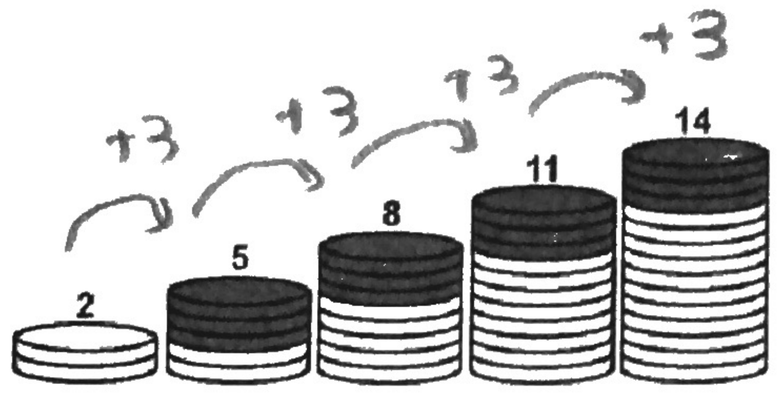
Name: Answer key Per: _____ Date: _____

Serafino • Algebra 2E

8A Arithmetic Sequences

Classwork/ HW Packet



I get bored and start stacking quarters, as so →
How many quarters will be in the 97th stack? .

An arithmetic sequence is a: list of numbers that increase or decrease by a constant rate of change → Linear! ☺

Let: **n** = position of the term a_n = term in the nth position **d** = common difference

Explicit Formulas: $a_n = a_1 + d(n-1)$ or $a_n = a_k + d(n-k)$

1st term, common difference

linear → like point slope form!

$a_k = a_1 + d(k-1)$

(Less popular: $a_n = dn + z$)

Finds a term, given a common difference and the "zeroth" term

Finds a term, given the first term and the common difference

Recursive Formula: $a_n = a_{n-1} + d$ (must provide a_1)

Finds a term in the sequence, based on the previous term (stupid)

asks: "what should you do the last term to get the next one?"

Provide Recursive and Explicit Formulas based on the first few terms of a sequence. Then find the 50th term.

1. 19, 25, 31, 37 … $d = 6$

n	1	2	3	4	50
a_n	19	25	31	37	313

Recursive: $a_n = a_{n-1} + 6$, $a_1 = 19$

Explicit: $a_n = 19 + 6(n-1)$

2. 5, –3, –11, –19 … $d = -8$

n	1	2	3	4	50
a_n	5	-3	-11	-19	-387

Recursive: $a_n = a_{n-1} - 8$, $a_1 = 5$

Explicit: $a_n = 5 - 8(n-1)$

3. 8, 9.5, 11, 12.5 ... n= 1.5

n	1	2	3	4	50
a_n	8	9.5	11	12.5	81.5

Recursive: $a_n = a_{n-1} + 1.5$; $a_1 = 8$

Explicit: $a_n = 8 + 1.5(n-1)$

Given one formula, give the first 4 terms and 50th term in the sequence, and the other formula.

4. Recursive Formula: $a_n = a_{n-1} + 3$; $a_1 = 13$

n	1	2	3	4	50
a_n	13	16	19	22	160

Explicit: $a_n = 13 + 3(n-1)$

5. Recursive Formula: $a_n = a_{n-1} - 5$; $a_1 = -4$

n	1	2	3	4	50
a_n	-4	-9	-14	-19	-249

Explicit: $a_n = -4 - 5(n-1)$

6. Explicit Formula: $a_n = -6 + 4(n-1)$

n	1	2	3	4	50
a_n	-6	-2	2	6	190

Recursive: $a_n = a_{n-1} + 4$; $a_1 = -6$

7. Explicit Formula: $a_n = 2.5 + 0.5(n-1)$

n	1	2	3	4	50
a_n	2.5	3	3.5	4	27

Recursive: $a_n = a_{n-1} + 0.5$; $a_1 = 2.5$

Now, I will only give you one term in the sequence and the common difference. Then fill in the rest.

