Name: Answer key No. 0 Per: 1 Date: 4/26/16
Serafino ▪ Algebra 2E M T W R F

7D

Evaluating & Using Logarithms

Notes / Practice

$2^3 = 8$

$\log_2(8) = 3$

Okay, well, we can solve exponential equations by getting a common base.
If $4^x = 8$, *then we rewrite as* $2^{2x} = 2^3$ *and* $2x = 3$ *and* $x = 3/2$
But what about when we can't get a common base? Like, when $3^x = 20$ or $8^x = 6$?
Well, we can at least ESTIMATE what it would be, decimal wise. Then check with your calculator.

$3^x = 20$ *Hint: We know* $3^2 = 9$ *and* $3^3 = 27$... $x \approx 2.73$

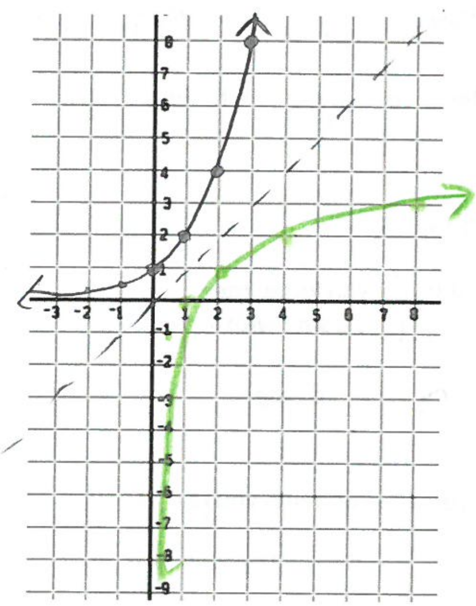
$8^x = 6$ *Hint: We know* $8^1 = 8$ ... $x \approx .9$

$2 = \frac{1}{10}$ *Hint: We know* $2^0 = 1$ ... $x \approx 3.32$

$\frac{1}{4} = 20$ *Hint: We know* $4^2 = 16$ ... $x \approx -2.15$

Estimating/guessing is fine, but we need a way to solve. When we solved equations with multiplication, we divided. When we solved quadratics, we used square roots. If we want to solve exponential equations, we need to somehow undo an exponent.

Growth: $f(x) = 2^x$



$y = \log_2 x$

$f(x)$: *Exponential Functions*

Domain: $x \in \mathbb{R}$ (Exponent) Range: $y > 0$ (Evaluation)

Asymptote: $y = 0$

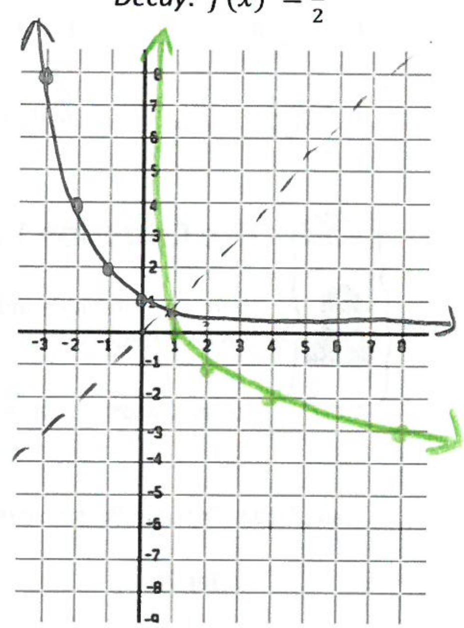
$f^{-1}(x)$: *Inverse of Exponential*

Domain: $x > 0$ (Evaluation) Range: $y \in \mathbb{R}$ (Exponent)

Asymptote: $x = 0$

$x = 2^y$ $x = \frac{1}{2}^y$ *How do we get y down? We stand on a log.*

Decay: $f(x) = \frac{1}{2}^x$



$y = \log_{\frac{1}{2}} x$

$\log_2 2 =$ 1 $\log_2 8 =$ 3

$\log_2 0 =$ ∅ $\log_2 \frac{1}{2} =$ −1

$\log_2 \frac{1}{4} =$ −2 $\log_2 1 =$ 0

$\log_2 7 \approx$ 2.8 $\log_2 -2 =$ ∅

$$\log_b x = y$$

"Log base b of x"

b: base (b > 0, ≠1) x: argument (x>0)
What exponent, y, turns b into x?

