Pre-Test Unit 4: Exponential Functions KEY

You may use a calculator on parts of the test.

Evaluate the following rational roots. NO CALCULATOR. *(4 pts; 2 pts for correct process, 2 pts for correct answer)*

1. $16^{\frac{3}{4}}$

   8

2. $125^{\frac{2}{3}}$

   25

Determine if the following statements are true or not. Justify your answer. NO CALCULATOR. *(4 pts; 2 pts for correct answer, 2 pts for justification)*

3. $x^{\frac{3}{7}} = \sqrt[7]{x^3}$

   True, denominator is the root.

4. $y^{\frac{8}{3}} = \sqrt[8]{y^3}$

   False, denominator should be the root.

Determine the appropriate value to make the equation true. Justify your answer. NO CALCULATOR. *(4 pts; no partial credit)*

5. $z^{\frac{5}{6}} = \sqrt[?]{z^5}$

   6

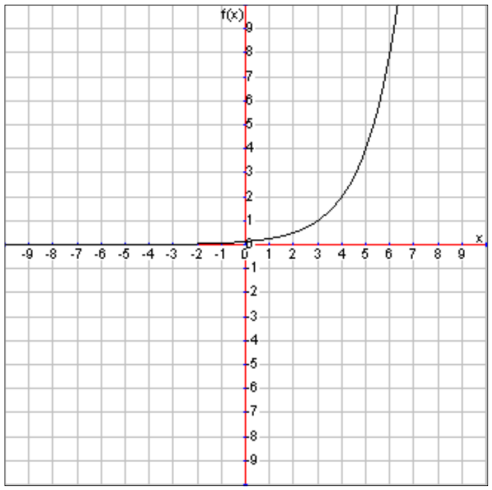
6. $64^{\frac{3}{8}} = 2^{?}$

   3

Graph the following functions by filling out the tables. NO CALCULATOR. *(4 pts; 2 pts for correct table, 2 pts for graph correctly based on table)*

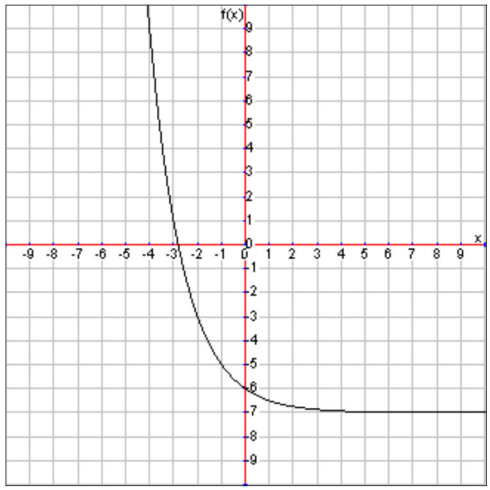
7. $f(x) = 2^{x-3}$

x	1	2	3	4	5
$f(x)$	0.25	0.5	1	2	4



8. $f(x) = \left(\frac{1}{2}\right)^x - 7$

x	-2	-1	0	1	2
$f(x)$	-3	-5	-6	-6.5	-6.75



Find the average rate of change over the interval $[-2, 2]$ ***for the following functions. YOU MAY USE A CALCULATOR.*** *(4 pts; no partial credit)*

9. $f(x) = 2^{x+2} - 7$

$\frac{9--6}{2--2} = \frac{15}{4}$

10. $f(x) = 3^{x+2}$

$\frac{81-1}{2--2} = \frac{80}{4} = 20$

Describe the transformation that would take place given the parent function $f(x) = 2^x$***. YOU MAY USE A CALCULATOR.*** *(4 pts; partial credit at teacher discretion)*

11. $f(2x)$

Gets two times closer to the x-axis.

12. $f(x + 3)$

Translates left three.

13. $4 * f(x)$

Gets four times further from the y-axis.

14. $f(x) - 5$

Translates down five.

15. $g(x) = 2^{-x}$

Reflects about the x-axis.

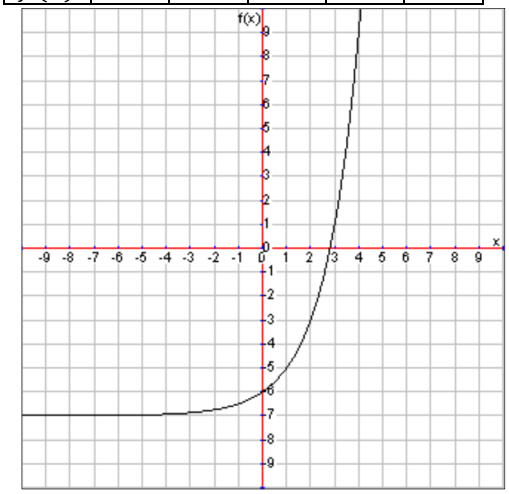
16. $g(x) = -2^x$

Reflects about the y-axis.

Create an equation for the following graph, table, or situation. YOU MAY USE A CALCULATOR. *(4 pts; partial credit at teacher discretion)*

17. $f(x) = 2^x - 7$

x	0	1	2	3	4
$f(x)$	-6	-5	-3	1	9



18. A woman invests $\$10{,}000$ at a compound interest rate of 2%.

$f(t) = 10000(1.02)^t$

Answer the following questions about the function $p(t) = 500(0.97)^t$. YOU MAY USE A CALCULATOR. *(4 pts; no partial credit)*

19. If the function $p(t)$ models the population of an endangered species after t years, what is their current population and growth rate? *(2 pts each)*

Current population: 500 Growth rate: -3%

20. What would we expect their population to be in 10 years to the nearest whole number?

≈ 369

21. What was their approximate population 10 years ago to the nearest whole number?

≈ 678

Charleston has a population of about $11,000$ people (not counting college students) and is growing at a rate of about 0.5%. Mattoon's population change can be modeled by the following function $m(t) = 18000(0.99)^t$ after t years. Answer the following questions. YOU MAY USE A CALCULATOR. *(4 pts; no credit without explanation)*

22. Which city has the higher growth rate and how do you know?

Charleston because Mattoon has a negative growth rate while Charleston's is positive.

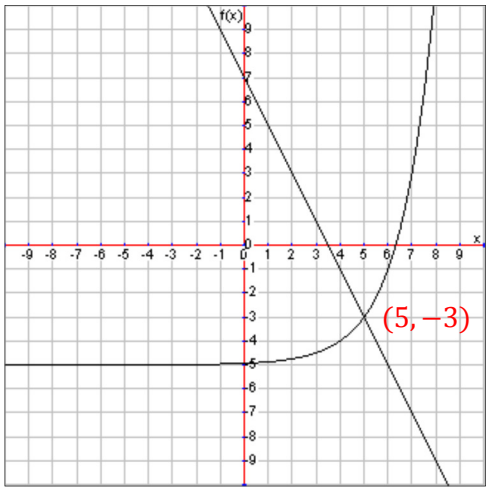
23. Which city has the higher initial value and how do you know?

Mattoon has an initial value of $18{,}000$ people while Charleston only has $11{,}000$.

Solve the following system of equations. You may graph the functions if that will help. YOU MAY USE A CALCULATOR. *(4 pts; 2 pts for each solution when there are two solutions)*

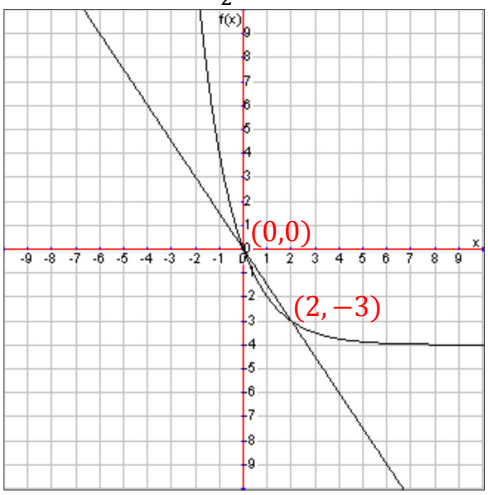
24. $f(x) = 2^{x-4} - 5$

$g(x) = -2x + 7$



25. $f(x) = \left(\frac{1}{2}\right)^{x-2} - 4$

$g(x) = -\frac{3}{2}x$



Lesson 4.0 Unit 4 Homework Answer Key

Evaluate the following exponents operations giving your answer as a fraction where necessary.

1. $5^3 \times 5^{-4} = \frac{1}{5}$
2. $(12^9)(12^{-7}) = 144$
3. $\frac{(t^{-5})(t^4)}{t^2} = \frac{1}{t^3}$
4. $\frac{4^3}{4^{-7}} \times 4^{-10} = 1$
5. $\frac{f^5}{f^{-1}} = f^6$
6. $(y^{-4})^{-5} = y^{20}$
7. $(2^3)^{-6} \times (2^2)^7 = \frac{1}{16}$
8. $12^2 \times 12^{-4} = \frac{1}{144}$
9. $\frac{(k^{-3})^2}{k^4} = \frac{1}{k^{10}}$
10. $\frac{4^{-2}}{4} = \frac{1}{64}$
11. $(5^{-3})^2 \times 5^9 = 125$
12. $(0^{-4})^{10} = \emptyset$

Determine if the following equations are true. Justify your answer.

13. $12^{-2} \times 12^7 = 12^{-8} \times 12^3$
False; $12^5 \neq \frac{1}{12^5}$
14. $\frac{x^{-5}}{x^{-3}} = \frac{x^5}{x^7}$
True; $\frac{1}{x^2} = \frac{1}{x^2}$
15. $(t^{-5})^2 = (t^{-2})^5$
True; $\frac{1}{t^{10}} = \frac{1}{t^{10}}$
16. $(5^{10})^2 = (5^{-5})^{-4}$
True; $5^{20} = 5^{20}$
17. $\frac{6^{-6} \times 6^8}{6^4} = \frac{6^{-2}}{6^0}$
True; $\frac{1}{6^2} = \frac{1}{6^2}$
18. $m^7 \times m^7 = (m^{-7})^2$
False; $m^{14} \neq \frac{1}{m^{14}}$
19. $\frac{k^{-6}}{k^2} = k^2 \times k^{-10}$
True; $\frac{1}{k^8} = \frac{1}{k^8}$
20. $\frac{(7^{-4})^2}{7^3} = 7 \times 7^{12}$
False; $\frac{1}{7^{11}} \neq 7^{13}$
21. $\frac{3 \times 3^4}{3^{10}} = (3^5)^{-1}$
True; $\frac{1}{3^5} = \frac{1}{3^5}$

Determine the appropriate exponent to make the equation true.

