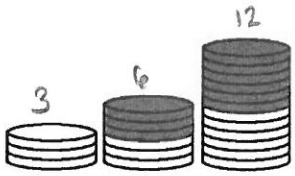
Name: Answer Key Per: _____ Date: _____
Serafino • Algebra 2E

8B Geometric Sequences

Classwork/ HW Packet



I get bored again and start stacking quarters. This time, I start with a stack of 3 and each day, the stack doubles in size. How many quarters will I have on the 19th day?

$r = 3$ $r = -\frac{1}{2}$

This is an example of a **geometric sequence.** Others include: 3, 9, 27, 81… or 20, –10, 5, –5/2 …

A geometric sequence is a: list of numbers that grows/decays exponentially (multiply by the same ratio each time)

Let: **n** = position of the term a_n = term in the nth position **r** = common ratio

Explicit Formula: $a_n = a_1 \cdot r^{(n-1)}$ or $a_n = a_k \cdot r^{(n-k)}$

Recursive Formula: $a_n = a_{n-1} \cdot r$ (must provide a_1)

Arithmetic or Geometric Sequence? Tell me which, if any, and give me d or r.

1. 1, –5, 25, –125 …
   geo $r = -5$
2. 120, 60, 20, 5 …
   neither
3. 5, –5, –15, –25 …
   arith $d = -10$
4. 1, 3, 6, 10, …
   neither
5. 90, 60, 40…
   geo $r = \frac{2}{3}$
6. 10, 100, 1000, 10000 …
   geo $r = 10$

Provide Recursive and Explicit Formulas based on the first few terms of a sequence. Then find the 50th term.

7. 1, 5, 25, 125 … $r = 5$

n	1	2	3	4	7
a_n	1	5	25	125	15625

Recursive: $a_n = a_{n-1} \cdot 5$, $a_1 = 1$

Explicit: $a_n = 1(5)^{n-1}$

8. 2, –6, 18, –54 … r = -3

n	1	2	3	4	10
a_n	2	−6	18	−54	−39366

Recursive: $a_n = a_{n-1}(-3)$, $a_1 = 2$

Explicit: $a_n = 2(-3)^{n-1}$

9. 2000, 500, 125, 31.25 … r = $\frac{1}{4}$

n	1	2	3	4	8
a_n	2000	500	125	125/4	125/16

Recursive: $a_n = a_{n-1}\left(\frac{1}{4}\right)$, $a_1 = 2,000$

Explicit: $a_n = 2(-3)^{n-1}$

Given one formula, give the first 4 terms and 50th term in the sequence, and the other formula.

10. Recursive Formula: $a_n = a_{n-1} \cdot 3$; $a_1 = 7$

n	1	2	3	4	8
a_n	7	21	63	189	15,309

Explicit: $a_n = 7(3)^{n-1}$

11. Recursive Formula: $a_n = a_{n-1} \cdot ½$; $a_1 = 32$

n	1	2	3	4	10
a_n	32	16	8	4	1/16

Explicit: $a_n = 32\left(\frac{1}{2}\right)^{n-1}$

12. Explicit Formula: $a_n = 6 \cdot -2^{n-1}$ r = -2

n	1	2	3	4	9
a_n	6	−12	24	−48	1536

Recursive: $a_n = a_{n-1}(-2)$, $a_1 = 6$

13. Explicit Formula: $a_n = -2 \cdot 3^{n-1}$ r = 3

n	1	2	3	4	11
a_n	−2	−6	−18	−54	−118098

Recursive: $a_n = a_{n-1}(3)$, $a_n = -2$

Now, I will only give you one term in the sequence and the common ratio.

14. $a_6 = 64$ $r = 2$

n	1	2	3	4	12
a_n	2	4	8	16	4096

Recursive: $a_n = a_{n-1}(2)$

$a_1 =$ $a_1 = 2$

Explicit: $a_n = 2(2)^{n-1}$

15. $a_4 = 100$ $r = 1/3$

n	1	2	3	4	5
a_n	2700	900	300	100	100/3

Recursive: $a_n = a_{n-1}\left(\frac{1}{3}\right)$

$a_1 =$ 2700

Explicit: $a_n = 2700\left(\frac{1}{3}\right)^{n-1}$

16. $a_{11} = -1$ $r = -1$

n	1	2	3	4	50
a_n	−1	1	−1	1	1

Recursive: $a_{n-1}(-1)$

$a_1 =$ -1

Explicit: $a_n = -1(-1)^{n-1}$

I also can only give you two terms in the geometric sequence. DON'T "CHEAT" – use the formula.