$\log_{\frac{1}{2}} 2 =$ −1 $\log_{\frac{1}{2}} 8 =$ −3

$\log_{\frac{1}{2}} 0 =$ ∅ $\log_{\frac{1}{2}} \frac{1}{2} =$ 1

$\log_{\frac{1}{2}} \frac{1}{4} =$ 2 $\log_{\frac{1}{2}} 1 =$ 0

$\log_{\frac{1}{2}} 7 \approx$ −2.8 $\log_{\frac{1}{2}} -2 =$ ∅

Note each point on the graph, (x, y)

1. Rewrite the following expressions from exponential to logarithmic form, and vice versa.

$6^2=36$	$\frac{1}{3}^4=\frac{1}{81}$	$3^0=1$	$4^3=64$	$25^{\frac{1}{2}}=5$	$\frac{1}{27}^{-\frac{1}{3}}=3$	$12^{-2}=\frac{1}{144}$
$\log_6 36=2$	$\log_{\frac{1}{3}}\frac{1}{81}=4$	$\log_3 1=0$	$\log_4 64=3$	$\log_{25}5=\frac{1}{2}$	$\log_{\frac{1}{27}}3=-\frac{1}{3}$	$\log_{12}\frac{1}{144}=-2$

This may sound crazy, but try it: Look at the top row… say, "___ to the____ power is_____". Then look at the bottom row and say the exact same thing while pointing at the numbers, with your pencil. It'll help you learn the layout of a log.

2. Evaluate the following expressions <u>without a calculator</u>. Try to do them in your head. If necessary, you can solve with common base. Life will become easier if you can eyeball these, so at least try!

a. $\log_{\frac{1}{2}}32$ -5 b. $\log_3 27$ 3 c. $\log_4 -2$ $\emptyset$ d. $\log_{16}32$ $5/4$ e. $\log_3 1$ 0

f. $\log_1 2$ $\emptyset$ g. $\log_9\frac{1}{3}$ $-1/2$ h. $\log_4 64$ 3 i. $\log_3\frac{1}{81}$ -4 j. $\log_{\frac{1}{2}}2$ -1

k. $\log_{\frac{1}{16}}8$ $-3/4$ l. $\log_5\frac{1}{5}$ -1 m. $\log_9 27$ $3/2$ n. $\log_8\frac{1}{2}$ $-1/3$ o. $\log_{\frac{1}{27}}\frac{1}{9}$ $2/3$

p. $\log_{\frac{1}{8}}\frac{1}{4}$ $2/3$ q. $\log_{27}27$ 1 r. $\log_4 2$ $1/2$ s. $\log_{64}16$ $2/3$ t. $\log_1 3$ $\emptyset$

The Common Log: log x

If you don't see a base in subscript, <u>it's 10!</u>

There are "invisible/understood" numbers in math. Even when they're not there, you know them. The exponent on 3 is _1_; the index of √5 is _2_; the coefficient of x is _1_. Same with logs.

No calculator: a. log 100 2 b. log 1,000 3 c. log 1 0 d. $\log\frac{1}{10}$ -1

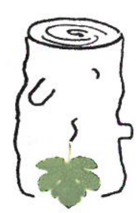
Calculator: Try to make sense of the number returned by your calculator. Compare it to exponents you know.

a. log 75 f. log 8 g. log 17 h. log 250

≈ 1.875 ≈ 0.903 ≈ 1.23 ≈ 2.3979

Natural Log: ln x

If you see an "ln" with no base, it's just a <u>log with base e</u>!