22. $2^5 \times 2^{\boxed{-8}} = 2^{-6} \times 2^3$
23. $\frac{p^6}{p^{-2}} = \frac{p^{\boxed{10}}}{p^2}$
24. $(3^{-4})^3 = (3^{-2})^{\boxed{6}}$
25. $(5^{12})^{-2} = (5^3)^{\boxed{-8}}$
26. $\frac{b^{-2} \times b^8}{b^5} = \frac{b^{\boxed{4}}}{b^3}$
27. $9^2 \times 9^{-8} = (9^{\boxed{-2}})^3$
28. $\frac{h^{-2}}{h^{\boxed{0}}} = h^3 \times h^{-5}$
29. $\frac{(6^2)^{\boxed{3}}}{6^6} = 6^{-8} \times 6^8$
30. $\frac{3^{-4}}{3^{\boxed{-6}} \times 3^9} = (3^7)^{-1}$

Lesson 4.1

Evaluate the following exponents operations giving your answer as a fraction where necessary.

1. $4^{\frac{3}{2}} = 8$
2. $4^{\frac{5}{2}} = 32$
3. $9^{\frac{3}{2}} = 27$
4. $9^{\frac{5}{2}} = 243$
5. $8^{\frac{2}{3}} = 4$
6. $27^{\frac{2}{3}} = 9$
7. $16^{\frac{2}{4}} = 4$
8. $16^{\frac{3}{4}} = 8$
9. $81^{\frac{1}{4}} = 3$
10. $81^{\frac{2}{4}} = 9$
11. $81^{\frac{3}{4}} = 27$
12. $16^{\frac{5}{4}} = 32$

Determine if the following equations are true. Justify your answer.

13. $5^{\frac{2}{3}} = (\sqrt[3]{5})^2$
True $5^{\frac{2}{3}} = 5^{\frac{2}{3}}$
14. $16^{\frac{5}{4}} = (\sqrt[5]{16})^4$
False $16^{\frac{5}{4}} \neq 16^{\frac{4}{5}}$
15. $16^{0.75} = (\sqrt[4]{16})^3$
True $16^{\frac{3}{4}} = 16^{\frac{3}{4}}$
16. $x^{\frac{11}{6}} = (\sqrt[11]{x})^6$
False $x^{\frac{11}{6}} \neq x^{\frac{6}{11}}$
17. $y^{\frac{3}{7}} = (\sqrt[3]{y})^7$
False $y^{\frac{3}{7}} \neq y^{\frac{7}{3}}$
18. $z^{\frac{12}{5}} = (\sqrt[5]{z})^{12}$
True $z^{\frac{12}{5}} = z^{\frac{12}{5}}$
19. $a^{\frac{2}{3}} = (\sqrt[3]{a})^2$
True $a^{\frac{2}{3}} = a^{\frac{2}{3}}$
20. $b^{\frac{5}{3}} = (\sqrt[5]{b})^3$
False $b^{\frac{5}{3}} \neq b^{\frac{3}{5}}$
21. $c^{\frac{7}{11}} = (\sqrt[11]{c})^7$
True $c^{\frac{7}{11}} = c^{\frac{7}{11}}$

Determine the appropriate exponent to make the equation true.

22. $\sqrt[\boxed{5}]{2^3} = 2^{\frac{3}{5}}$
23. $\sqrt[\boxed{4}]{x^7} = x^{\frac{7}{4}}$
24. $\sqrt[\boxed{8}]{7^6} = 7^{\frac{3}{4}}$
25. $\sqrt[7]{4^{\boxed{4}}} = 4^{\frac{4}{7}}$
26. $\sqrt[4]{x^{\boxed{3}}} = x^{\frac{3}{4}}$
27. $\sqrt[10]{z^{\boxed{4}}} = z^{\frac{2}{5}}$
28. $\sqrt[3]{6^8} = 6^{\boxed{\frac{8}{3}}}$
29. $\sqrt[9]{b^2} = b^{\boxed{\frac{2}{9}}}$
30. $\sqrt[7]{3^7} = 3^{\boxed{1}}$

Lesson 4.2

Evaluate the following using the given functions.

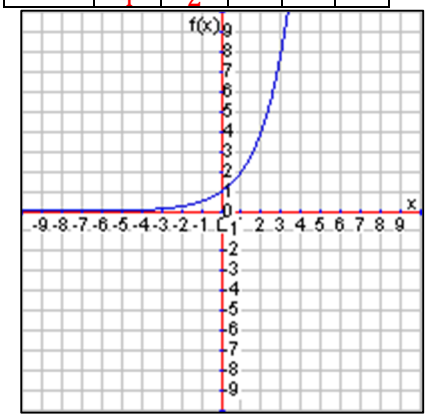
$$f(x) = 4^x \qquad g(x) = 2^{-x} \qquad h(x) = 4 * \left(\frac{1}{2}\right)^x$$

1. $f(-2) = \frac{1}{16}$
2. $f(-1) = \frac{1}{4}$
3. $f(0) = 1$
4. $f(1) = 4$
5. $g(-1) = 2$
6. $g(0) = 1$
7. $g(1) = \frac{1}{2}$
8. $g(2) = \frac{1}{4}$
9. $h(2) = 1$
10. $h(3) = \frac{1}{2}$
11. $h(4) = \frac{1}{4}$
12. $h(5) = \frac{1}{8}$

Graph the following exponential functions.

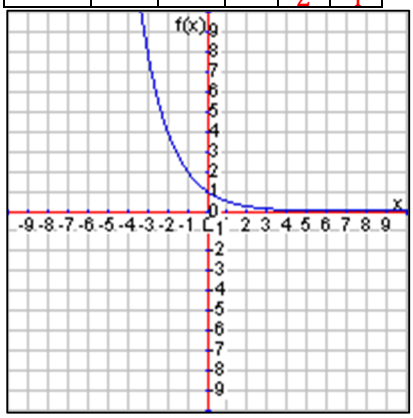
13. $f(x) = 2^x$

x	-2	-1	0	1	2
$f(x)$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4



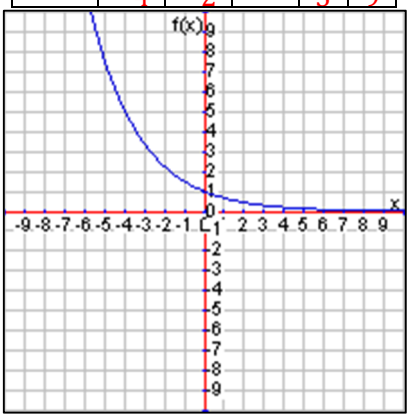
14. $f(x) = 0.5^x$

x	-2	-1	0	1	2
$f(x)$	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$



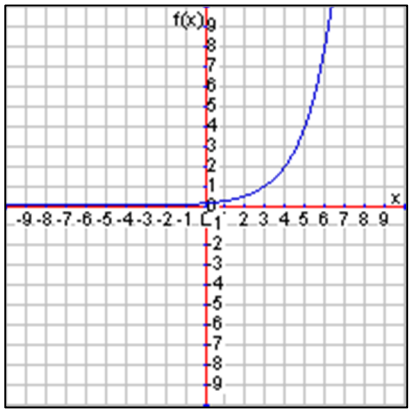
15. $f(x) = \left(\frac{2}{3}\right)^x$

x	-2	-1	0	1	2
$f(x)$	$2\frac{1}{4}$	$1\frac{1}{2}$	1	$\frac{2}{3}$	$\frac{4}{9}$



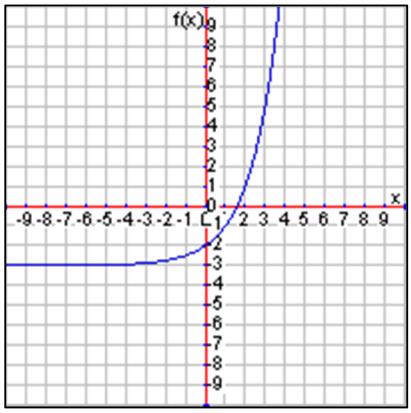
16. $f(x) = 2^{x-3}$

x	1	2	3	4	5
$f(x)$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4



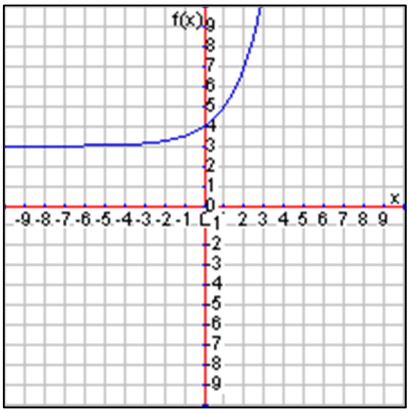
17. $f(x) = 2^{x} - 3$

x	-2	-1	0	1	2
$f(x)$	$-2\frac{3}{4}$	$-2\frac{1}{2}$	-2	-1	1



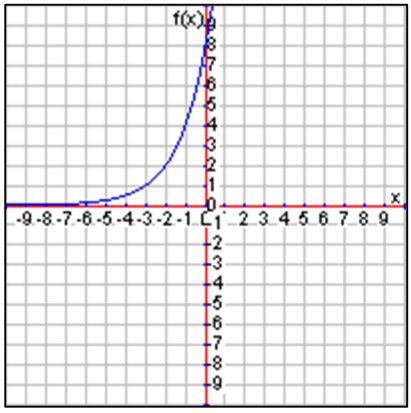
18. $f(x) = 2^{x} + 3$

x	-2	-1	0	1	2
$f(x)$	$3\frac{1}{4}$	$3\frac{1}{2}$	4	5	7



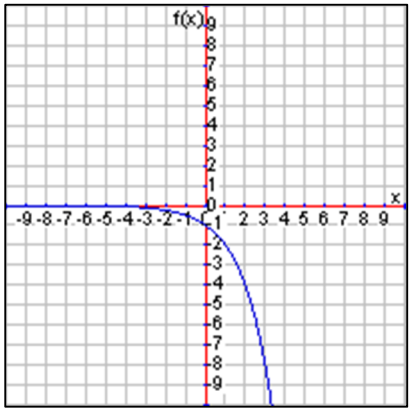
19. $f(x) = 2^{x+3}$

x	-5	-4	-3	-2	-1
$f(x)$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4



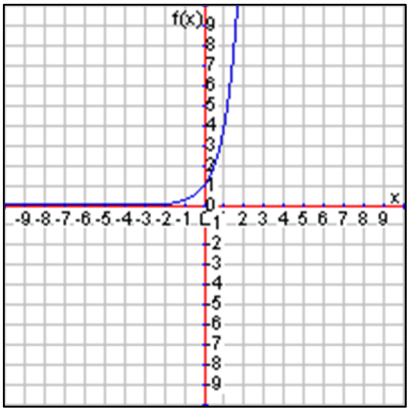
20. $f(x) = -(2^{x})$

x	-2	-1	0	1	2
$f(x)$	$-\frac{1}{4}$	$-\frac{1}{2}$	-1	-2	-4



21. $f(x) = 2^{2x}$

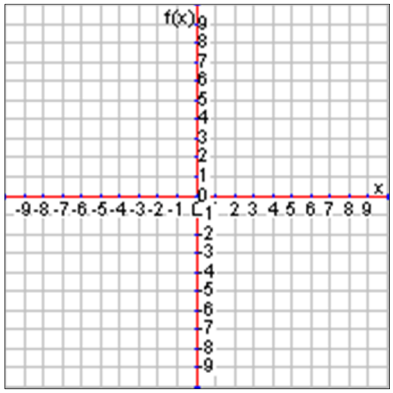
x	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1
$f(x)$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4



