

## Key

1)  
3 decisions  
3 blanks

$$\rightarrow \frac{6}{\swarrow} \cdot \frac{5}{\downarrow} \cdot \frac{4}{\searrow} = 120$$

Any of the  
6 people.
The runner  
who finishes  
first can't  
finish second.
Must be 4 runners left  
since first and second  
are picked.

2)  
3 decisions  
3 blanks

$$\rightarrow \frac{4}{\swarrow} \cdot \frac{3}{\downarrow} \cdot \frac{2}{\searrow} = \cancel{24} \quad 24$$

4 hats
3 gloves
2 pairs of shoes

3a)

Letter      Letter      Letter [ 26 letters in the alphabet ]

↓            ↓            ↓

6 decisions

6 blanks

$$\rightarrow \frac{26}{\uparrow} \cdot \frac{10}{\text{digit}} \cdot \frac{26}{\uparrow} \cdot \frac{10}{\text{digit}} \cdot \frac{26}{\uparrow} \cdot \frac{10}{\text{digit}} = 17,576,000$$

There are 10 digits { 0, 1, 2, 3 ... 9 }

b)

Must be S            Any letter [ 26 letters in the alphabet ]

↓                    ↙                    ↘

6 decisions

6 blanks

$$\rightarrow \frac{1}{\uparrow} \cdot \frac{10}{\text{digit}} \cdot \frac{26}{\uparrow} \cdot \frac{10}{\text{digit}} \cdot \frac{26}{\uparrow} \cdot \frac{10}{\text{digit}} = 676,000$$

There are 10 digits { 0, 1, 2, 3 ... 9 }

c)

Must be S            Any letter [ 26 letters in the alphabet ]

↓                    ↙                    ↘

6 decisions

6 blanks

$$\rightarrow \frac{1}{\uparrow} \cdot \frac{10}{\text{digit}} \cdot \frac{26}{\uparrow} \cdot \frac{10}{\text{digit}} \cdot \frac{26}{\uparrow} \cdot \frac{1}{\uparrow} = 67,000$$

Look at Restrictions first

digit
digit
Must be the digit 8.  
Only one way to get the digit 8

d)

6 decisions  
 6 blanks  
 Look at  
 Restrictions first

Must be 5      5 can't be used so 25 letters remain, then 24

$$\rightarrow \frac{1}{\uparrow} \cdot \frac{1}{\uparrow} \cdot \frac{25}{\uparrow} \cdot \frac{9}{\uparrow} \cdot \frac{24}{\uparrow} \cdot \frac{8}{\nwarrow} = 43,200$$

Must be 6      digit 6 is      Now 8 digits remain  
 One choice      used, therefore  
                          9 digits remain  
                          to pick from

4)

a)

3 blanks  

$$\rightarrow \frac{5}{\uparrow} \cdot \frac{5}{\uparrow} \cdot \frac{5}{\uparrow} = 125$$

Each blank can be any of the 5 digits

b)

3 blanks  

$$\rightarrow \frac{5}{\uparrow} \cdot \frac{5}{\nearrow} \cdot \frac{3}{\uparrow} = 75$$

Any of the      Only the digits 1, 3, 5  
 five digits      are odd, so 3 choices

c)

Fill in the restrictions first!

Only 3 digits are left for the middle spot

3 blanks  

$$\rightarrow \frac{4}{\uparrow} \cdot \frac{3}{\nwarrow} \cdot \frac{3}{\uparrow} = 36$$

Only 4 left      Only the digits 1, 3, 5  
 since either      are odd, so 3 choices  
 1, 3, or 5 is  
 used at the end  
 and you can't repeat digits

d part a)

3 blanks  

$$\rightarrow \frac{5}{\uparrow} \cdot \frac{6}{\nwarrow} \cdot \frac{6}{\nwarrow} = 180$$

Cannot be 0      Any of the 6 digits  
 Since it would  
 NOT be a 3 digit  
 integer

d part b)

Can be any of the 6 digits

↓

$$3 \text{ blanks} \rightarrow \frac{5}{\uparrow} \cdot \frac{6}{\uparrow} \cdot \frac{3}{\uparrow} = 90$$

Cannot be 0      Only the digits 1, 3, 5  
 So, 5 digits      are odd, so 3 choices  
 to pick from

d)

(c) it is odd and repetition is NOT allowed?