17. $a_4 = -8$ $a_5 = -40$ $r =$ 5

n	1	2	3	4	7
a_n	−8/125	−8/25	−8/5	−8	−1000

Recursive: $a_n = a_{n-1}(5)$

$a_1 =$ $a_n = -8/125$

Explicit: $a_n = \frac{-8}{125}(5)^{n-1}$

18. $a_6 = -96$ $a_9 = 768$ $r =$ −2

n	1	2	3	4	7
a_n	3	−6	12	−24	192

Recursive: $a_n = a_{n-1}(-2)$

$a_1 =$ 3

Explicit: $a_n = 3(-2)^{n-1}$

Sometimes, there are TWO options for r.

19. $a_3 = 36$ $a_5 = 324$ r = ± 3

n	1	2	3	4	8
a_n	4	12	36	108	8748
	4	−12	36	−108	−8748

Recursive: $a_n = a_{n-1}(\pm 3)$

$a_1 =$ 4

Explicit: $a_n = 4(\pm 3)^{n-1}$

20. $a_2 = 12$ $a_6 = 192$ r = ± 2

	n	1	2	3	4	8
r=2	a_n	6	12	24	48	768
r=−2		−6	12	−24	−48	−768

Recursive (1): $a_n = a_{n-1}(2)$

$a_1 =$ 6

Recursive (2): $a_n = a_{n-1}(-2)$

$a_1 =$ −6

Explicit (1): $a_n = 6(2)^{n-1}$ (2) $a_n = -6(-2)^{n-1}$

Find if a number is a member of a sequence. If it is, tell me which term. If not, between which two terms?

21. 4, 8, 16, …

Is 1024 a member? yes, 9th

Is 16,380 a member? no! btw 12th & 13th

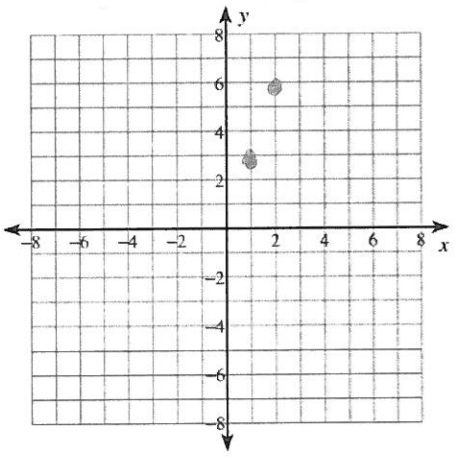
22. −20,000, 10,000, −5,000, 2,500 ….