Find the average rate of change over the interval $[-2, 2]$**.**

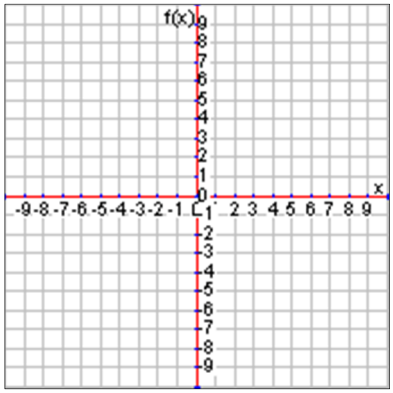
22. $f(x) = 2^{x+2}$ ROC: $\frac{15}{4}$

x	-2	-1	0	1	2
$f(x)$	1	2	4	8	16



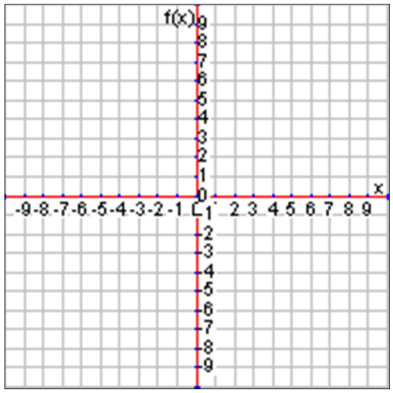
23. $f(x) = 2^{x+2} + 3$ ROC: $\frac{15}{4}$

x	-2	-1	0	1	2
$f(x)$	4	5	7	11	19



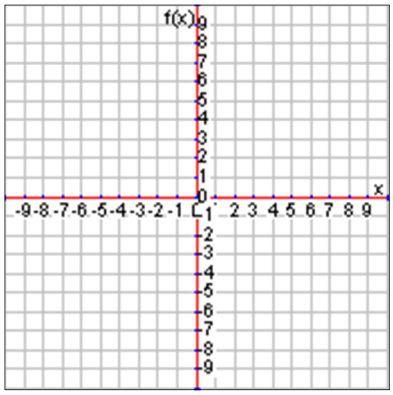
24. $f(x) = 2^{x+2} - 3$ ROC: $\frac{15}{4}$

x	-2	-1	0	1	2
$f(x)$	-2	-1	1	5	13



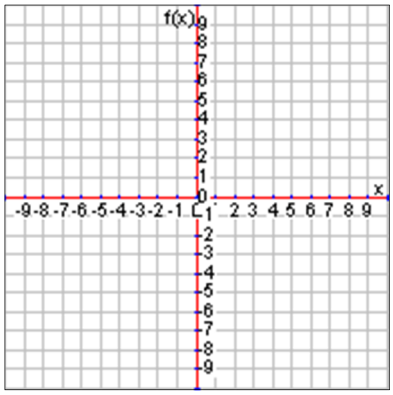
25. $f(x) = 2^{x-2}$ ROC: $\frac{15}{64}$

x	-2	-1	0	1	2
$f(x)$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1



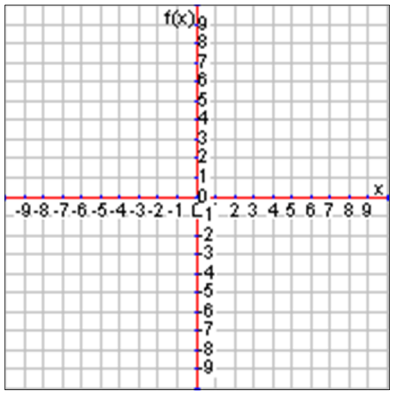
26. $f(x) = 2^{x}$ ROC: $\frac{15}{16}$

x	-2	-1	0	1	2
$f(x)$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4



27. $f(x) = 2^{x+4}$ ROC: 15

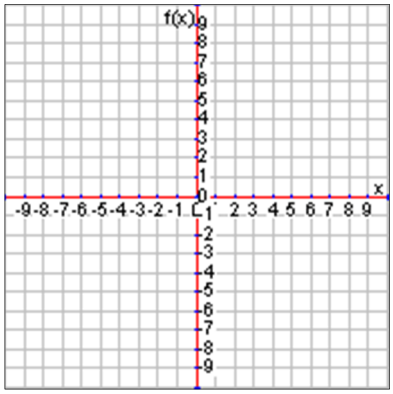
x	-2	-1	0	1	2
$f(x)$	4	8	16	32	64



Find the average rate of change over the given intervals of the following function.

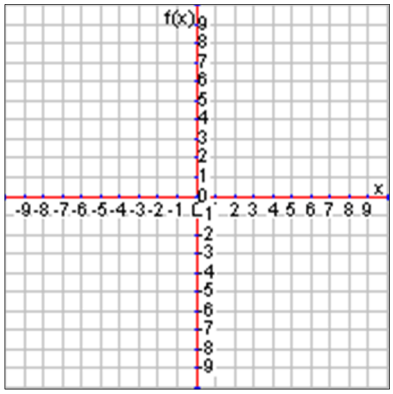
28. $f(x) = \frac{1}{2}^x$ on $[-4, -2]$ ROC: -6

x	-4	-2	0	2	4
$f(x)$	16	4	1	$\frac{1}{4}$	$\frac{1}{16}$



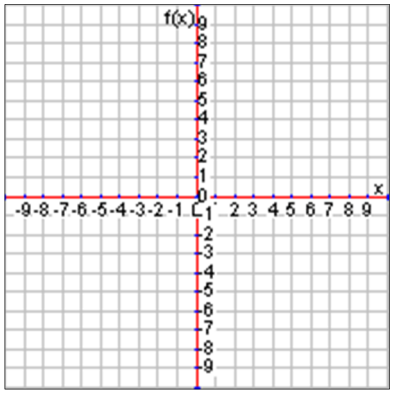
29. $f(x) = \frac{1}{2}^x$ on $[-2, 2]$ ROC: $-\frac{15}{16}$

x	-2	-1	0	1	2
$f(x)$	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$



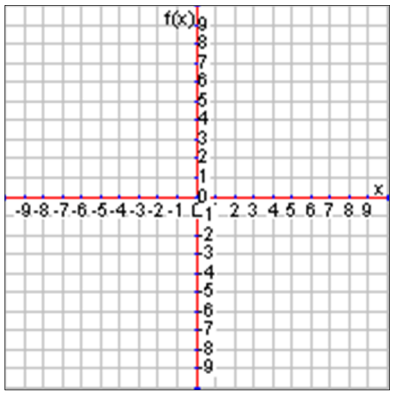
30. $f(x) = \frac{1}{2}^x$ on $[2, 4]$ ROC: $-\frac{3}{32}$

x	-4	-2	0	2	4
$f(x)$	16	4	1	$\frac{1}{4}$	$\frac{1}{16}$



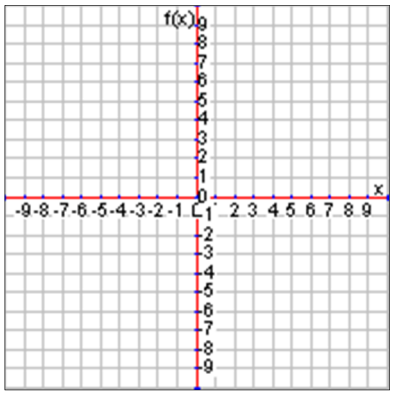
31. $f(x) = 2^x$ on $[-4, -2]$ ROC: $\frac{3}{32}$

x	-4	-2	0	2	4
$f(x)$	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16



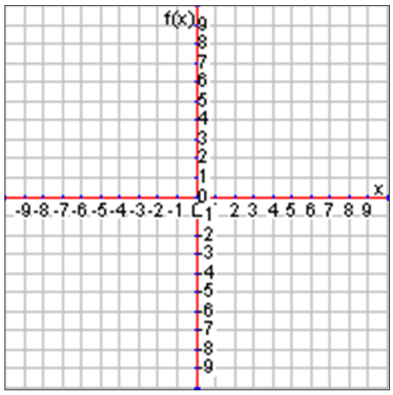
32. $f(x) = 2^x$ on $[-2, 2]$ ROC: $\frac{15}{16}$

x	-2	-1	0	1	2
$f(x)$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4



33. $f(x) = 2^x$ on $[2, 4]$ ROC: 6

x	-4	-2	0	2	4
$f(x)$	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16

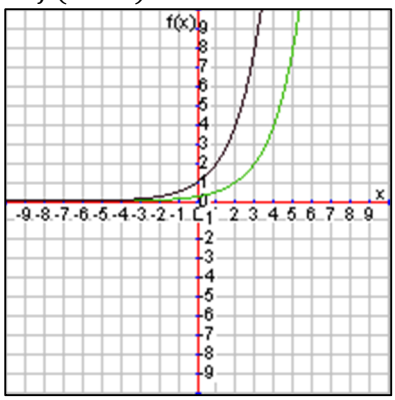


Lesson 4.3

Describe the transform that occurs using the given functions. Then write the new function's equation and draw a quick sketch of the graph.

