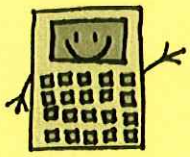
Name: Answer key Per: _____ Date: _____

Serafino ▪ Algebra 2E

7R Exponential & Logarithmic Functions Review

Quest Review



PART 1: Exponential Growth & Decay Models

1. You put $100 in savings, to earn 5.8% annual interest, compounded semiannually.
   a. How much will you have in 2 years? $112.11
   b. When will my initial investment triple? ≈ 19 years
   c. What interest rate would give you $500 in 3 years? 61.53%

2. Mrs. Serafino's grandmother has been shrinking, annually. Three years ago, she was 59.2 inches tall. This year, on her 101st birthday, she was 57.4 inches tall.
   a. What equation models her height? $h = 59.2(0.98976)^t$
   b. At what rate is she shrinking? 1.02% per year
   c. How tall will she be when she's 103? 56.23 inches
   d. How old was she when she was as tall as Mrs. Serafino (63 inches?) 92 ☺

3. Right now I have $500 in an account that has been compounding interest continually.
   a. What interest rate would I need to double my money in 5 years? 13.86%
   b. Using that interest rate, how much will I have in 10 years if I double the amount I put in initially? $4,000

4. You have $25,000 to invest for the next 20 years; you can choose Bank A, which compounds 2.5% interest continuously or Bank B, which compounds 3% interest monthly. In 20 years, which bank gives you more money, and how much more? Bank B: $4300.84 more!

5. A well-bred Samoyed dog could be as much as $11,000. If I invest in an account that yields 4% interest continuously, how much would I have to put in now to buy the dog in 8 years? $7487.64 now

6. A $35,000 diamond depreciates continuously at a rate of 4.3%.
   a. How much will the diamond be worth in 30 years? $9,634.78
   b. When will the diamond be worth 1/10th of how much it's worth now? 53.5 years from now

7. You borrow money from a friend. How many years, to the nearest year, will it take for your original loan to double if you're charged and annual interest rate of 15%, compounded monthly? 4.65 years

** use graphing calculator

8. In 1996 the population of Russia was 148 million and the population of Nigeria was 104 million. If the populations of Russia and Nigeria grow continuously at annual rates of -0.62% and 3.0%, respectively, when will Nigeria have a greater population than Russia? After 9.861 years

9. A scientist places 7.35 grams of a radioactive element in a dish. After d days, the number of grams of the element remaining in the dish is given by the function $R(d) = 7.35\left(\frac{1}{2}\right)^{\frac{d}{2}}$.
Which statement(s) is/are true about the equation when it is rewritten? **Circle ALL that apply.**

A. An approximately equivalent equation is $R(d) = 7.35(0.250)^d$
B. An approximately equivalent equation is $R(d) = 7.35(0.707)^d$
C. The element decays by 0.250 grams per day.
D. The element decays by 0.707 grams per day.
E. About 30% of the element remains from one day to the next day.
F. About 70% of the element remains from one day to the next day.

10. An important physics application of an exponential growth/decay is *Newton's Law of Cooling*. This law states that for a cooling substance after t minutes after being heated can be modeled by $T = (T_0 - T_R)e^{-rt} + T_R$ where temperature T_0 is initial temperature, T_R is room temperature and r is the cooling rate of the substance.

You are cooking a delicious stew. When you take it off the stove, its temperature is 212°F. The room temperature is 70°F, and the cooling rate of the stew is 4.6% per minute.

a. How long will it take to cool the stew to a serving temperature of 100°F? 33 minutes

b. How much longer would it take if you cooked the strew in an 78° room? ≈ 7 minutes longer

11. The data at the right shows the cooling temperatures of a freshly brewed cup of coffee after it is poured from the brewing pot into a serving cup.