Fill in the restrictions first!

2 digits are used from the 6

Therefore, 4 digits to pick from

↙

$$3 \text{ blanks} \rightarrow \frac{4}{\uparrow} \cdot \frac{4}{\uparrow} \cdot \frac{3}{\uparrow} = 48$$

Cannot be 0      Only the digits 1, 3, 5  
 and either      are odd, so 3 choices  
 1, 3, or 5 is  
 used at the end  
 Therefore, 4 digits  
 remain

5a)

$$6 \text{ blanks} \rightarrow \frac{6}{\uparrow} \cdot \frac{5}{\uparrow} \cdot \frac{4}{\uparrow} \cdot \frac{3}{\uparrow} \cdot \frac{2}{\uparrow} \cdot \frac{1}{\uparrow} = 6! = 720$$

Note: All letters must be used → NO Repetition

b)

$$6 \text{ blanks} \rightarrow \frac{1}{\uparrow} \cdot \frac{5}{\uparrow} \cdot \frac{4}{\uparrow} \cdot \frac{3}{\uparrow} \cdot \frac{2}{\uparrow} \cdot \frac{1}{\uparrow} = 5! = 120$$

One way for T  
 to be picked

c)

Remember → Fill in the restrictions first

4 letters are left for these 4 blanks

↙ ↓ ↓ ↘

$$6 \text{ blanks} \rightarrow \frac{4}{\uparrow} \cdot \frac{4}{\uparrow} \cdot \frac{3}{\uparrow} \cdot \frac{2}{\uparrow} \cdot \frac{1}{\uparrow} \cdot \frac{2}{\uparrow} = 192$$

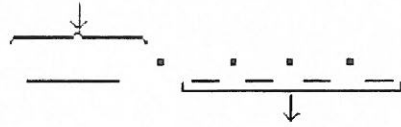
4 consonants      2 vowels to  
 to pick from      pick from

d)

Think of TR as one big blank

↓

2! ways you can arranging TR



↓ Now, arrange these 4 blanks with the big TR blank → 5 blanks

↓

$$2! \cdot 5! = 240$$

6a)

Any of the 6 can be used

↓

$$\frac{3}{\uparrow} \cdot \frac{6}{\uparrow} \cdot \frac{3}{\uparrow} = 54$$

Must be less

than 4 {0,1,2,3}

but can't be 0

So, 3 choices

Must be even

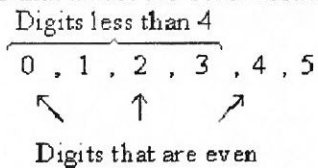
{0,2,4} can be used

3 choices

b)

Remember, fill in restrictions first, but in this question you have overlapping restrictions.

(restrictions that affect the other restrictions)



So, break up the question into parts – this will eliminate a restriction.

Last digit is 4

↓

$$\frac{3}{\uparrow} \cdot \frac{4}{\swarrow} \cdot \frac{1}{\swarrow}$$

Must be 4 digits

1, 2 or 3.