$f(x) = 2^x$ $\quad$ $g(x) = \left(\frac{1}{2}\right)^x$ $\quad$ $h(x) = 1^x$

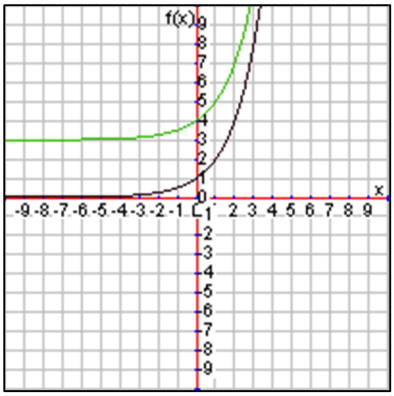
1. $f(x-2)$



Translate right 2 units

$n(x) = 2^{x-2}$

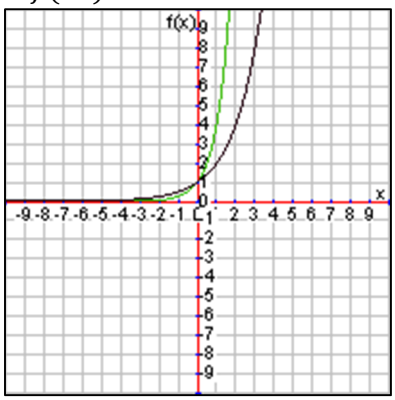
2. $f(x)+3$



Translate up 3 units

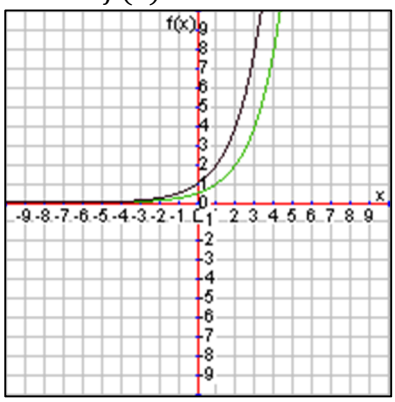
$n(x) = \left(\frac{1}{2}\right)^x + 3$

3. $f(2x)$



Stretched twice as close to the y-axis; $n(x) = 2^{2x}$

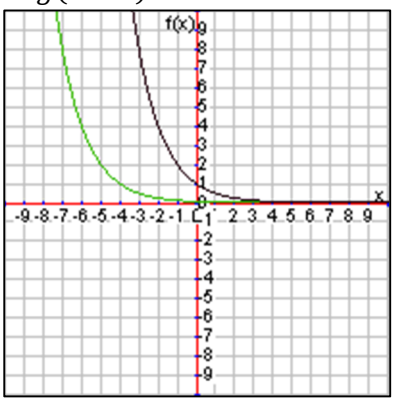
4. $0.5 * f(x)$



Stretched half as far from the x-axis

$n(x) = \frac{1}{2} * 2^x = 2^{x-1}$

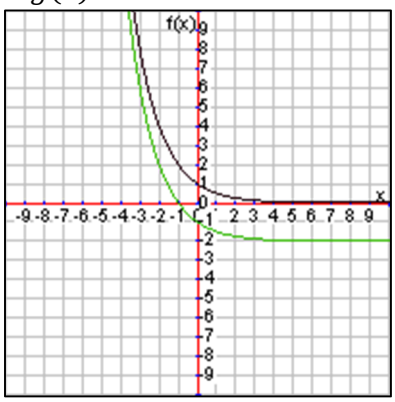
5. $g(x+4)$



Translate left 4 units

$n(x) = \left(\frac{1}{2}\right)^{x+4}$

6. $g(x)-2$



Translate down 2 units

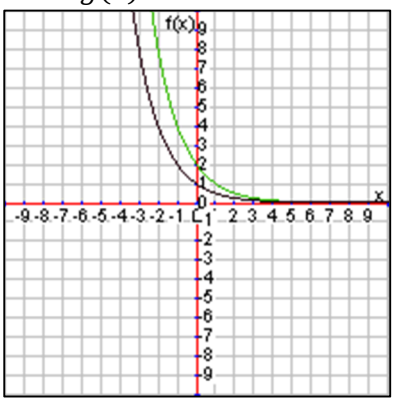
$n(x) = \left(\frac{1}{2}\right)^x - 2$

7. $g(0.5x)$

Stretched half as close to the y-axis

$n(x) = \left(\frac{1}{2}\right)^{0.5x}$

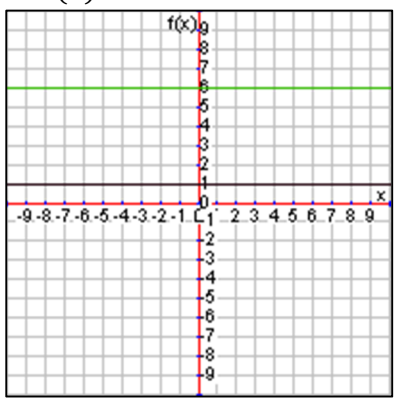
8. $2 * g(x)$



Stretched twice as far from the x-axis

$n(x) = 2 * \left(\frac{1}{2}\right)^x = \left(\frac{1}{2}\right)^{x-1}$

9. $h(x)+5$

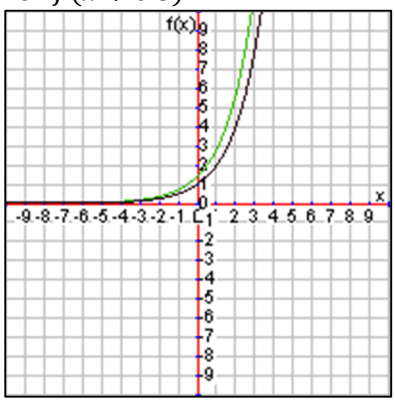


Translated up 5 units

$n(x) = 1^x + 5 = 6$

$f(x) = 2^x$ $g(x) = \left(\frac{1}{2}\right)^x$ $h(x) = 1^x$

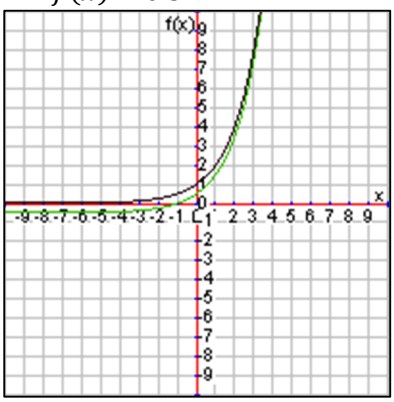
10. $f(x + 0.5)$



Translates left 0.5 units

$n(x) = 2^{x+0.5}$

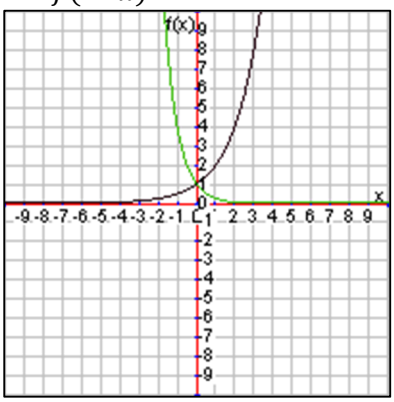
11. $f(x) - 0.5$



Translates down 0.5 units

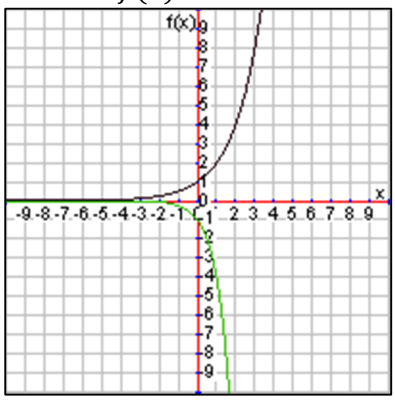
$n(x) = 2^x - 0.5$

12. $f(-2x)$



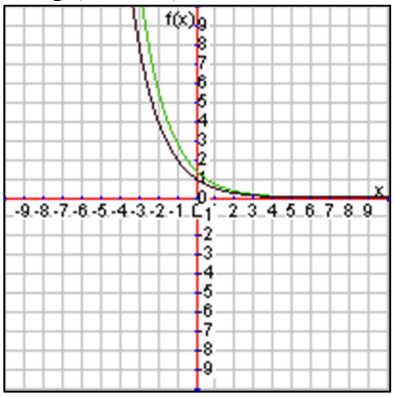
Reflected across the y-axis and Stretched twice as close to the y-axis; $n(x) = 2^{-2x}$

13. $-2 * f(x)$



Reflected across the x-axis and stretched twice as far away from the x-axis; $n(x) = -2 * 2^x = -2^{x+1}$

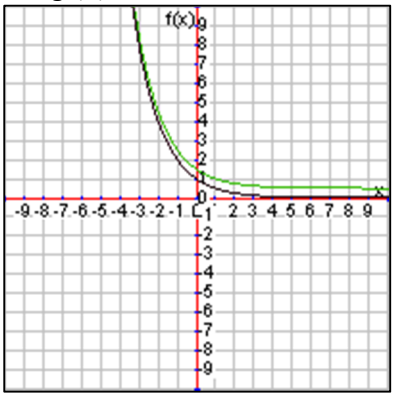
14. $g(x - 0.5)$



Translated right 0.5 units

$n(x) = \left(\frac{1}{2}\right)^{x-0.5}$

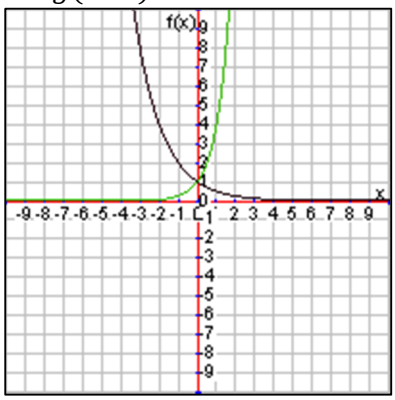
15. $g(x) + 0.5$



Translated up 0.5 units

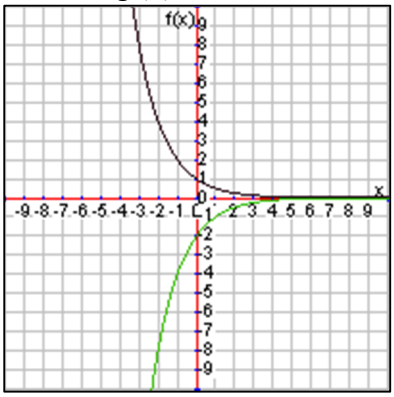
$n(x) = \left(\frac{1}{2}\right)^x + 0.5$

16. $g(-2x)$



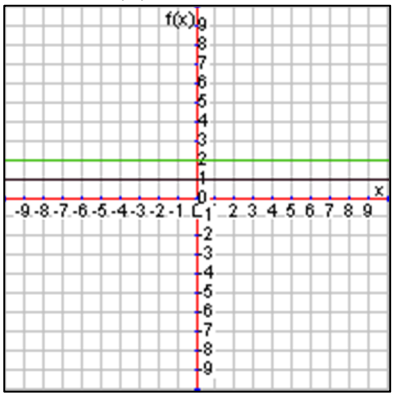
Reflected across the y-axis and stretched twice as far away from the y-axis; $n(x) = \left(\frac{1}{2}\right)^{-2x}$

17. $-2 * g(x)$



Reflected across the x-axis and stretched twice as far away from the x-axis; $n(x) = -2 * \left(\frac{1}{2}\right)^x = -\left(\frac{1}{2}\right)^{x-1}$

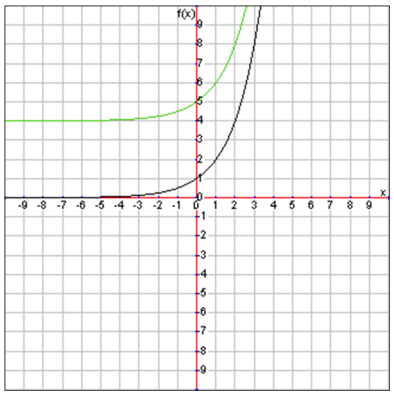
18. $2 * h(x)$



Stretched twice as far away from the x-axis; $n(x) = 2 * 1^x = 2$

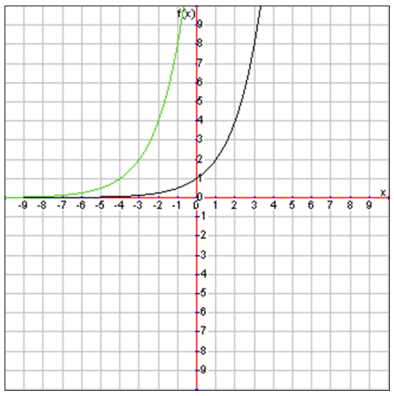
Describe the transform denoted by $g(x)$ using the function $f(x) = 2^x$ as the parent function. Write $g(x)$ in terms of $f(x)$ and then do a quick sketch of the graph of $g(x)$.