Time (min)	Temp (°F)
5	169.3
11	149.2
15	141.7
22	125.4
25	123.5

a. What exponential regression model equation represents this data? $y = 180.6388(.98418)^x$

b. What is the initial temperature of the coffee? ≈ 180.6°

c. How long does it take for the coffee to cool to 130°? ≈ 20.6 minutes

d. Extrapolate: how warm is the coffee after one hour? ≈ 69.38°F

e. In 1992, a woman sued McDonald's after being severely burned when her coffee spilled on her. The woman was awarded over 2.7 million dollars. As a result of this famous case, many restaurants now serve coffee at a temperature around 155º. How long should restaurants wait (after pouring the coffee from the pot) before serving coffee, to ensure that the coffee is not hotter than 155º? 9.6 minutes

f. If the temperature in the room is 76° F, what will happen to the temperature of the coffee, after being poured from the pot, over an extended period of time? It will reach an HA of $y = 76°$

PART 2: EVALUATING & SOLVING & GRAPHING WITHOUT A CALCULATOR:

12. Evaluate the following logarithms. No calculator.

a. $\log_3 9$ (2)

b. $\log_x \sqrt{(x)^3}$ (3/2)

c. $\log_3 -3$ (und)

d. $\log_4 \frac{1}{16}$ (-2)

e. $\log 1{,}000$ (3)

f. $\log_{\frac{1}{9}} 3$ ($-\frac{1}{2}$)

g. $\log_8 16$ (4/3)

h. $\log 1$ (0)

i. $\ln \frac{1}{e}$ (-1)

13. Solve the Following Equations. No Calculator.

a. $25^{2x+1} = 125$ ($x = \frac{1}{4}$)

b. $4^{2x+3} = 1$ ($x = -3/2$)

c. $\log_5(x-3)^2 = 2$ ($x = 8$)

d. $\frac{1}{9}^x = 27$ ($x = -3/2$)

e. $\log_2 \frac{3}{4}x = 5$ ($x = 128/3$)

f. $\log(x^2 + 20) = 2$ ($x = \pm 4\sqrt{5}$)

g. $16^{x+1} = 8^{\frac{1}{2}x}$ ($x = -8/5$)

h. $\log_4(2x-5) = \log_4(x-6)$ (no solution)

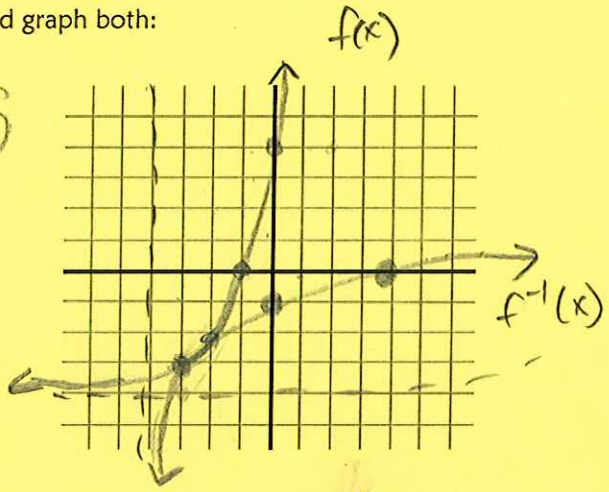
i. $\log_x(4x-3) = 2$ ($x = 3$)

14. For each equation, get the inverse, state the domain and range and graph both:

a. $f(x) = (2)^{x+3} - 4$ $\quad f^{-1}(x) =$ ($\log_2(x+4) - 3$)

Domain: ($x \in \mathbb{R}$) $\quad$ Domain: ($x > -4$)