Can't be 0

or

Last digit is 2

↓

$$\frac{2}{\uparrow} \cdot \frac{4}{\swarrow} \cdot \frac{1}{\swarrow}$$

Must be 4 digits

1 or 3 left

or

Last digit is 0

↓

$$\frac{3}{\uparrow} \cdot \frac{4}{\swarrow} \cdot \frac{1}{\swarrow}$$

Must be 4 digits

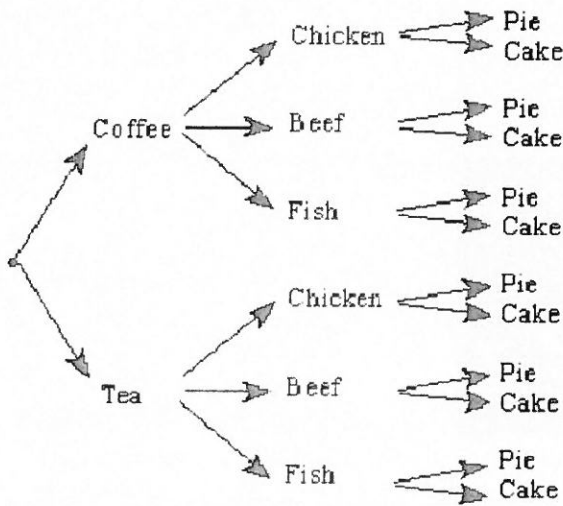
1, 2 or 3 left

$$= 12 + 8 + 12$$

$$= 32$$

7a)

Draw a picture — this is known as a TREE DIAGRAM



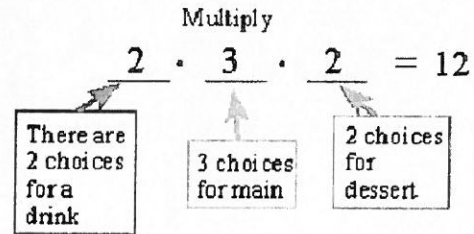
Follow each path, we end up with 12 different meals that you can order.

b)

Drawing a picture each time would be a lot of work

Let's use the Fundamental Counting Principle

Think of this question as making 3 choices.  
So, let's use 3 blanks.



8)

3 medals  $\rightarrow$  3 blanks

$$\underline{8} \cdot \underline{7} \cdot \underline{6} = 336$$

7 are left to finish second

6 are left to finish third

9)

4 letters must be used therefore 4 choices to make

4 blanks

$$\underline{1} \cdot \underline{26} \cdot \underline{26} \cdot \underline{26} = 17,576$$

Must begin with C, so only 1 to pick from

There are 26 letters in the alphabet, so 26 choices for each blank. NOTE: The question did not say you couldn't repeat letters

10)

$$\underline{1} \cdot \underline{25} \cdot \underline{24} \cdot \underline{23} = 13,800$$

Must be the letter C

Can't be a C

2 are used only 24 left

3 are used so only 23 are left

11)

6 choices  $\rightarrow$  6 blanks

3 letters

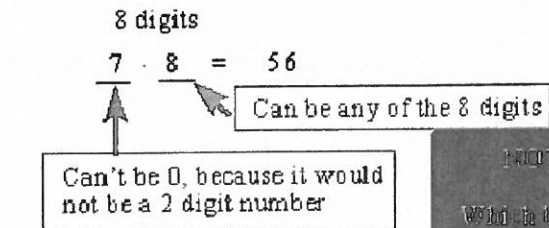
a)  $\underline{26} \cdot \underline{26} \cdot \underline{26} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} = 17,576,000$

b)  $\underline{26} \cdot \underline{25} \cdot \underline{24} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} = 15,600,000$

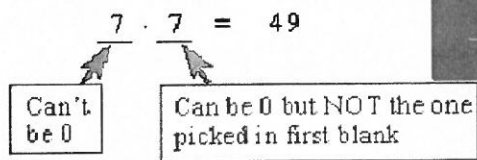
c)  $\underline{26} \cdot \underline{25} \cdot \underline{24} \cdot \underline{9} \cdot \underline{10} \cdot \underline{10} = 14,040,000$

12)

(a) repetitions are allowed



(b) repetitions are NOT allowed



**NOTE**

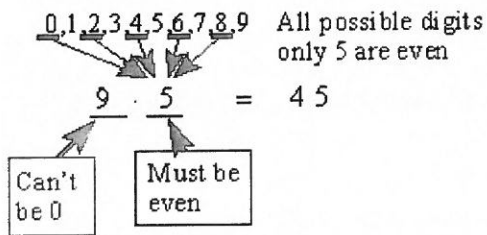
Which blank do you start with?