19. $g(x) = 2^x + 4$



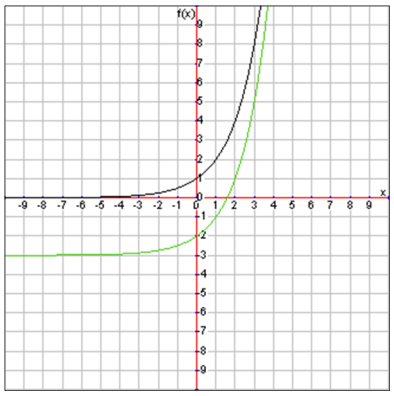
Translated up 4 units
$g(x) = f(x) + 4$

20. $g(x) = 2^{x+4}$



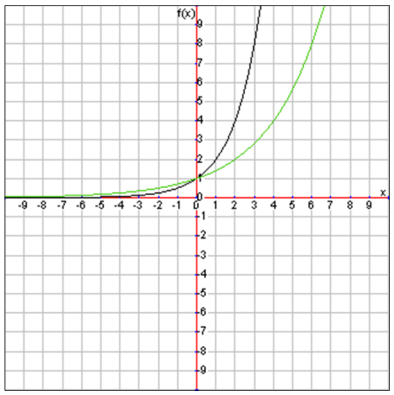
Translated left 4 units
$g(x) = f(x + 4)$

21. $g(x) = 2^x - 3$



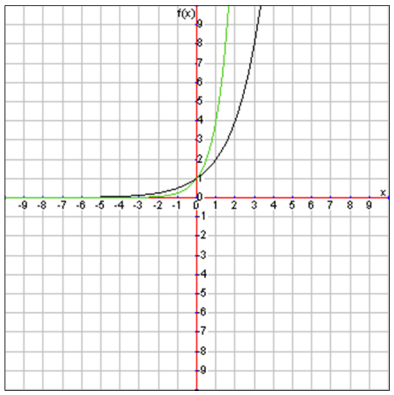
Translated down 3 units
$g(x) = f(x) - 3$

22. $g(x) = 2^{0.5x}$.



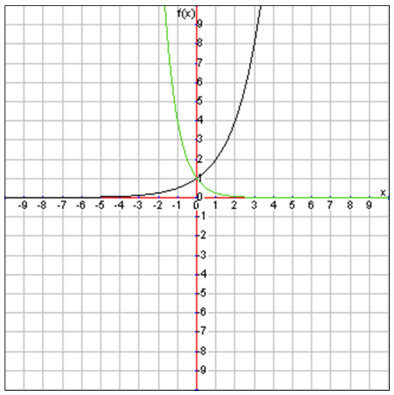
Stretched half as close to the y-axis
$g(x) = f(0.5x)$

23. $g(x) = 2^{2x}$



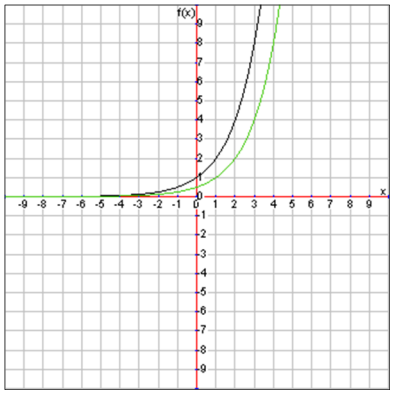
Stretched twice as close to the y-axis
$g(x) = f(2x)$

24. $g(x) = 2^{-2x}$



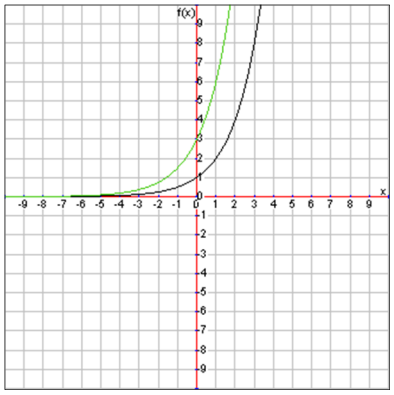
Stretched twice as close to the y-axis and reflected across the y-axis; $g(x) = f(-2x)$

25. $g(x) = 0.5 * 2^x$



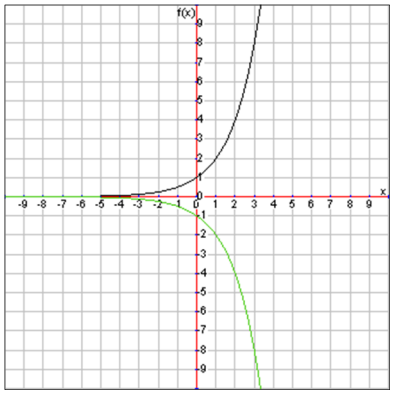
Stretched half as far away from the x-axis; $g(x) = 0.5 * f(x)$

26. $g(x) = 3 * 2^x$



Stretched three times as far away from the x-axis; $g(x) = 3 * f(x)$

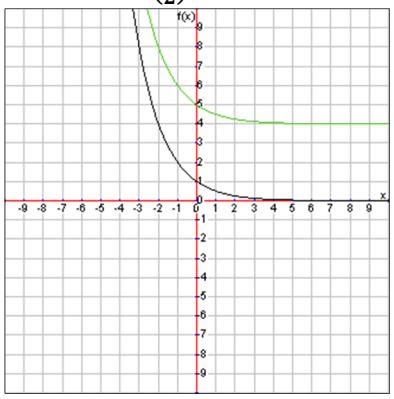
27. $g(x) = -1 * 2^x$



Reflected across the x-axis
$g(x) = -f(x)$

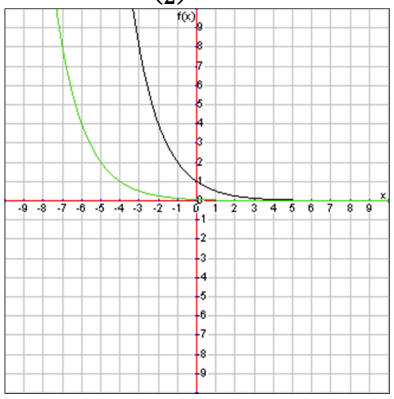
Describe the transform denoted by $g(x)$ using the function $f(x) = \left(\frac{1}{2}\right)^x$ as the parent function. Write $g(x)$ in terms of $f(x)$ and then do a quick sketch of the graph of $g(x)$.

28. $g(x) = \left(\frac{1}{2}\right)^x + 4$



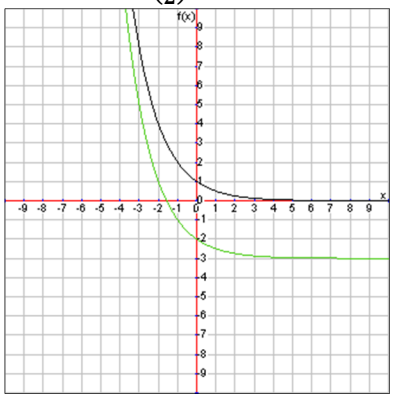
Shifted up 4 units
$g(x) = f(x) + 4$

29. $g(x) = \left(\frac{1}{2}\right)^{x+4}$



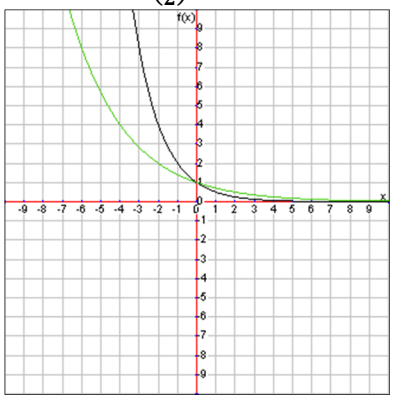
Shifted left 4 units
$g(x) = f(x + 4)$

30. $g(x) = \left(\frac{1}{2}\right)^x - 3$



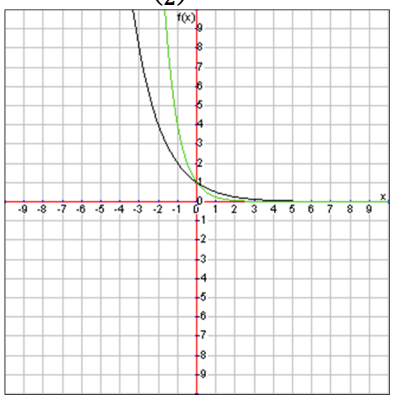
Shifted down 3 units
$g(x) = f(x) - 3$

31. $g(x) = \left(\frac{1}{2}\right)^{0.5x}$



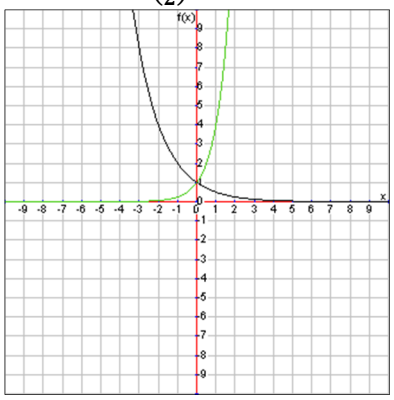
Stretched half has close to the y-axis
$g(x) = f(0.5x)$

32. $g(x) = \left(\frac{1}{2}\right)^{2x}$



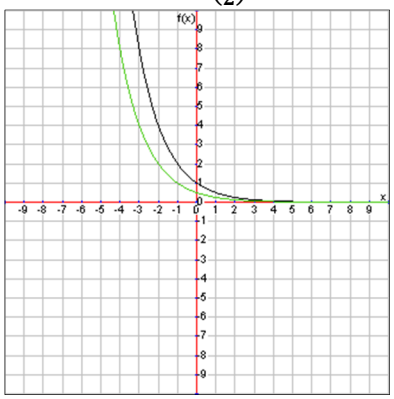
Stretched twice as close to the y-axis
$g(x) = f(2x)$