$a_n = a_1 + d(n-1)$

8. $a_1 = 10$ $d = 5$

n	1	2	3	4	50
a_n	10	15	20	25	255

Recursive: $a_n = a_{n-1} + 5$, $a_1 = 10$

Explicit: $a_n = 10 + 5(n-1)$

Use: $a_n = a_1 + d(n-1)$

$a_2 = a_1 + d(2-1)$

$7 = a_1 + 3(2-1)$

$7 = a_1 + 3$ $\quad a_1 = 4$

9. $a_2 = 7$ $d = 3$

n	1	2	3	4	50
a_n	4	7	10	13	151

Recursive: $a_n = a_{n-1} + 3$, $a_1 = a_1 = 4$

Explicit: $a_n = 4 + 3(n-1)$

$a_n = a_1 + d(n-1)$

$7 = a_1 - 1(4-1)$

$7 = a_1 - 3$ $\quad a_1 = 10$

10. $a_4 = 7$ $d = -1$

n	1	2	3	4	50
a_n	10	9	8	7	-39

Recursive: $a_n = a_{n-1} - 1$, $a_1 = a_1 = 10$

Explicit: $a_n = 10 - 1(n-1)$

I also can only give you two terms in the arithmetic sequence. You will have to find the common difference, and then fill out the rest.

11. $a_3 = 15$ $a_4 = 19$ $d = 4$

n	1	2	3	4	50
a_n	7	11	15	19	323

Recursive: $a_n = a_{n-1} + 4$, $a_1 = 7$

Explicit: $a_n = 7 + 4(n-1)$

Use either one to get a_1

$a_n = a_1 + d(n-1)$

$15 = a_1 + 4(3-1)$

$15 = a_1 + 8$

12. $a_3 = 8$ $a_5 = 14$ d = 3

n	1	2	3	4	50
a_n	2	5	8	11	149

$8 = a_1 + 3(3-1)$
$8 = a_1 + 6$

Recursive: $a_n = a_{n-1} + 3$
$a_1 = 2$

Explicit: $a_n = 2 + 3(n-1)$

13. $a_8 = 25$ $a_{20} = 61$ $d = 3$ check using a_{20}!

n	1	2	3	4	50
a_n	4	7	11	15	151

$25 = a_1 + 3(8-1)$
$25 = a_1 + 21$

Recursive: $a_n = a_{n-1} + 3$
$a_1 = 4$

Explicit: $a_n = 4 + 3(n-1)$

14. $a_{14} = 50$ $a_{22} = 34$ $\frac{-16}{8} = -2$ * check using a_{22}!

n	1	2	3	4	50
a_n	76	74	72	70	-22

$50 = a_1 - 2(14-1)$
$50 = a_1 - 26$ $a_1 = 76$

Recursive: $a_n = a_{n-1} - 2$
$a_1 = 76$

Explicit: $a_n = 76 - 2(n-1)$

Finding if a number is a member of a sequence....

15. –15, 5, 25, 45, 65.... $a_n = -15 + 20(n-1)$

Is 75 a member? no $75 = -15 + 20(n-1)$
$90 = 20n - 20$
$\frac{110}{20} = \frac{20n}{20}$
$n = 5.5$ ... so no! (b/w 5th + 6th)

Is 285 a member? yes, 16th

$285 = -15 + 20(n-1)$
$\frac{300}{20} = \frac{20(n-1)}{20}$
$15 = n-1$

16. 6, 17, 28, 39, 50 ... $325 = 6 + 11(n-1)$

Is 325 a member? yes, 30th

Is 761 a member? no, b/w 69th, 70th

17. 87, 83, 79, 75 ... $35 = 87 - 4(n-1)$
$-52 = -4(n-1)$
$13 = n-1$

Is 35 a member? yes, 14th

Is –279 a member? no, b/c $-279 = 87 - 4(n-1)$

18. 2, 15, 28, 41 ... $145 = 2 + 13(n-1)$

Is 145 a member? yes, 12th

Is 528 a member? no, b/w 41st and 42nd

Let's put it all together!