Things grow continually (as they often do, in nature), we use a log with a base of e called the "Natural Log" (logarithmus naturali). Recall that e ≈

No Calculator: a. $\ln e$ 1 b. $\ln 1$ 0 c. $\ln\frac{1}{e}$ -1 d. $\ln e^2$ 2 e. $\ln\sqrt[3]{e}$ $\frac{1}{3}$

Calculator e. ln 2.71828 f. ln 27.1828 g. ln 45 h. ln 0.8

≈ 0.999 ≈ 3.303 ≈ 3.8067 ≈ -0.2231

same as $\log_2 9$

Now we FINALLY can use logs for what they were designed to do – help us solve equations!

TYPE I: An exponential equation: Isolate the base, and take the log both sides. Bring the exponent down and get the x alone. ★ No need to check for extraneous because the domain of exponentials is all real numbers.

Examples: a) $2^x + 1 = 10$

$-1 \quad -1$

$2^x = 9$

$\log 2^x = \log 9$

$x \log 2 = \log 9$

$x = \dfrac{\log 9}{\log 2}$

$\boxed{x \approx 3.1699}$

b) $\dfrac{2 \cdot 4^{x-3}}{2} = \dfrac{9}{2}$

$4^{x-3} = 4.5$

$(x-3)\log 4 = \log 4.5$

$x - 3 = 1.0849...$

$\boxed{x \approx 4.08496}$

c) $\dfrac{2e^x}{2} = \dfrac{18}{2}$ $\quad e^x = 9$

$x \cancel{\ln e} = \ln 9$

same as 1

$\boxed{x \approx 2.1972}$

Problem Set I:

Solve the following with a calculator

a) $\dfrac{5 \cdot 6^{3m}}{5} = \dfrac{20}{5}$

$\log 6^{3m} = \log 4$

$3m = \log 4 / \log 6$

$3m = .7737...$

$\boxed{m \approx 0.2579}$

b) $8^{-5m} - 5 = 53$

$+5 \quad +5$

$-5m = \log 58 / \log 8$

$\boxed{m \approx -0.3905}$

d) $16^{n-7} + 5 = 24$

$-5 \quad -5$

$16^{n-7} = 19$

$n - 7 \log 16 = \log 19$

$n - 7 = \log 19 / \log 16$

$\boxed{n \approx 8.06198}$

e) $-e^{-3.9n-1} - 1 = -3$

$e^{-3.9n-1} + 1 = 3$

$e^{-3.9n-1} = 2$

$-3.9n - 1 = \ln 2$

$\boxed{n \approx -0.4341}$

f) $3.4e^{2x-2} - 9 = -4$

$+9 \quad +9$

$3.4e^{2x-2} = 5$

$2x - 2 = \ln 1.4705...$

$\boxed{x \approx 1.1928}$

TYPE II: A logarithmic equation with ONE log on one side

Isolate the log and rewrite as an exponential equation. ★ You MUST check for extraneous here, because the domain of a log is NOT all real numbers. The argument MUST be greater than 0.

No Calc: a) $\dfrac{2\log_2(x+2)}{2} = \dfrac{10}{2}$

rewrite!

$2^5 = x + 2$

$32 = x + 2$

$\boxed{x = 30}$ ✓

b) $\log(x^2 + 19) = 2$

$10^2 = x^2 + 19$

$81 = x^2$

$\boxed{x = 9, -9}$ ✓ ✓

c) $\log_3(3x - 7) - 5 = -2$

$+5 \quad +5$

$3^3 = 3x - 7$

$27 = 3x - 7$

$\boxed{x = 34/3}$

Calc: a) $1.9\log_3(2x) = 85$

$\log_3 2x = 44.7368$

$3^{44.736...} = 2x$

$\boxed{x \approx 1.106 \times 10^{21}}$

b) $\log(x^3 - 1) = 2.7$

$10^{2.7} = x^3 - 1$

$\sqrt[3]{502.187...} = \sqrt[3]{x^3}$

$\boxed{x \approx 7.94856}$

c) $\ln\left(\frac{1}{2}x - 4\right) = 5$

$e^5 = \frac{1}{2}x - 4$

$\boxed{x \approx 304.8263}$

Problem Set II: No Calc:

a) $-\cancel{10} + \log_3(n+3) = -\cancel{10}$

$\log_3(n+3) = 0$

$n+3 = 1$

$\boxed{n = -2}$ ✓

c) $2\log_7(-2r) = 0$

$\log_7(-2r) = 0$

$1 = -2r$

$\boxed{r = -\frac{1}{2}}$ ✓

e) $-2\log_5(7x) = 2$

$\log_5(7x) = -1$

$5^{-1} = 7x$

$\frac{1}{5} = 7x$

$\boxed{x = \frac{1}{35}}$ ✓

b) $2 - \log(-m) = 4$

$-\log(-m) = 2$

$\log(-m) = -2$

$10^{-2} = -m$

$\frac{1}{100} = -m$

$\boxed{m = \frac{-1}{100}}$

d) $-6\log_3(x-3) = -24$

$\log_3(x-3) = 3$

$27 = x - 3$

$\boxed{x = 30}$

★ Careful! this is NOT $\log(x-5) = 12$

Calc: f) $\frac{2\log_5(3x-10)}{2} = \frac{15}{2}$

$5^{7.5} = 3x - 10$

$\boxed{x \approx 58{,}234.270}$

g) $\log x - 5 = 12$

$+5 \quad +5$

$\log x = 17$

$10^{17} = x$

$\boxed{x \approx 1 \times 10^{17}}$

h) $-1.8\ \ln(-2x) = 61$

$\ln(-2x) = 33.8885\ldots$

$e^{33.88} = -2x$

$\boxed{x \approx -2.61 \times 10^{14}}$

TYPE III: A logarithmic equation with same–based log on BOTH sides: Set the arguments equal to one another.
★ You MUST again, check for extraneous solutions. No Calc:

a. $\log_3(4-x) = \log_3(x+8)$

$4 - x = x + 8$

$-4 = 2x$

$\boxed{x = -2}$ ✓

c. $\log_3 \frac{2x}{x-3} = 1$

$\cancel{3^1 = \frac{2x}{x-3}}$

$3(x-3) = 2x$

$3x - 9 = 2x$

$\boxed{x = 9}$ ✓

b. $\log_4(x^2-4) = \log_4(3x)$

$x^2 - 4 = 3x$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1)$

$x = 4, \cancel{-1}$

extraneous

$\boxed{x = 4}$

d. $\log_5(x+1) = \log_5(2x+7)$

$x + 1 = 2x + 7$

$-6 = x$

$x = \cancel{-6}$

extraneous

$\boxed{\text{No solution}}$

Problem Set III: Solve without a calculator:

2. $\log_4(x+2) = \log_4(55)$

3. $\log_2(2x+1) = \log_2(15)$

5. $\log_3(x+2) = \log_3(3x-5)$ **6.** $\log_7(x+3) = \log_7(5x-8)$

7. $\log_5(-x+1) = \log_5(5+x)$ **8.** $\log_8(2x+4) = \log_8(60)$

9. $\log_4(x+1) = \log_4(10)$ **10.** $\log_4(3x+1) = \log_4(2x)$

$x+2=55$, $\boxed{x=53}$ ✓

3. $2x+1=15$, $\boxed{x=7}$ ✓

5. $x+2=3x-5$, $\boxed{x=7/2}$ ✓

6. $x+3=5x-8$, $\boxed{x=11/4}$ ✓

7. $-x+1=5+x$ $\boxed{x=-2}$
$-2x=4$

8. $\boxed{x=28}$

9. $\boxed{x=9}$

10: $3x+1=2x$
$x=\cancel{-1}$ $\boxed{\text{No solution}}$

Applications of Logarithms: Now we can use logarithms to solve exponential growth & decay problems!

4. You're googling random things one day and you see there is such a thing as a hot tub boat. Literally... a floating hot tub. This baby right retails for $42,000. You want it. You invest $5,000 to in a bank account that compounds 3.5% annual interest monthly.

a. What equation models your money's growth?

$\boxed{f(x) = 5{,}000\left(1+ .035/12\right)^{12x}}$

b. How much money will you have after 30 years?