33. $g(x) = \left(\frac{1}{2}\right)^{-2x}$



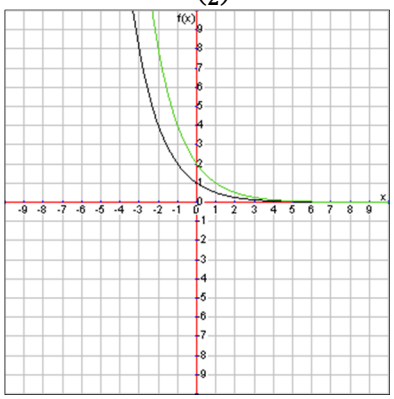
Stretched twice as close to the y-axis and reflected across the y-axis; $g(x) = f(-2x)$

34. $g(x) = 0.5 * \left(\frac{1}{2}\right)^x$



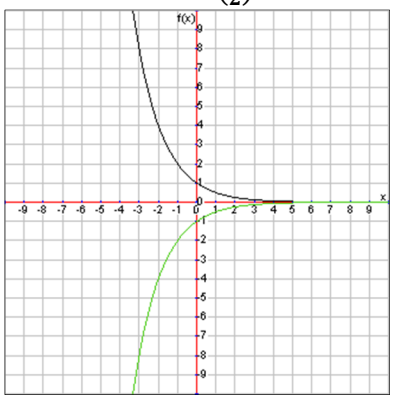
Stretched half as far away from the x-axis; $g(x) = 0.5 * f(x)$

35. $g(x) = 2 * \left(\frac{1}{2}\right)^x$



Stretched twice as far away from the x-axis; $g(x) = 2 * f(x)$

36. $g(x) = -1 * \left(\frac{1}{2}\right)^x$

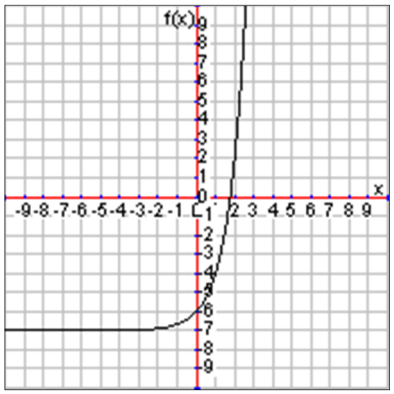


Reflected across the x-axis
$g(x) = -f(x)$

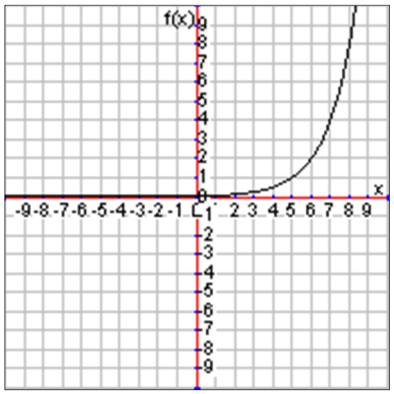
Lesson 4.4

Create an exponential function for the given graph, table, or description.

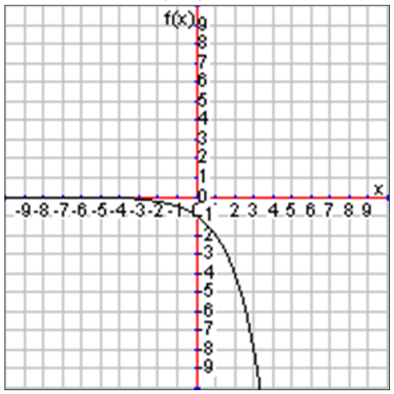
1. $f(x) = 3^x - 7$



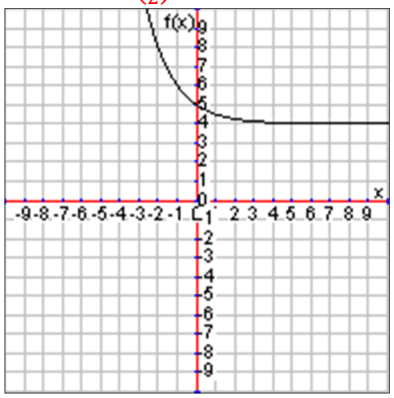
2. $f(x) = 2^{x-5}$



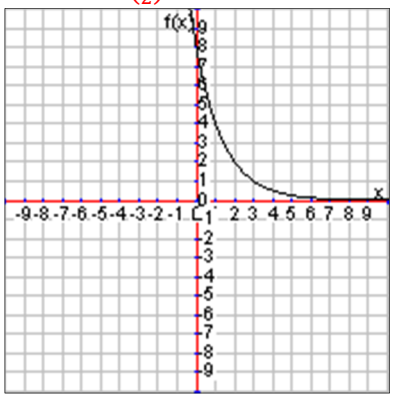
3. $f(x) = -(2^x)$



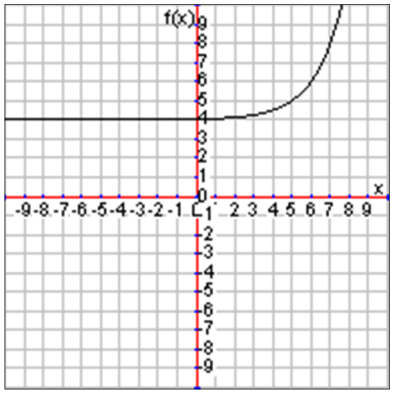
4. $f(x) = \left(\frac{1}{2}\right)^x + 4$



5. $f(x) = \left(\frac{1}{2}\right)^{x-3}$



6. $f(x) = 2^{x-5} + 4$



7. $f(x) = 2^x$

x	-2	-1	0	1
$f(x)$	0.25	0.5	1	2

8. $f(x) = 3^{x+2}$

x	-1	0	1	2
$f(x)$	3	9	27	81

9. $f(x) = 2^{x-1} - 4$

x	0	1	2	3
$f(x)$	-3.5	-3	-2	0

10. $f(x) = 2^x + 3$

x	-2	-1	0	1
$f(x)$	3.25	3.5	4	5

11. $f(x) = 2^x - 6$

x	-1	0	1	2
$f(x)$	-5.5	-5	-4	-2

12. $f(x) = \left(\frac{1}{2}\right)^x$

x	-1	0	1	2
$f(x)$	2	1	0.5	0.25

13. Your parents offer to pay you exponentially to study for your Algebra test. They say that if you study for one hour you'll get $6, two hours gets you a total of $7, three hours $9, four hours $13, etc. What equation are they using to come up with those values?

$$f(x) = 2^{x-1} + 5$$

14. In the game Fruit Slicer you get more points if you can slice multiple fruits in a single swipe. If you only get one fruit in a swipe you get 10 points, two fruits in a swipe gives you 50 points, three fruits gives you 250 points, four fruits 1250 points, etc. What equation does Fruit Slicer use to assign point values?

$$f(x) = 10 * 5^{x-1} = 2 * 5^x$$

15. In general the stock return has averaged about 10% growth per year since its beginning. This is true even through the Great Depression! You have been investing in the stock market for a while, and your stock is currently worth $100,000. What equation models this situation?

$$p(t) = 100{,}000(1.1)^t$$

16. Viruses spread exponentially. In fact, if you start with just 1 person infected with the zombie virus, after one day a total of 10 people would be zombies. After two days a total of 100 people would be zombies. After three days a total of 1000 people would be zombies. What equation models this situation?

$$f(x) = 10^x$$

17. India's current is approximately 1.2 billion, almost as big as China! If India's population is growing at a rate of 2% per year, what equation would model their population growth?

$$p(t) = 1.2(1.02)^t$$

18. There are approximately 25,000 polar bears worldwide. Some scientists estimate that the polar bear population has been declining at a yearly reduction rate of 1%. What equation would model this situation?

$$p(t) = 25{,}000(.99)^t$$

Solve the following problems using the polar bear example above.

19. Approximately what will the population of polar bears be in 20 years?

$$\approx 20{,}448$$

20. Approximately what will the population of polar bears be in 50 years?

$$\approx 15{,}125$$

21. Approximately what was the polar bear population 20 years ago?

$$\approx 30{,}566$$

22. Approximately what was the polar bear population 50 years ago?

$$\approx 41{,}322$$

23. Using technology, graph the polar bear equation. Approximately when was the polar bear population about 100,000 bears?

Approximately 138 years ago

24. Using technology, graph the polar bear equation. Approximately when will the polar bear population be <u>less than</u> 10,000 bears?

In approximately 92 years

Solve the following problems using the given investment information.

A man currently has stock worth a total of $\$5{,}000{,}000$. *It has been growing at an average rate of* 10% *per year.*

25. What equation would model this situation?

$$p(t) = 5{,}000{,}000(1.1)^t$$

26. If the man started investing 60 years ago and did not put any more money in, how much did he start with?

$\$16{,}421.35$

27. Assume the man was 20 years old when he put in that initial amount. Due to the miracles of technology this 80 year old man is still healthy and active enough to work. If he can work until he is 90 years old, how much will his investment be worth then?

$\$12{,}968{,}712.30$

28. What domain would make sense for this problem? Why?

Between 20 and 90 years old because that is how long he was investing

29. Using technology, graph the man's investment equation. Approximately when was his investment worth about $\$1{,}000{,}000$?

Approximately 17 years ago

30. Using technology, graph the man's investment equation. Approximately how old would the man be when his investment would be worth <u>greater than</u> $\$8{,}000{,}000$?

In approximately 5 years

Lesson 4.5

Your financial advisor presents you with four plans for retirement as follows. All dollar amounts are given in millions of dollars. For example, 0.025 million is really $\$25,000$. Answer the following questions about those retirement plans.

Plan A:

Put in an initial investment of $\$0.025$ million and get a return rate of 5%.

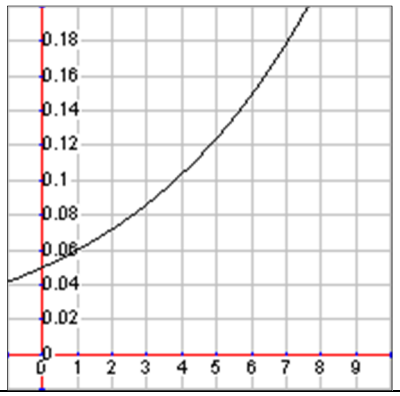
Plan B:

$$g(t) = 0.01(1.15)^t$$

Plan C:

Years	0	1	2	3	4
Money	0.02	0.022	0.0242	0.02662	0.029282

Plan D:



y-int: $(0, 0.05)$

1 year: $(1, 0.06)$

2 years: $(2, 0.072)$

1. List the retirement plans from the highest growth rate to the lowest growth rate.

D, B, C, A

2. List the retirement plans from the lowest initial investment to the highest initial investment.

B, C, A, D

3. How long will it take each retirement plan to be worth $\$1{,}000{,}000$? *(Hint: You will have to graph each plan.)*

Plan A will take 76 years, Plan B will take 33 years, Plan C will take 42 years, Plan D will take 16.5 years

4. Fill out the following table evaluating each plan at specific points in time.

	Retire after 20 years	**Retire after 30 years**	**Retire after 40 years**	**Retire after 50 years**
Plan A	$66,332	$108,049	$176,000	$286,685
Plan B	$163,665	$662,118	$2,678,635	$10,836,574
Plan C	$134,550	$348,988	$905,185	$2,347,817
Plan D	$1,916,880	$11,868,816	$73,488,578	$455,021,908

5. Which plan do you think is the best? Why do think that? What aspect of the function makes it the best retirement plan?