- the one that has restrictions
- the can't be 0

13)

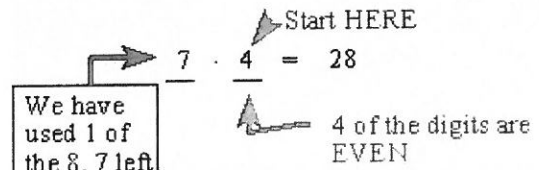
$$\underline{64} \cdot \underline{20} \cdot \underline{3} = 3840$$

14)

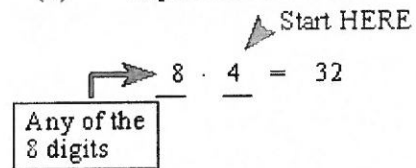


15)

(a) Repetitions are NOT allowed



(b) Repetitions are allowed



16)

(a) repetitions are allowed  
 Start HERE 5 digits to use

$$\frac{4}{\text{Can't be 0}} \cdot \frac{5}{\text{Can be anything}} = 20$$

(b) repetitions are NOT allowed  
 Start HERE

$$\frac{4}{\text{Can't be 0}} \cdot \frac{4}{\text{We have used 1 of the 5. So 4 left}} = 16$$

18)

$$\frac{7}{\text{7 Letters}} \cdot \frac{6}{\text{We have used 1, so 6 left}} \cdot \frac{5}{\text{5 are left and so on}} \cdot \frac{4}{\text{4 are left and so on}} \cdot \frac{3}{\text{3 are left and so on}} \cdot \frac{2}{\text{2 are left and so on}} \cdot \frac{1}{\text{1 are left and so on}} = 7! = 5040$$

*7! is read 7 factorial*  
 It's an easy way of writing  
 7 · 6 · 5 · 4 · 3 · 2 · 1

Example

$$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$$

Check your calculator, it probably has an n! button.

20) 6!

23) 2! 7! = 10080

21) 4!

24) 4! 3! = 144

17)

(a) repetition is allowed

$$\frac{9}{\text{Can't be 0}} \cdot \frac{10}{\text{Can be any of the 10 digits}} \cdot \frac{10}{\text{Can be any of the 10 digits}} \cdot \frac{5}{\text{Only 5 odd digits}} = 4500$$

(b) NO repetition of digits

Start filling in the blanks at the spots you have the most restrictions

$$\frac{5}{\text{Must be ODD}} =$$

$$\frac{8}{\text{Can't be 0 and one digit is already used}} \cdot \frac{5}{\text{Must be ODD}} =$$

$$\frac{8}{\text{2 out of the 10 digits are used therefore only 8 left}} \cdot \frac{8}{\text{2 out of the 10 digits are used therefore only 8 left}} \cdot \frac{7}{\text{3 out of the 10 are used, 7 left}} \cdot \frac{5}{\text{Must be ODD}} = 2240$$

19)

PROBLEM — 5 consonants  
 — 2 vowels

Start here and here

$$\frac{5}{\text{Consonant}} \cdot \frac{2}{\text{Vowel}} =$$

$$\frac{5}{\text{2 of the 7 letters are used, so 5 are left}} \cdot \frac{5}{\text{2 of the 7 letters are used, so 5 are left}} \cdot \frac{4}{\text{3 are used so 4 are left and so on}} \cdot \frac{3}{\text{3 are used so 4 are left and so on}} \cdot \frac{2}{\text{3 are used so 4 are left and so on}} \cdot \frac{1}{\text{3 are used so 4 are left and so on}} \cdot \frac{2}{\text{3 are used so 4 are left and so on}} = 1200$$

22) 3!

25) 4! 5! 6! 3! = 12441600

