} \quad 2x = 3$$

$$9^x = 27$$

$$3^{2x} = 3^3$$

**3/2 weeks**

b) 1.458 billion bacteria?

$$2(9)^x = 1458$$

$$9^x = 729$$

$$9^x = 9^3$$

1 bill = 1000 mill  
 $\frac{1000}{1.458} = x \text{ mil}$

**3 weeks**

**Ex 3: Mmmm... Doughnuts:** Homer won a contest and got a truck full of doughnuts. He's been eating half of them every week. Now, he only has 2 dozen left.

a) If he started with 6,144 donuts, when did he win the contest?

$$6144\left(\frac{1}{2}\right)^x = 24 \quad x = -8$$

$$\left(\frac{1}{2}\right)^x = 256$$

$$2^{-x} = 2^8$$

**8 weeks ago**

b) He doesn't want to be greedy, so he'll give his last 3 to Marge, Bart and Lisa. When will that be?

$$\frac{24\left(\frac{1}{2}\right)^x = 3}{24}$$

$$\left(\frac{1}{2}\right)^x = \frac{1}{8}$$

$$2^{-x} = 2^{-3}$$

**3 weeks**



**Classwork/Homework:** Solve the exponential equations without a calculator.

1.  $2^x = 2^{3x-4}$

$$x = 3x - 4$$

$$4 = 2x$$

$$\boxed{x = 2}$$

2.  $5^{x+1} = 25$

$$5^{x+1} = 5^2$$

$$x+1 = 2$$

$$\boxed{x = 1}$$

3.  $4^{x+2} = 8$

$$2^{2(x+2)} = 2^3$$

$$2(x+2) = 3$$

$$2x + 4 = 3$$

$$2x = -1$$

$$\boxed{x = -1/2}$$

4.  $2^{3x-4} = 8^{x-1}$

$$2^{3x-4} = 2^3(x-1)$$

$$3x-4 = 3x-3$$

$$-4 \neq -3$$

$\boxed{\text{No solution}}$

base 2

base 4

5.  $4^{x-1} = \frac{1}{64}$

$2^{2(x-1)} = 2^{-6}$

$2(x-1) = -6$

$x-1 = -3$

$\boxed{x = -2}$

$4^{x-1} = 4^{-3}$

$x-1 = -3$

$\boxed{x = -2}$

6.  $3^x = 3\sqrt{3} = 3^1 \cdot 3^{1/2}$

$$3^x = 3^{3/2}$$

$$\boxed{x = 3/2}$$

7.  $5^{3x-2} = 125^{2x}$

$$5^{3x-2} = 5^3(2x)$$

$$3x-2 = 6x$$

$$-2 = 3x$$

$$\boxed{x = -2/3}$$

8.  $36^x = 216^{x-3}$

$$6^{2x} = 6^{3(x-3)}$$

$$2x = 3x-9$$

$$\boxed{x = 9}$$

9.  $32^{x-2} = 64$

$$2^5(x-2) = 2^6$$

$$5x-10 = 6$$

$$\frac{5x}{5} = \frac{16}{5}$$

$$\boxed{x = \frac{16}{5}}$$

10.  $100^x = 1000^{2x-1}$

$$10^{2x} = 10^3(2x-1)$$

$$2x = 6x-3$$

$$3 = 4x$$

$$\boxed{x = 3/4}$$

11)  $2^{2x} = 8$

$$2^{2x} = 2^3$$

$$\frac{2x}{2} = \frac{3}{2}$$

$$\boxed{x = 3/2}$$

12)  $5^{2x-1} = 125$

$$5^{2x-1} = 5^3$$

$$2x-1 = 3$$

$$2x = 4$$

$$\boxed{x = 2}$$

13)  $3^{2x-1} = 81$

$$3^{2x-1} = 3^4$$

$$2x-1 = 4$$

$$2x = 5$$

$$\boxed{x = 5/2}$$

14)  $4^{3x} = 128$

$$2^{2 \cdot 3x} = 2^7$$

$$6x = 7$$

$$\boxed{x = 7/6}$$

15)  $5^{2x} = 625$

$$25^x = 25^2$$

$$\boxed{x = 2}$$

16)  $8^{\frac{1}{2}x} = 32$

$$2^{3 \cdot \frac{1}{2}x} = 2^5$$

$$\frac{3}{2}x = 5$$

$$x = 10/3$$

19)  $4^{x^2} = 64$

$$\sqrt{x^2} = \sqrt{64}$$

$$\pm$$

$$x = \pm \sqrt{64}$$

22)  $3^{x-5} = 27$

$$3^{x-5} = 3^3$$

$$x-5=3$$

$$x=8$$

25)  $\left(\frac{1}{3}\right)^{x+2} = 9^{3x}$

$$3^{-1(x+2)} = 3^{2 \cdot 3x}$$

$$-x-2 = 6x$$

$$-2 = 7x$$

$$x = -2/7$$

28)  $125^{3-2x} = 5^{x-1}$

$$5^{3(3-2x)} = 5^{x-1}$$

$$9-6x = x-1$$

$$10 = 7x$$

$$x = 10/7$$

31)  $5^x = 25\sqrt{5}$

$$5^x = 5^2 \cdot 5^{1/2}$$

$$x = 2\frac{1}{2}$$

$$x = 5/2$$

17)  $4^{x^2+2x+1} = 16$