Plan D would be the best because it will make the most money over time. The high rate makes it the best retirement plan.

6. Fill out the following table showing when each plan is worth the same as every other plan in the future. If they are not equal to each other in the future (only in the past), then put Ø for no solution. *(Hint: You will have to graph each plan on the same coordinate plane.)*

	Plan A		
Plan B	10 years	**Plan B**	
Plan C	$4-5$ years	$15-16$ years	**Plan C**
Plan D	Ø	Ø	Ø

You are deciding between different amounts of student loans and your college presents you with four possible plans each with different rates at which the loan is paid off. All dollar amounts for the remaining debt are given in thousands of dollars. For example, 30.5 thousand is really $\$30,500$. Answer the following questions about those student loan plans.

Plan A:

Take out $\$250$ thousand and have a payoff rate of -10%.

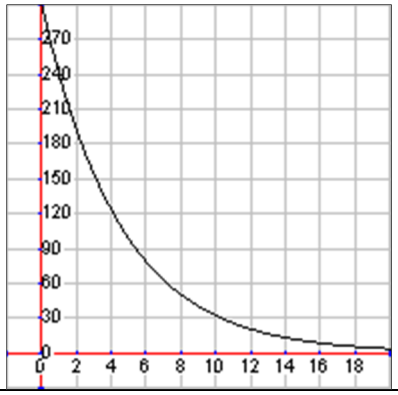
Plan B:

$$g(t) = 100(0.85)^t$$

Plan C:

Years	0	1	2	3	4
Money	150	135	121.5	109.35	98.415

Plan D:



y-int: $(0, 300)$

1 year: $(1, 240)$

2 years: $(2, 192)$

7. List the student loan plans from the fastest payoff rate to the slowest payoff rate.

D, B, A/C

8. List the student loan plans from the lowest initial debt to the highest initial debt.

B, C, A, D

9. How long will it take each student loan plan to be paid down to $\$1{,}000$? *(Hint: Graph each plan or guess and check years.)*

Plan A will take $52-53$ years, Plan B will take $28-29$ years, Plan C will take $47-48$ years, and Plan D will take $25-26$ years

10. Fill out the following table evaluating each plan at specific points in time.

	Remaining debt after 10 years	Remaining debt after 15 years	Remaining debt after 20 years	Remaining debt after 25 years
Plan A	$87,169.61	$51,472.78	$30,394.16	$17,947.45
Plan B	$19,687.44	$8,735.42	$3,875.95	$1,719.78
Plan C	$52,301.77	$30,883.67	$18,236.50	$10,768.47
Plan D	$32,212.26	$10,555.31	$3,458.77	$1,133.37

11. Which plan do you think is the best? Why do think that?

Plan D would be the best because it will pay off sooner than the other plans.

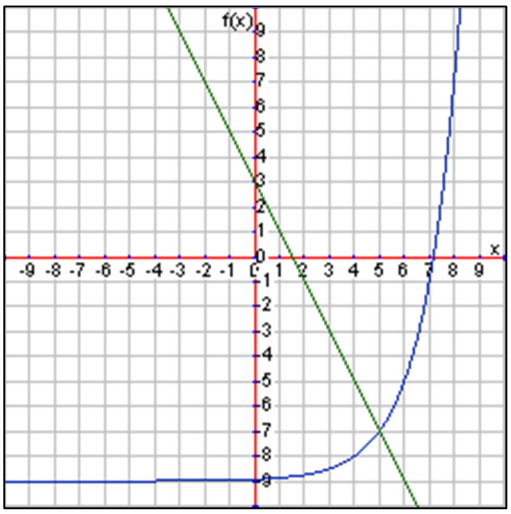
12. Fill out the following table showing when each plan is worth the same as every other plan in the future. If they are not equal to each other in the future (only in the past), then put Ø for no solution. *(Hint: You will have to graph each plan on the same coordinate plane.)*

	Plan A		
Plan B	Ø	**Plan B**	
Plan C	Ø	Ø	**Plan C**
Plan D	1.5 years	18 years	6 years

Solve the following systems of equations.

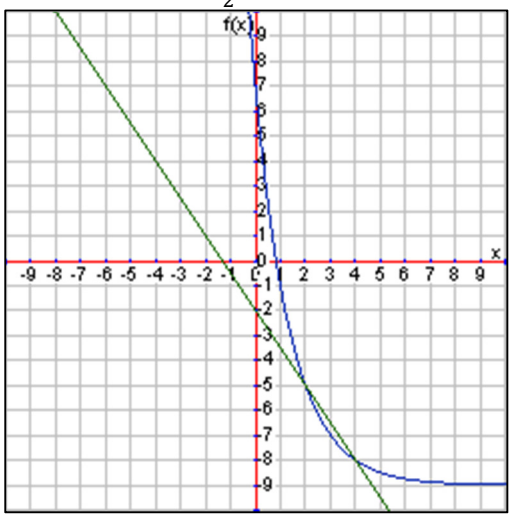
13. $f(x) = 2^{x-4} - 9$

$g(x) = -2x + 3$ $(5, -7)$



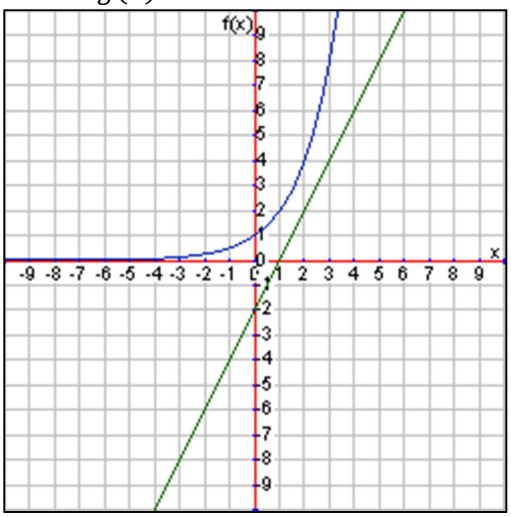
14. $f(x) = \left(\frac{1}{2}\right)^{x-4} - 9$

$g(x) = -\frac{3}{2}x - 2$ $(2, -5)$ and $(4, -8)$



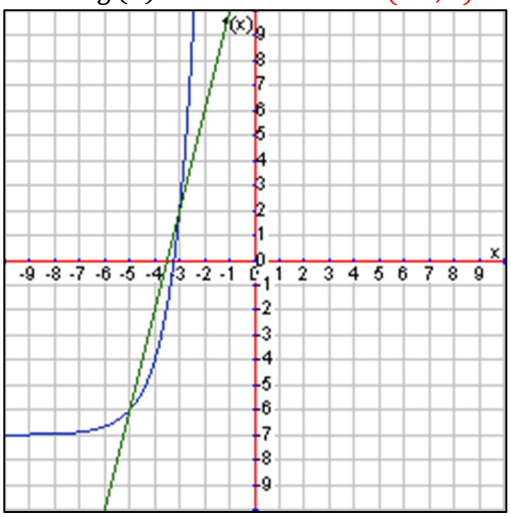
15. $f(x) = 2^x$

$g(x) = 2x - 2$ No solution



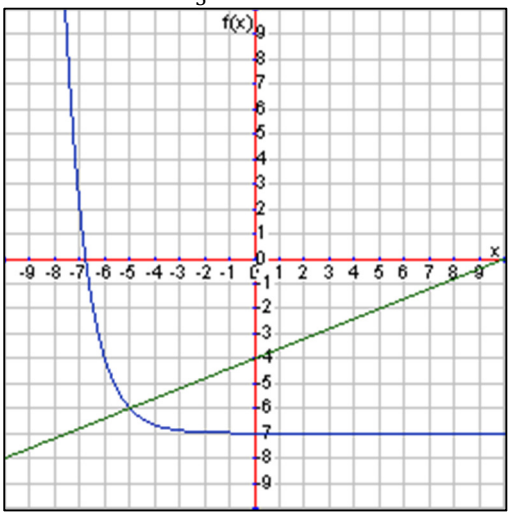
16. $f(x) = 3^{x+5} - 7$

$g(x) = 4x + 14$ $(-3, 2)$ and $(-5, -6)$



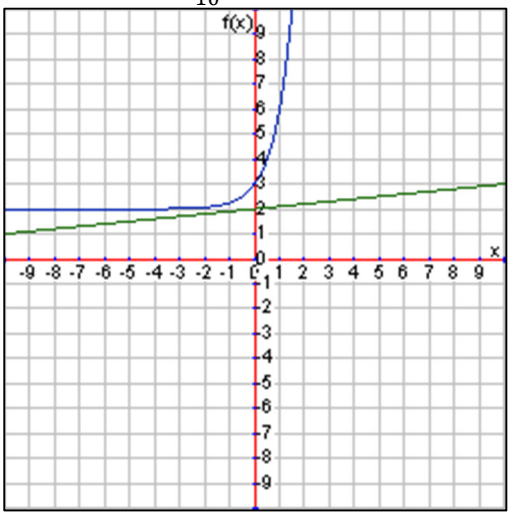
17. $f(x) = \left(\frac{1}{3}\right)^{x+5} - 7$

$g(x) = \frac{2}{5}x - 4$ $(-5, -6)$



18. $f(x) = 4^x + 2$

$g(x) = \frac{1}{10}x + 2$ No solution



Review Unit 4: Exponential Functions KEY

You may use a calculator on parts of this review.

Evaluate the following rational roots. NO CALCULATOR.

1. $16^{\frac{1}{4}} = 2$

2. $64^{\frac{2}{3}} = 16$

3. $4^{\frac{5}{2}} = 32$

Determine if the following statements are true or not. Justify your answer. NO CALCULATOR.

4. $g^{\frac{4}{3}} = \sqrt[4]{g^3}$

   False; $g^{\frac{4}{3}} \neq g^{\frac{3}{4}}$

5. $9^{\frac{5}{2}} = \sqrt[2]{9^5}$

   True; $9^{\frac{5}{2}} = 9^{\frac{5}{2}}$

6. $16^{\frac{1}{2}} = \sqrt[3]{8^2}$

   True; $4 = 4$

Determine the appropriate value to make the equation true. Justify your answer. NO CALCULATOR.