Is −625 a member? no

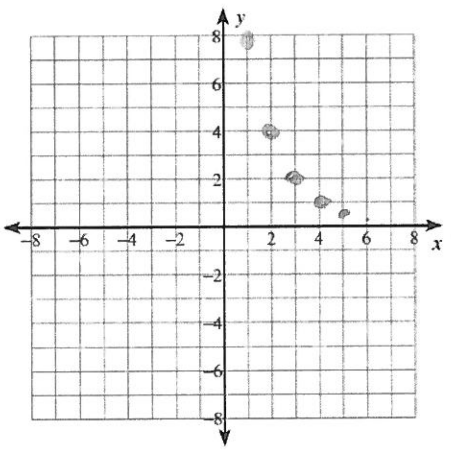
Is 625 a member? yes, 6th

23. Again, graphing is easy. Just plot the points. Don't connect. Start with n = 1.

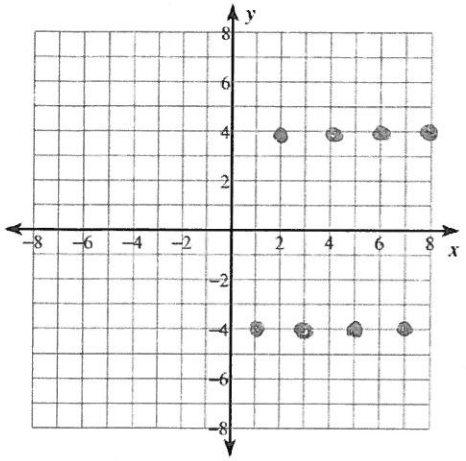
a. $a_n = 3(2)^{n-1}$



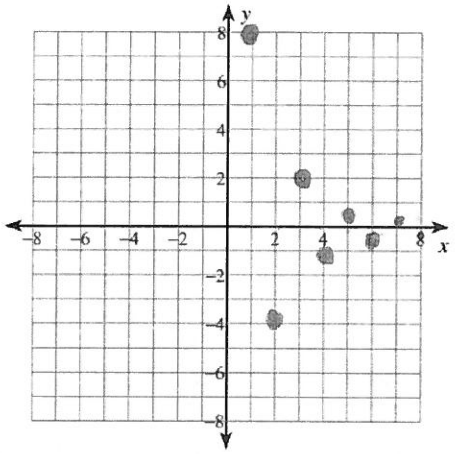
b. $a_n = 8(0.5)^{n-1}$



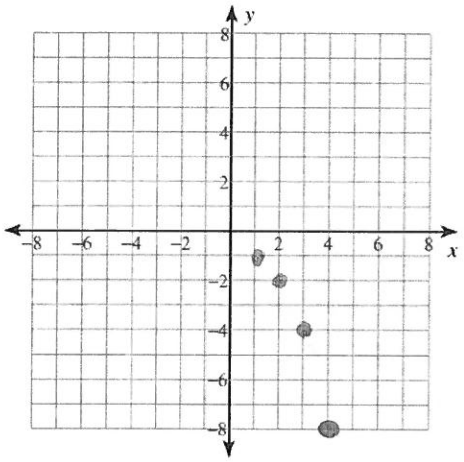
c. $a_n = -4(-1)^{n-1}$



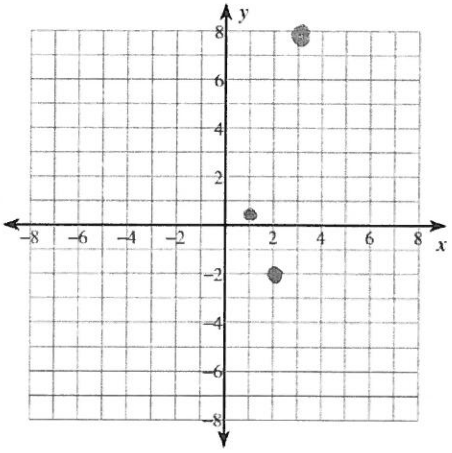
d. $a_n = 8(-0.5)^{n-1}$



e. $a_n = -1(2)^{n-1}$



f. $a_n = 1/2(-4)n^{-1}$



Put it all together! Do the following on a separate piece of paper

1. Circle the letters that are geometric sequences. Give the explicit formula and use it to identify the 8th term.

a. 2, 4, 8, 12 …

b. 100, 150, 225, … $a_n = 100\left(\frac{3}{2}\right)^{n-1}$; $\frac{54675}{32}$

c. 3, –9, 27, … $a_n = 3(-3)^{n-1}$; -6561

d. 21, 16, 11, 6 …

e. 5/3, 5/9, 5/27, … $a_n = \frac{5}{3}\left(\frac{1}{3}\right)^{n-1}$; $\frac{5}{6561}$

d. –5, 5, –5 … $a_n = -5(-1)^{n-1}$; 5

2. Stay fresh! In # 1, put a star next to the arithmetic sequence. Write the explicit formula:

$a_n = 21 - 5(n-1)$

3. Get the first term in each sequence:

a. The 5th term is 16, 000,000, the common ratio is –20. $a_1 = 100$

b. The 6th term is -64. The common ratio is –2. $a_1 = 2$

4. In each of the geometric sequences, find r and fill in the missing terms (indicate ± if there are two possibilities).

a. 8, ± 16, 32, ± 64 ...

b. 1600, 400, 100, 25, 6.25 ...

c. $\pm \frac{14}{3}$, 14, ± 42, 126 ...

d. –2, 8, –32, 128, –512, 2048...

5. Given the sequence: 17, –34, 68, –136, ...

Recursive: $a_n = a_{n-1}(-2)$, $a_1 = 17$ Explicit: $a_n = 17(-2)^{n-1}$ a_{13}: 69,632

6. The a_{12} = is 177147/4096 and a_{15} = 45395/311 (Keep in fraction form. Let your calculator do the work)

First 5 terms: $\frac{1}{2}$ $\frac{3}{4}$ $\frac{9}{8}$ $\frac{27}{16}$ $\frac{81}{32}$

Recursive: $a_n = a_{n-1}\left(\frac{3}{2}\right)$, $a_n = \frac{1}{2}$ Explicit: $a_n = \left(\frac{1}{2}\right)\left(\frac{3}{2}\right)^{n-1}$ a_5: $\frac{81}{32}$

7. The 3rd term in a sequence is 9/8. The 6th term is 243/8.

a. How many possibilities exist for r?
one, $r = 3$

b. How many possibilities exist for a_1?
one, $a_1 = 1/8$

c. Write all possible explicit formulas that would capture this sequence.
$a_n = \frac{1}{8}(3)^{n-1}$