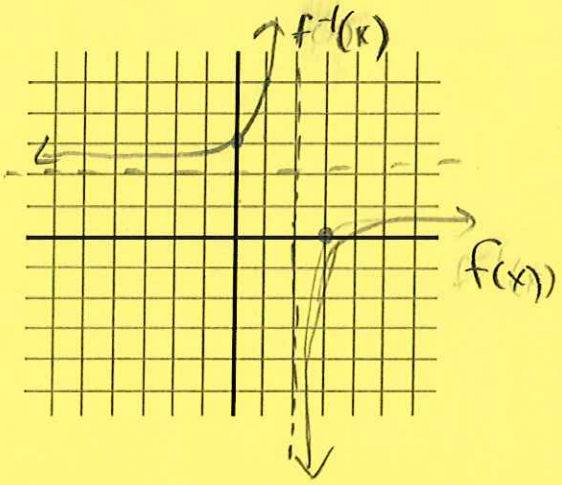
Range: ($y > -4$) $\quad$ Range: ($y \in \mathbb{R}$)



b. $f(x) = \log_3(x-2)$ $\quad f^{-1}(x) =$ ($3^x + 2$)

Domain: ($x > 2$) $\quad$ Domain: ($x \in \mathbb{R}$)

Range: ($y \in \mathbb{R}$) $\quad$ Range: ($y > 2$)



Solving Equations

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LESSON 7.6

Practice A

For use with pages 515–522

Tell whether the *x*-value is a solution of the equation.

1. $\ln x = 3, x = e^3$
2. $\ln x = 4, x = 4e$
3. $\ln x = 5, x = 5^e$
4. $\log 5x = 2, x = 20$
5. $\ln 4x = 4, x = \frac{e^4}{4}$
6. $\log x = 7, x = e^7$
7. $e^x = 4, x = 4$
8. $e^x = 6, x = \ln 6$
9. $10^x = 2, x = \log 3$
10. $3e^x = 9, x = \ln 3$
11. $10^{x+2} = 10, x = -1$
12. $10^{x-3} + 10 = 0, x = 2$

Solve the equation by equating exponents.

13. $3^{x+1} = 3^{x+1}$ (x ∈ ℝ)
14. $4^{2x} = 4^{x+2}$ (2)
15. $7^x = 7^{2x-3}$ (3)
16. $e^{3x} = e^{2x+1}$ (1)
17. $e^{x+3} = e^{3x+2}$ (1/2)
18. $e^{3-x} = e^{2x-6}$ (3)
19. $9^x = 3^{x+4}$ (4)
20. $16^{x+1} = 4^{4x+1}$ (1/2)
21. $8^{x-1} = 2^{x+2}$ (5/2)

Solve the equation by taking the appropriate log of each side.

22. $2^x = 16$ (4)
23. $5^x = 22$ (1.9206)
24. $e^x = 3$ (1.0986)
25. $e^{3x} = 6$ (0.5973)
26. $4^x - 2 = 7$ (1.161)
27. $10^{4x+1} + 3 = 17$ (.0365)

Solve the equation. Check for extraneous solutions.

28. $\log x = \log 5$ (5)
29. $\ln(3x - 1) = \ln x$ (1/2)
30. $\log_3 5x = \log_3(x + 4)$ (1)
31. $\log_7(5 - x) = \log_7 3x$ (5/4)
32. $\ln(4x - 1) = \ln(x - 6)$ (no solution)
33. $\ln(7 - 2x) = \ln(3x + 3)$ (4/5)

Solve the equation by exponentiating each side.

34. $\log x = 3$ (1000)
35. $\log_2(x - 1) = 4$ (17)
36. $\log_3 3x = 5$ (81)
37. $\log_7(3 - 2x) = 3$ (−170)
38. $\ln(x + 7) = 4$ (47.598)
39. $\ln(1 - 5x) = 0$ (0)

40. **Multiple Choice** You deposit $400 in an account that pays 3.75% annual interest compounded monthly. About how long does it take for the balance to quadruple?

A. 26.2 years B. 32.5 years C. 37 years

In Exercises 41–43, use the following information.

Compounding Interest You deposit $500 in an account that pays 4% annual interest. How long does it take the balance to reach the following amounts?

41. $1000 when interest is compounded quarterly (17.4 yrs)
42. $1500 when interest is compounded yearly (28 yrs)
43. $2000 when interest is compounded continuously (34.7 yrs)

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LESSON 7.6

Practice B

For use with pages 515–522

Solve the exponential equation. Check for extraneous solutions. Round the result to three decimal places if necessary.