$f(30) = \boxed{\$14{,}266.44}$

$\log 8.4 = 12x \log(1+\ldots)$
or $\log 8.4 = x \log(1+\ldots)^{12}$

c. How many years would it take for you to have the $42,000? ✱ Can solve 2 ways!

$42000 = 5000(1+.035/12)^{12x}$
$8.4 = (1+.035/12)^{12x}$

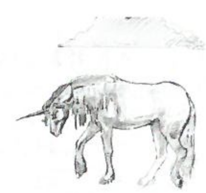
$\boxed{x \approx 60.8952 \text{ years}}$

d. How much money would you have to put in NOW to have $42,000 in ten years?

$42000 = a(1+.035/12)^{12 \cdot 10}$

$\boxed{a = \$29{,}611.98}$

5. There are currently 300 unicorns in Serafinotopia. However, an evil force is causing their population to decline at a rate of 15% per year.



a. What equation models the population of the unicorns?

$\boxed{y = 300(0.85)^x}$

b. If the trend continues, how will there be in 2 years?

$f(2) = \boxed{217 \text{ unicorns}}$

c. How many years will it take for there to be 100 left?

$300(.85)^x = 100$
$x \log .85 = 1/3$

$x \approx \boxed{6.7599 \text{ years}}$

6. A test was given to a group of participants, and they were NOT given a chance to practice it. Then they tested them over a few months. The score results decayed in a natural logarithmic model: $s(t) = 75 - 6\ln(t + 1)$, where s is the % score on a test and t is time in hours.

   a. What was the original score at the time the material was taught? After 6 hours? After 1 day?

Just evaluate :) $\boxed{S(0) = 75\% \quad S(6) = 64.25\% \quad S(24) = 55.93\%}$

   b. How long did people only remember half of the original material of the material?

$75 - 6\ln(t+1) = 50$

$-6\ln(t+1) = -25$

$\ln(t+1) = 4.16$

$e^{4.16} = t+1$

$t \approx \boxed{63.5 \text{ hours}}$ (2 days, 15 hours, 30 min)

7. Off a small shore in Alaska, the Beluga population is diminishing due to hunting and fossil fuel extraction. ☹
In the 1980's, there were ≈ 1,300. A decade ago, the number was down to the low hundreds. Realizing they could soon be extinct, law enforcers enacted policies to help preserve them. It's working!!
In 2007, the population was 284. In 2009, it was 311.

   a. What equation models the yearly growth for the belugas?

2007 → $284(r)^2 = 311$ ← 2009

$r^2 = 1.095\ldots$

$r = 1.04645\ldots$

$\boxed{y = 284(1.04656\ldots)^x}$

   b. If the trend continues, how many belugas should there be in 5 years?

$f(5) = 356.388$

$\boxed{356 \text{ Belugas}}$

   c. How many years will it take for there to be as many as there were in the 1980s?

$284(1.04656\ldots)^x = 1300$

$x \log 1.04656\ldots = \log 4.5774$

$x \approx 33.499$ yers from 2007, so in $\boxed{2040, \text{ 24 years from now}}$

8. Some people put $20,000 put in a bank that compounded 2% interest continuously when you were born. Those people loved you, they did not trust you, so you cannot access that money until you turn 30.

   a. How much money do you have NOW (that you can't touch?)

$20{,}000e^{.02 \cdot 16} = \boxed{\$27{,}542.56}$

   b. How much money will you receive when you turn 30?

$f(30) = \boxed{\$36{,}442.38}$

   c. If they wanted you to have $50,000 by the time you were 30, how much money SHOULD they have invested initially?

$\dfrac{a e^{.02 \cdot 30}}{e^{.02 \cdot 3}} = \dfrac{50{,}000}{e^{.02 \cdot 30}}$

$a = \boxed{\$27{,}440.58}$

   d. If they only had $20,000 to invest, what interest rate would they have needed to get you to $50,000 by the time you were 30?

$20{,}000e^{r \cdot 30} = 50{,}000$

$30r = \ln 2.5$

$r = 0.030543$

Intrest rate: $\boxed{3.054\%}$