8. The 3rd term in a sequence is –6. The 5th term is –13.5.

a. How many possibilities exist for r?
two, $r = \pm \frac{3}{2}$

b. How many possibilities exist for a_1?
one, $a_1 = -\frac{8}{3}$

c. Write all possible explicit formulas that would capture this sequence.
$a_n = -\frac{8}{3}\left(\frac{3}{2}\right)^{n-1}$ or $a_n = -\frac{8}{3}\left(-\frac{3}{2}\right)^{n-1}$

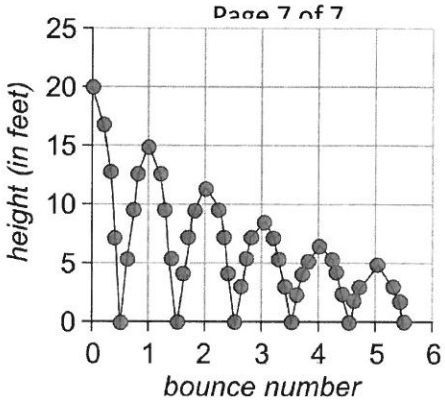
9. The 4rd term in a sequence is –4. The 8th term is –5,184.

a. How many possibilities exist for r?
two, $r = \pm 6$

b. How many possibilities exist for a_1?
two, $a_1 = \pm \frac{1}{54}$

c. Write all possible explicit formulas that would capture this sequence.
$a_n = -\frac{1}{54}(6)^{n-1}$ or $a_n = \frac{1}{54}(-6)^{n-1}$

10. A ball is dropped from 20 feet above the ground and bounces, as such.



a) What is the common ratio from one bounce to the other?

$r = 0.759836$

b) What explicit formula gives the height of any bounce?

$a_n = 15(0.759836)^{n-1}$

c) How high will the ball bounce on the 12th bounce?

$a_{12} = 6.73115$ ft (or 8.77 inches)

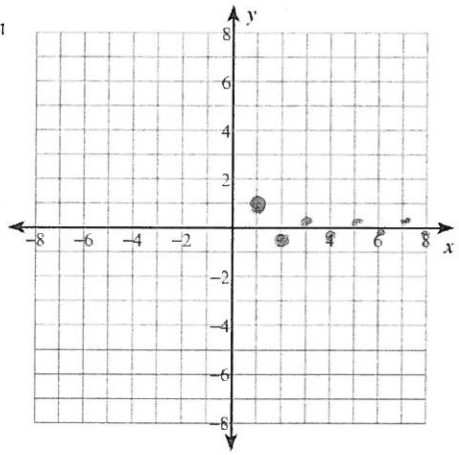
11. Is the given number a member of the sequence? If so, what term? If not, between which two terms does it lie? Solve with logs or without logs, but <u>do not "cheat" by multiplying or dividing a bunch of times or, so help me, on quizzes I will give you the worst fractional ratios I can imagine and I will not allow you to use a calculator. You have been warned.</u>

a. 9, 63, 441, ... .151263? yes, 6th .... 1,058,849? no

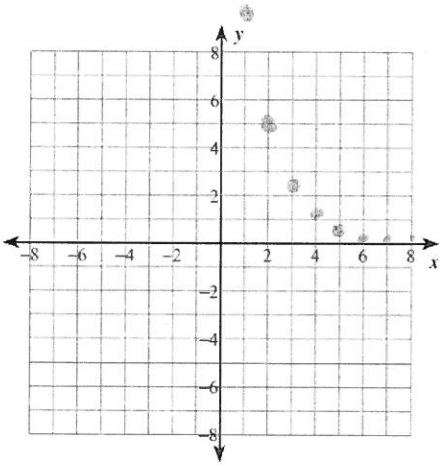
b. 4,000, 2,000, 1,000... ...325? no ...125? yes, 6th

12. Graph the sequences:

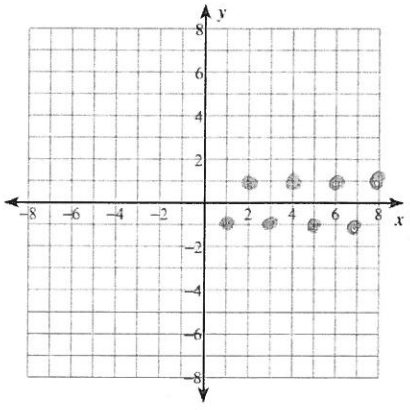
a. $a_n = 1(-0.5)^{n-1}$



b. $a_n = 10(0.5)^{n-1}$



c. $a_n = -1(-1)^{n-1}$



d. $a_n = 8(-0.5)^{n-1}$