1. Circle the letters that are arithmetic sequences. Give the explicit formula and identify the 100th term.

+2 +4 +4

X a. 2, 4, 8, 12 ...

b. 19.5, 19.9, 20.3, 20.7 ...

$a_n = 19.5 + .4(n-1)$

$a_{100} = 59.1$

X c. 3, 9, 27, 81 ...

d. 21, 20, 19, 18 ...

$a_n = 21 - 1(n-1)$

$a_{100} = -78$

e. 4/3, 7/3, 10/3, 13/3 ...

$a_n = \frac{4}{3} + 1(n-1)$

$a_{100} = 301/3$

+4 +5 +5

d. 5, 9, 14, 19 ...

2. In each of the arithmetic sequences, fill in the missing term(s).

+12

a. 8, 20, 32, ... (+12; 2; d=12)

b. –14, 8, 2, ... ($-\frac{12}{2}$; d=-6)

12/3

c. 2, 6, 10, 14, ...

d. 3, 9, 15, 21, ... (d=6; $\frac{18}{3}$)

e. 65, 54, 43, 32, 21, ... ($\frac{44}{4}$ d=11)

3. Provide Recursive and Explicit Formulas and the requested term: 4, 11, 18, 25, ...

Recursive: $a_n = a_{n-1} + 7$, $a_1 = 4$ Explicit: $a_n = 4 + 7(n-1)$ a_{20}: 137

4. The 5th term of a sequence is 34 and the common difference is –3.

$34 = a_1 - 3(5-1)$

$34 = a_1 - 12$

$a_1 = 46$

First 5 terms: 46 43 40 37 34

Recursive: $a_n = a_{n-1} - 3$, $a_1 = 46$ Explicit: $a_n = 46 - 3(n-1)$ a_{20}: -14

65

5. The 9th term is –20 and the 14th term is 45. $d=13$ $45 = a_1 + 13(14-1)$

5

First 5 terms: -124 -111 -98 -85 -72

Recursive: $a_n = a_{n-1} + 13$, $a_1 = -124$ Explicit: $a_n = -124 + 13(n-1)$ a_{20}: 123

6. Is 233 a member of the sequence?: 0, 3, 6, 9, 12… ? If yes, say which term. If no, explain why.

$233 = 0 + 3(n-1)$ no, n is not a natural number. 233 is b/w the 78th / 79th term

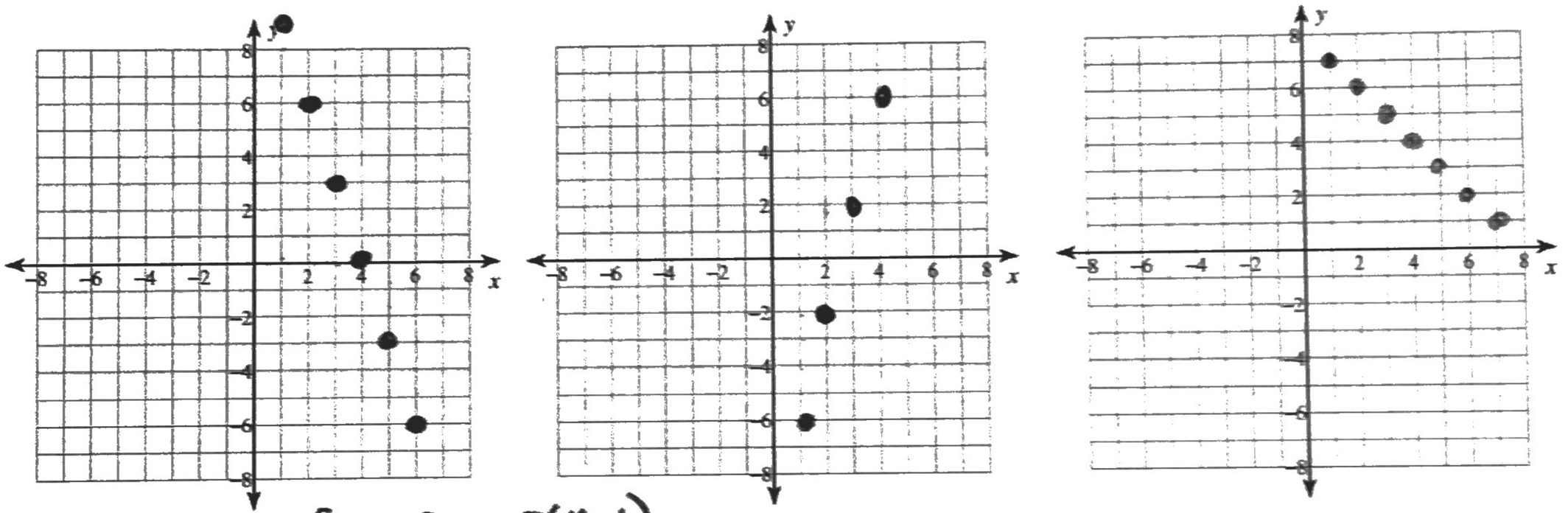
7. Graphing sequences is insanely easy. Just… uh…. plot the points. Done.