7. $y^{\frac{5}{2}} = \sqrt[\boxed{2}]{y^5}$

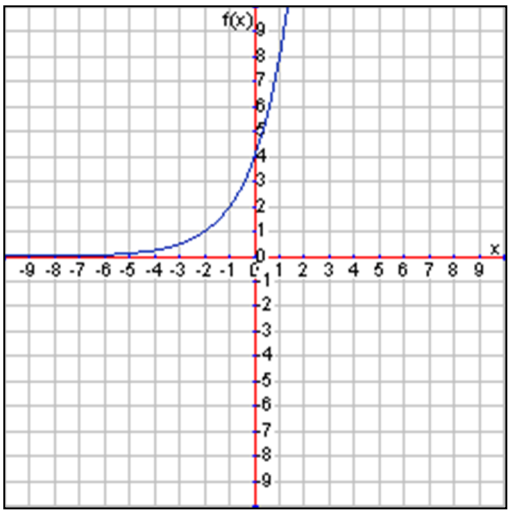
8. $16^{\frac{3}{4}} = 2^{\boxed{3}}$

9. $27^{\frac{2}{3}} = \sqrt[\boxed{2}]{3^4}$

Graph the following functions by filling out the tables. NO CALCULATOR.

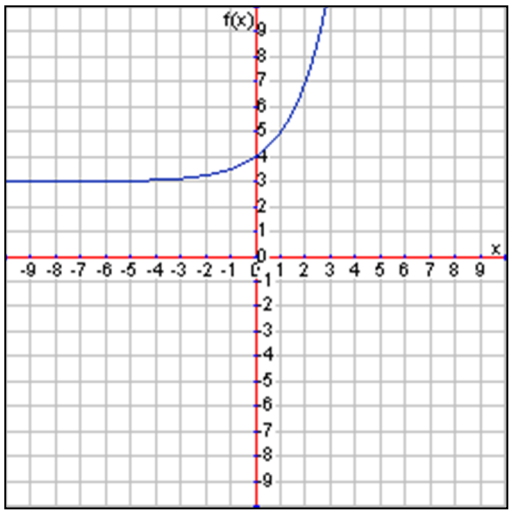
10. $f(x) = 2^{x+2}$

x	-4	-3	-2	-1	0
$f(x)$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4



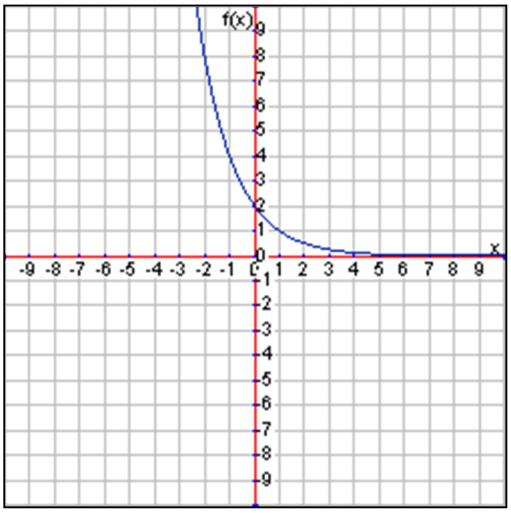
11. $f(x) = 2^x + 3$

x	-2	-1	0	1	2
$f(x)$	$3\frac{1}{4}$	$3\frac{1}{2}$	4	5	7



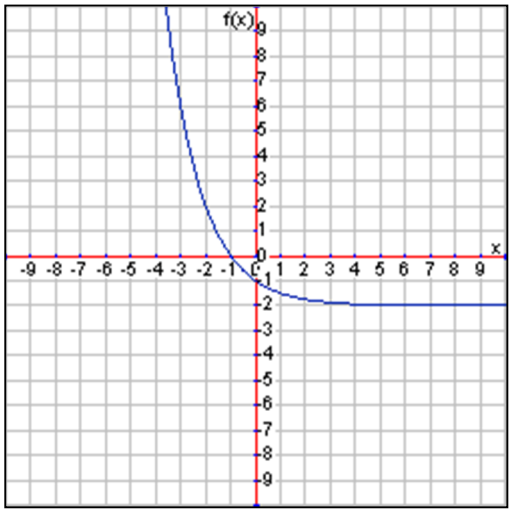
12. $f(x) = \left(\frac{1}{2}\right)^{x-1}$

x	-1	0	1	2	3
$f(x)$	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$



13. $f(x) = \left(\frac{1}{2}\right)^{x} - 2$

x	-2	-1	0	1	2
$f(x)$	2	0	-1	$-1\frac{1}{2}$	$-1\frac{3}{4}$



Find the average rate of change over the interval $[-2,2]$ ***for the following functions. YOU MAY USE A CALCULATOR.***

14. $f(x) = \left(\frac{1}{2}\right)^{x-3} - 3$

$-\frac{15}{2}$

15. $f(x) = 3^{x+2} - 2$

20

16. $f(x) = 2^{x+3}$

$\frac{15}{2}$

Describe the transformation that would take place given the parent function $f(x) = 2^x$***. YOU MAY USE A CALCULATOR.***

17. $f(3x)$

Stretched 3 times closer to the y-axis

18. $\frac{1}{2} * f(-x)$

Stretched half as far from the x-axis and reflected across the y-axis

19. $f(x-2)+4$

Translated right 2 units and up 4 units

20. $-f(x) - 1$

Reflected across the x-axis and translated down 1 unit

21. $g(x) = 2^x + 4$

Translated up 4 units

22. $g(x) = 2^{-x-3}$

Reflected across the y-axis and translated right 3 units

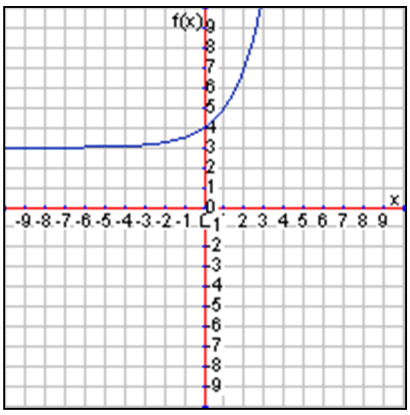
Create an equation for the following graph, table, or situation. YOU MAY USE A CALCULATOR.

23.

x	0	1	2	3	4
$f(x)$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2

$f(x) = 2^{x-3}$

24.



$f(x) = 2^x + 3$

25. A man had $\$250{,}000$ at an interest rate of 2%.

$p(t) = 250{,}000(1.02)^t$

Answer the following questions about the function $p(t) = 100{,}000(0.85)^t$. YOU MAY USE A CALCULATOR.

26. If the function $p(t)$ models the amount of money owed to pay of a loan in t years, what is the current loan value and payment rate?

$\$100{,}000$ and -15%

27. What would we expect their loan value to be in 10 years to the nearest dollar?

$\$19{,}687$

28. What was their approximate loan value 10 years ago to the nearest dollar?

$\$507{,}938$

The current cicada population is $120{,}000$ and is growing at a rate of 15%. The June bug population can be modeled by the function $p(t) = 8000(1.2)^t$ after t years. Answer the following questions. YOU MAY USE A CALCULATOR.

29. Which population is currently the highest and how do you know?

The cicada population is currently the highest because it's current population is $120{,}000$ and the June bug population is currently $8{,}000$.

30. Which population is growing at the fastest rate and how do you know?

The June bug population is currently growing at the fastest rate because it's growth rate is 20% and the cicada growth rate is 15%.

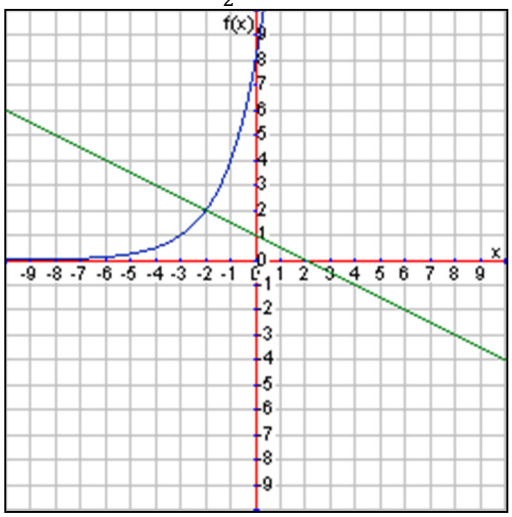
31. Which population will eventually surpass the other and how do you know?

The June bug population will eventually surpass the other because it has a higher growth rate.

Solve the following system of equations. You may graph the functions if that will help. YOU MAY USE A CALCULATOR.

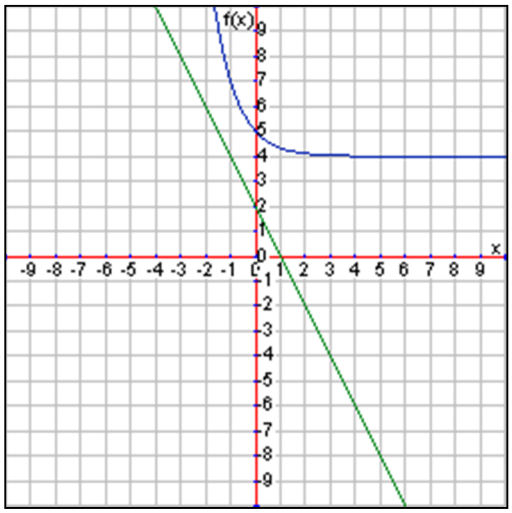
32. $f(x) = 2^{x+3}$

$g(x) = -\frac{1}{2}x + 1$ $(-2, 2)$



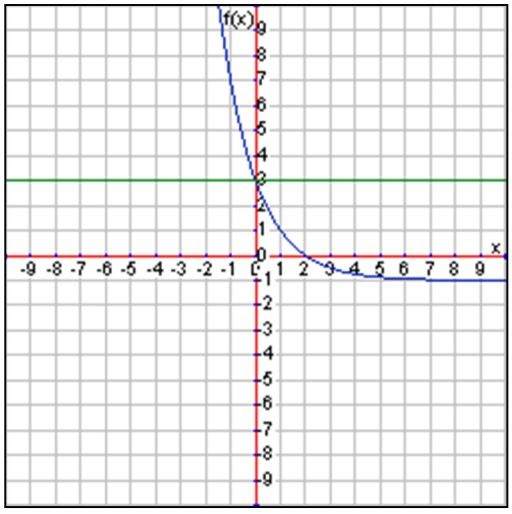
33. $f(x) = 3^{-x} + 4$

$g(x) = -2x + 2$ No solution



34. $f(x) = \left(\frac{1}{2}\right)^{x-2} - 1$

$g(x) = 3$ $(0, 3)$



35. $f(x) = 2^x - 4$

$g(x) = \frac{3}{2}x - 3$ $(0, -3)$ and $(2, 0)$