1. $e^x = 1$ (0)
2. $e^x = 4$ (1.386)
3. $e^x + 1 = 7$ (1.792)
4. $5^x = 12$ (1.544)
5. $4^x - 6 = 4$ (1.661)
6. $3^{4x} = 27$ (0.75)
7. $e^{2x} = 4$ (0.693)
8. $3e^{3x} = 12$ (0.462)
9. $10^{2x-3} + 3 = 19$ (2.102)
10. $3e^x + 7 = 9$ (−.405)
11. $10^{x+2} - 12 = 22$ (−.469)
12. $10^{-x+4} + 7 = 5$ (no solution)
13. $3^{-3x+1} = 3^{x-9}$ (5/2)
14. $8^{2x} = 8^{x+7}$ (7)
15. $7^{2x-3} - 4 = 14$ (2.243)
16. $4e^{3x} = 1$ (−0.462)
17. $e^{5x+2} = e^{3x+12}$ (5)
18. $3e^{3-x} = 15$ (.391)
19. $9^{2x} = 3^{2x+4}$ (2)
20. $25^{x-4} = 5^{3x+1}$ (−9)
21. $8^{x-1} = \left(\frac{1}{2}\right)^{2x-1}$ (415)
22. $3(2^{x+6}) = 17$ (−3.497)
23. $5^{0.5x} + 12 = 21$ (2.73)
24. $-5e^x - 3 = 24$ (no solution)
25. $\frac{3}{4}e^{3x} - 8 = -6$ (6.327)
26. $\frac{2}{3}(4^{3x}) - 5 = -2$ (0.362)
27. $10^{2x+1} + 2 = 2$ (no solution)

Solve the logarithmic equation. Check for extraneous solutions. Round the result to three decimal places if necessary.

28. $\log x = 3$ (1000)
29. $\ln x = 4$ (54.598)
30. $\log_3 x = 5$ (243)
31. $\log_7(2 - x) = \log_7 5x$ (1/3)
32. $\ln(3x - 3) = \ln(x - 6)$ (∅)
33. $\ln(5 - 2x) = \ln(5x + 3)$ (2/7)
34. $\log_4 3x = 6$ (1365.3)
35. $\log_2(3x - 1) = 8$ (85.67)
36. $7 - \log_3 8x = 2$ (30.375)
37. $2\log_7(1 - 2x) = 12$ (−58.824)
38. $3\ln x - 7 = 4$ (39.121)
39. $\ln(1 - 3x) + 3 = 9$ (−134.14)
40. $\log 7x + 4 = 5$ (1617)
41. $4 + \log_9(3x - 7) = 6$ (27.33)
42. $\log_2 2x + \log_2 x = 5$
43. $\log_6(2x - 6) + \log_6 x = 2$
44. $\ln 3x - \ln 2 = 4$
45. $\ln(-5x + 3) = \ln 2x + 2$

46. **Multiple Choice** You deposit $500 in an account that pays 3.25% annual interest compounded monthly. About how long does it take for the balance to quadruple?

A. 36.3 years B. 42.7 years C. 45.1 years

In Exercises 47–49, use the following information.

Compounding Interest You deposit $700 in an account that pays 2.75% annual interest. How long does it take the balance to reach the following amounts?

47. $1000 when interest is compounded quarterly (13.0 yrs)
48. $1500 when interest is compounded yearly (28.1 yrs)
49. $2000 when interest is compounded continuously (38.2 yrs)

50. **Rocket Velocity** Disregarding the force of gravity, the maximum velocity v of a rocket is given by $v = t \ln M$ where t is the velocity of the exhaust and M is the ratio of the mass of the rocket with fuel to its mass without fuel. A solid propellant rocket has an exhaust velocity of 2.3 kilometers per second. Its maximum velocity is 7.2 kilometers per second. Find its mass ratio M. (22.88)