$a_n = 7 - 1(n-1)$

a. 9, 6, 3, 0, …

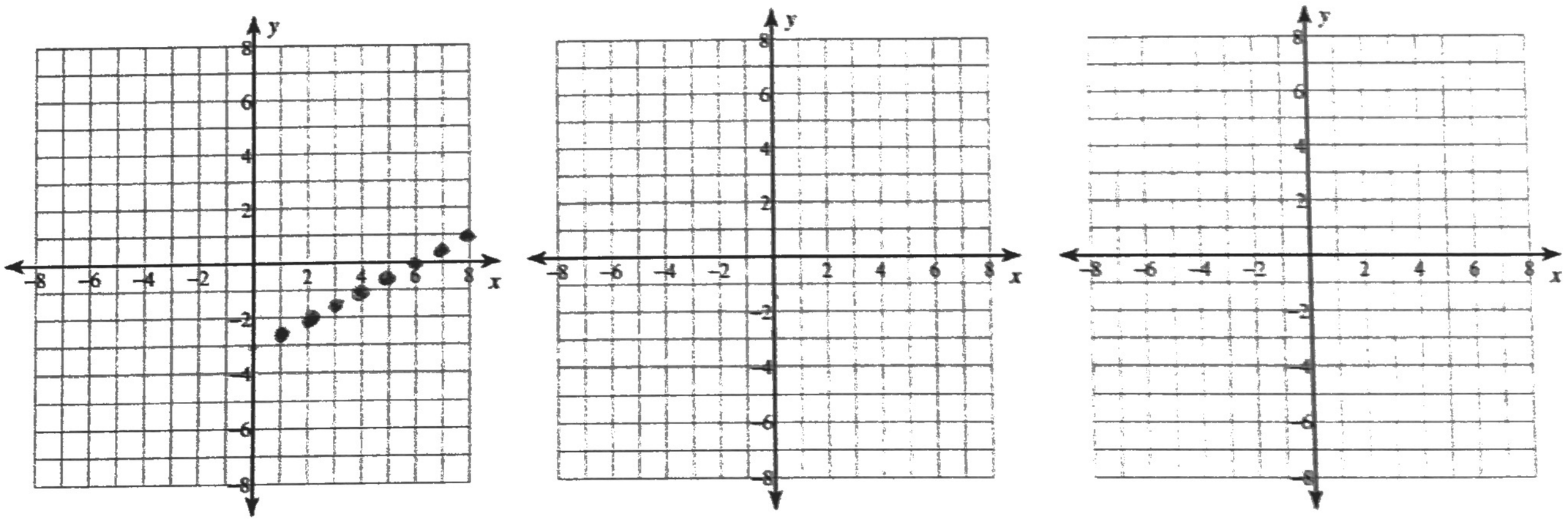
b. –6, –2, 2, …

c. $a_n = -n + 8$



$a_n = -2.5 + .5(n-1)$

d. $a_n = 1/2\ n - 3$



g. Look at your graphs above. Which are the only quadrants(s) you used? QI and IV

Why are those the only ones you need to graph sequences?

b/c n's domain is the natural #'s (integers greater than zero)