$$x^2+2x+1 = 2$$

$$x^2+2x-1 = 0$$

$$\sqrt{(x+1)^2} = \sqrt{2}$$

$$x = -1 \pm \sqrt{2}$$

20)  $3^x = 0$

$$\text{NO solution}$$

18)  $9^{\frac{1}{2}x} = \frac{1}{3}$

$$3^x = 3^{-1}$$

$$x = -1$$

21)  $3^{2x-1} = 3^1$

$$2x-1 = 1$$

$$2x = 2$$

$$x = 1$$

23)  $3^{2x-4} = 1$

$$2x-4 = 0$$

$$2x = 4$$

$$x = 2$$

24)  $9^x = 27$

$$3^{2x} = 3^3$$

$$x = \frac{3}{2}$$

26)  $\left(\frac{1}{4}\right)^{x-1} = 32^{x+3}$

$$2^{-2(x-1)} = 2^{5(x+3)}$$

$$-2x+2 = 5x+15$$

$$-13 = 7x$$

$$x = -13/7$$

27)  $18^{4x} = -18$

$$\text{NO solution}$$

29)  $4^{x-1} = \frac{1}{64}$

$$4^{x-1} = 4^{-3}$$

$$x = -2$$

30)  $\left(\frac{1}{4}\right)^x = 8^{x-1}$

$$2^{-2} = 2^{3(x-1)}$$

$$-2 = 3x-3$$

$$1 = 3x$$

$$x = 1/3$$

33)  $3^{x-4} = 9\sqrt{3}$

$$3^{x-4} = 3^2 \cdot 3^{1/2}$$

$$x-4 = 2\frac{1}{2} + \frac{1}{2}$$

$$x = 13/2$$

32)  $4^{2x} = 16\sqrt[3]{4}$

$$4^{2x} = 4^2 \cdot 4^{1/3}$$

$$2x = 2\frac{1}{3}$$

$$2x = \frac{7}{3}$$

$$x = 7/6$$

Skip around... to the "hard-looking" ones.

**Homework:** Practice, practice, practice!! Do you have to do all of these? I don't know. You are responsible for knowing how much practice you need to master a topic completely. I should be able to put any one of these on a quiz, and you can solve it quickly and easily.

1)  $3^{1-2n} = 3^{1-3n}$   $n=0$

3)  $4^{2a} = 1$   $a=0$

5)  $(\frac{1}{25})^{-k} = 125^{-2k-2}$   $k=-3/4$

7)  $6^{2m+1} = \frac{1}{36}$   $m=-3/2$

9)  $6^{-3x} = 36$   $x=-2/3$

11)  $64^b = 2^5$   $b=5/6$

13)  $(\frac{1}{4})^x = 16$   $x=-2$

15)  $4^{3a} = 4^3$   $a=1$

17)  $36^{3x} = 216^{2x+1}$  No solution

19)  $9^{2n+3} = 243$   $n=-1/4$

21)  $3^{3x-2} = 3^{3x+1}$  No solution

23)  $3^{-2x} = 3^3$   $x=-3/2$

25)  $5^{m+2} = 5^{-m}$   $m=-1$

27)  $(\frac{1}{36})^{b-1} = 216$   $b=-1/2$

29)  $6^{2-2x} = 6^2$   $x=0$

31)  $4 \cdot 2^{-3n-1} = \frac{1}{4}$   $n=1$

33)  $4^{3k-3} \cdot 4^{2-2k} = 16^{-k}$   $k=1/3$

35)  $9^{-2x} \cdot (\frac{1}{243})^{3x} = 243^{-x}$   $x=0$

37)  $64^{n-2} \cdot 16^{n+2} = (\frac{1}{4})^{3n-1}$   $n=3/8$

39)  $5^{-3n-3} \cdot 5^{2n} = 1$   $n=-3$

2)  $4^{2x} = \frac{1}{16}$   $x=-1$

4)  $16^{-3p} = 64^{-3p}$   $p=0$

6)  $625^{-n-2} = \frac{1}{125}$   $n=-5/4$

8)  $6^{2r-3} = 6^{r-3}$   $r=0$

10)  $5^{2n} = 5^{-n}$   $n=0$

12)  $216^{-3v} = 36^{3v}$   $v=0$

14)  $27^{-2n-1} = 9$   $n=-5/6$

16)  $4^{-3v} = 64$   $v=-1$

18)  $64^{x+2} = 16$   $x=-4/3$

20)  $16^{2k} = \frac{1}{64}$   $k=-3/4$

22)  $243^p = 27^{-3p}$   $p=0$

24)  $4^{2n} = 4^{2-3n}$   $n=2/5$

26)  $625^{2x} = 25$   $x=1/4$

28)  $216^{2n} = 36$   $n=1/3$

30)  $(\frac{1}{4})^{3v-2} = 64^{1-v}$  No solution

32)  $\frac{216}{6^{-2a}} = 6^{3a}$   $a=3$

34)  $32^{2p-2} \cdot 8^p = (\frac{1}{2})^{2p}$   $p=2/3$

36)  $3^{2m} \cdot 3^{3m} = 1$   $m=0$

38)  $3^{2-x} \cdot 3^{3m} = 1$   $m=-1$

40)  $4^{3r} \cdot 4^{-3r} = \frac{1}{64}$  No solution